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Some Measures of Picture Fuzzy Sets and Their Application in Multi-attribute Decision Making

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Abstract

To measure the difference of two fuzzy sets / intuitionistic sets, we can use the distance measure and dissimilarity measure between fuzzy sets. Characterization of distance/dissimilarity measure between fuzzy sets/intuitionistic fuzzy set is important as it has application in different areas: pattern recognition, image segmentation, and decision making. Picture fuzzy set (PFS) is a generalization of fuzzy set and intuitionistic set, so that it have many application. In this paper, we introduce concepts: difference between PFS-sets, distance measure and dissimilarity measure between picture fuzzy sets, and also provide the formulas for determining these values. We also present an application of dissimilarity measures in multi-attribute decision making.

Index Terms: Picture fuzzy set (PFS), difference between PFS-sets, distance measure and dissimilarity measure between picture fuzzy sets, multi-attribute decision making.

1. Introduction

In many practical problems, we need to compare two objects. Therefore, the question of the process and the way to compare those objects is important. There are some models to measure difference between objects, as a general axiomatic framework for the comparison of fuzzy set. (Bouchon et al. [1]). Fuzzy set and intuitionistic fuzzy set have been used a lot in practical math problems [6,8,9,11]. Distance measure between fuzzy sets and intuitionistic fuzzy sets is also important for many practical applications (Ejegwa et al. [4], Hatzimichailidis et al. [6], Lindblad et al. [8], Muthukumar et al. [12]). Besides, dissimilarity measure between fuzzy sets/intuitionistic fuzzy set is also studied and applied in various matters (Li [7], Faghihi [5], Nguyen [13],

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Mahmood [10]).

In 2014, Cuong introduced the concept of the picture fuzzy set (PFS-sets) [2], in which a given set is represented by three memberships: a degree of positive membership, a degree of negative membership, and a degree of neutral membership. After that, Son gave the applications of the picture fuzzy set in clustering problems in [15, 16, 17]. Nguyen et al. [14] use picture fuzzy sets to applied for Geographic Data Clustering. Van Dinh et al. [18] introduce the picture fuzzy set database. Cuong and Hai [3] studied some fuzzy logic operators for picture fuzzy sets. Nguyen et all [15] investigate the equivalence of two picture fuzzy set and apply them in clustering. But, difference between PFS-sets and dissimilarity between picture fuzzy sets (the concepts are important in application of picture fuzzy sets) are not yet been research.

In this paper, we introduce the concept of difference between PFS-sets, distance measure operators and dissimilarity measure operators between picture fuzzy sets. The rest of paper, in section 2, we recall the concept of picture fuzzy set and we introduce the new concept difference between PFS-sets. The function of distance measure between PFS-sets is defined in section 3. After that, we introduce the function of dissimilarity measure between PFS-sets in section 4. We also illustrate with numerical examples the above measures in decision making in section 5. In section 6, we apply the dissimilarity measure in the multiple-attribute decision making.

2. Basic Notions

Definition 1.

A picture fuzzy set (PFS) is defined by:

$$A = \{(u, \mu_A(u), \eta_A(u), \gamma_A(u)) | u \in U\}$$

where: μ_A is a positive membership function, η_A is neural membership function, γ_A is negative membership function of A, in there: $\mu_A(u), \eta_A(u), \gamma_A(u) \in [0,1]$ and

$$0 \leq \mu_A(u) + \eta_A(u) + \gamma_A(u) \leq 1, \text{ for all } u \in U.$$

- We denote $PFS(U)$ is a collection of picture fuzzy set on U. In which:

$$U = \{(u, 1, 0, 0) | u \in U\}$$

and:

$$\emptyset = \{(u, 0, 0, 1) | u \in U\}.$$

- For $A, B \in PFS(U)$, then:

+ Union of A and B is defined by:

$$A \cup B = \{(u, \mu_{A\cup B}(u), \eta_{A\cup B}(u), \gamma_{A\cup B}(u)) | u \in U\}$$

where:

$$\mu_{A\cup B}(u) = \max\{\mu_A(u), \mu_B(u)\},$$

$$\eta_{A\cup B}(u) = \min\{\eta_A(u), \eta_B(u)\},$$

$$\gamma_{A\cup B}(u) = \min\{\gamma_A(u), \gamma_B(u)\}.$$

+ Intersection of A and B is defined by:

$$A \cap B = \{(u, \mu_{A\cap B}(u), \eta_{A\cap B}(u), \gamma_{A\cap B}(u)) | u \in U\}$$

Where:

$$\mu_{A\cap B}(u) = \min\{\mu_A(u), \mu_B(u)\},$$

$$\eta_{A\cap B}(u) = \min\{\eta_A(u), \eta_B(u)\},$$

$$\gamma_{A\cap B}(u) = \max\{\gamma_A(u), \gamma_B(u)\}.$$

+ Subset: $A \subset B$ iff $\begin{cases} \mu_A(u) \le \mu_B(u) \\ \eta_A(u) \le \eta_B(u). \\ \gamma_A(u) \ge \gamma_B(u) \end{cases}$

Now, we define an operator called difference between picture fuzzy sets.

Definition 2.

An operator $- : PFS(U) \times PFS(U) \to PFS(U)$ is a difference between PFS-sets if it satisfies properties:

(D1) $A \subset B$ iff $A - B = \emptyset$,
(D2) If $B \subset C$ then $B - A \subset C - A$,
(D3) $(A \cap C) - (B \cap C) \subset A - B$,
(D4) $(A \cup C) - (B \cup C) \subset A - B$,

For all $A, B, C \in PFS(U)$.

Theorem 1.

The function $- : PFS(U) \times PFS(U) \to PFS(U)$ given by:

$$A - B = \{(u, \mu_{A-B}(u), \eta_{A-B}(u), \gamma_{A-B}(u)) | u \in U\},$$

where:

$$\mu_{A-B}(u) = \max\{(0, \mu_A(u) - \mu_B(u)\},$$

$$\eta_{A-B}(u) = \max\{0, \eta_A(u) - \eta_B(u)\},$$

$$\gamma_{A-B}(u) = \begin{cases} 1 - \mu_{A-B}(u) - \eta_{A-B}(u) \; if \; \gamma_A(u) > \gamma_B(u) \\ \min\begin{Bmatrix} 1 + \gamma_A(u) - \gamma_B(u), \\ 1 - \mu_{A-B}(u) - \eta_{A-B}(u) \end{Bmatrix} \text{ if } \gamma_A(u) \le \gamma_B(u) \end{cases} \quad (1)$$

is a difference between PFS-sets.

Proof.

It is easy to see that

$$0 \leq \mu_{A-B}(u) + \eta_{A-B}(u) + \gamma_{A-B}(u) \leq 1$$

for all $u \in U$.
We verify all condition in definition 2:

- With condition (D1).

+ $A \subset B \Longrightarrow A - B = \emptyset$ is obvious.
+ If $A - B = \emptyset$ then

$$\mu_{A-B}(u) = \max(0, \mu_A(u) - \mu_B(u)\} = 0,$$

$$\eta_{A-B}(u) = \max\{0, \eta_A(u) - \eta_B(u)\} = 0$$

so that $\mu_A(u) \leq \mu_B(u)$ and $\eta_A(u) \leq \eta_B(u)$;
Hence $\gamma_{A-B}(u)$ = 1 so that $\gamma_A(u) \geq \gamma_B(u)$.
It means that $A \subset B$.

- With condition (D2).

With $B \subset C$, we have

$$\mu_B(u) \leq \mu_C(u), \eta_B(u) \leq \eta_C(u) \text{ and } \gamma_B(u) \geq \gamma_A(u).$$

So that:

$$\mu_{B-A}(u) = \max(0, \mu_B(u) - \mu_A(u)\}$$

$$\leq \max(0, \mu_C(u) - \mu_A(u)\}$$

$$= \mu_{C-A}(u),$$

$$\eta_{B-A}(u) = \max(0, \eta_B(u) - \eta_A(u)\}$$

$$\leq \max(0, \eta_C(u) - \eta_A(u)\}$$

$$= \eta_{C-A}(u).$$

With negative membership function, we consider some cases:

If $\gamma_A(u) \leq \gamma_C(u) \leq \gamma_B(u)$ then

$$\gamma_{B-A}(u) = 1 - \mu_{B-A}(u) - \eta_{B-A}(u)$$

$$\geq 1 - \mu_{C-A}(u) - \eta_{C-A}(u) = \gamma_{C-A}(u)$$

If $\gamma_C(u) \leq \gamma_A(u) \leq \gamma_B(u)$ then

$$\gamma_{B-A}(u) = 1 - \mu_{B-A}(u) - \eta_{B-A}(u)$$

$$\geq 1 - \mu_{C-A}(u) - \eta_{C-A}(u).$$

So that

$$\gamma_{B-A}(u) \geq \min\left\{\begin{matrix} 1 + \gamma_A(u) - \gamma_C(u) \quad , \\ 1 - \mu_{C-A}(u) - \eta_{C-A}(u) \end{matrix}\right\} = \gamma_{C-A}(u).$$

If $\gamma_C(u) \leq \gamma_B(u) \leq \gamma_A(u)$ then

$$\gamma_{B-A}(u) = 1 - \mu_{B-A}(u) - \eta_{B-A}(u)$$

$$\geq 1 - \mu_{C-A}(u) - \eta_{C-A}(u)$$

and

$$\gamma_B(u) - \gamma_A(u) \geq \gamma_C(u) - \gamma_A(u).$$

So that:

$$\gamma_{B-A}(u) = \min\left\{\begin{matrix} 1 + \gamma_A(u) - \gamma_B(u), \\ 1 - \mu_{A-B}(u) - \eta_{A-B}(u) \end{matrix}\right\}$$

$$\geq \min\left\{\begin{matrix} 1 + \gamma_A(u) - \gamma_C(u), \\ 1 - \mu_{C-A}(u) - \eta_{C-A}(u) \end{matrix}\right\}$$

$$= \gamma_{C-A}(u).$$

- Similarity, it is possible to show that conditions (D3) and (D4) are also satisfied.

Example 1. Given $U = \{u_1, u_2, u_3\}$ and two PFS-sets:

$$A = \left\{\begin{matrix} (u_1, 0.7, 0.2, 0.1), \\ (u_2, 0.6, 0.1, 0.1), \\ (u_3, 0.6, 0.1, 0.2) \end{matrix}\right\}, \ B = \left\{\begin{matrix} (u_1, 0.6, 0.3, 0.1), \\ (u_2, 0.7, 0.05, 0.2), \\ (u_3, 0.4, 0.4, 0.1) \end{matrix}\right\}.$$

Then, computing by Eq.(1) in theorem 1, we have:

$$A - B = \begin{Bmatrix} (u_1, 0.1, 0.0, 0.9), \\ (u_2, 0.0, 0.05, 0.9), \\ (u_3, 0.2, 0.0, 0.8) \end{Bmatrix}$$

3. Distance Mesure of Picture Fuzy Sets

In this section, we define the distance measure between picture fuzzy sets.

Definition 3.

A function $D\ : PFS(U) \times PFS(U) \to [0, +\infty)$ is a distance measure between PFS-sets if it satisfies follow properties

(i) PF-dist 1: $D(A, B) = 0$ iff $A = B$,
(ii) PF-dist 2: $D(A, B)\ = D(B, A)$, for all $A, B \in PFS(U)$,
(iii) PF-dist 3: $D(A, C) \le D(A, B) + D(B, C)$, for all $A, B, C \in PFS(U)$.

There are many formulas that determine the distance between two picture fuzzy sets.

Theorem 2.

Given $U = \{u_1, u_2, \dots, u_n\}$ is an universe set. For $A, B \in PFS(U)$. We have some distance measure between picture fuzzy sets

(i) $$D_H(A, B) = \frac{1}{3n} \sum_{i=1}^{n} \begin{bmatrix} |\mu_A(u_i) - \mu_B(u_i)| \\ +|\eta_A(u_i) - \eta_B(u_i)| \\ +|\gamma_A(u_i) - \gamma_B(u_i)| \end{bmatrix} \tag{2}$$

(ii) $$D_E(A, B) = \frac{1}{n} \left\{ \sum_{i=1}^{n} \begin{bmatrix} (\mu_A(u_i) - \mu_B(u_i))^2 \\ +(\eta_A(u_i) - \eta_B(u_i))^2 \\ +(\gamma_A(u_i) - \gamma_B(u_i))^2 \end{bmatrix} \right\}^{\frac{1}{2}} \tag{3}$$

(iii) $$D_H^m(A, B) = \frac{1}{n} \sum_{i=1}^{n} \max \begin{bmatrix} |\mu_A(u_i) - \mu_B(u_i)|, \\ |\eta_A(u_i) - \eta_B(u_i)|, \\ |\gamma_A(u_i) - \gamma_B(u_i)| \end{bmatrix} \tag{4}$$

(iv) $$D_E^m(A, B) = \frac{1}{n} \left\{ \sum_{i=1}^{n} \max \begin{bmatrix} |\mu_A(u_i) - \mu_B(u_i)|^2, \\ |\eta_A(u_i) - \eta_B(u_i)|^2, \\ |\gamma_A(u_i) - \gamma_B(u_i)|^2 \end{bmatrix} \right\}^{\frac{1}{2}} \tag{5}$$

We easy to verify that the functions in theorem 2 are satisfies properties of distance measure between picture fuzzy sets (def. 3). In there, $D_E(A, B)$ is usually used to measure the distance of objects in geometry, $D_H(A, B)$ is used in the information theory.

Example 2. Assume that there are two patterns denoted by picture fuzzy sets on $U = \{u_1, u_2, u_3\}$ as follows:

$$A_1 = \begin{Bmatrix} (u_1, 0.609, 0.091, 0.298), \\ (u_2, 0.651, 0.231, 0.059), \\ (u_3, 0.792, 0.095, 0.106) \end{Bmatrix},$$

$$A_2 = \begin{Bmatrix} (u_1, 0.291, 0.365, 0.134), \\ (u_2, 0.438, 0.468, \ 0.065), \\ (u_3, 0.816, 0.169, 0.006) \end{Bmatrix}.$$

$$A_3 = \begin{Bmatrix} (u_1, 0.679, 0.215, 0.014), \\ (u_2, 0.239, 0.617, 0.086), \\ (u_3, 0.917, 0.045, 0.011) \end{Bmatrix}$$

Then we have the results:

+ Using the Eq.(2) we obtain:

$$D_H(A_1, A_2) = 0.156667;$$

$$D_H(A_1, A_3) = 0.174778;$$

$$D_H(A_2, A_3) = 0.139667;$$

+ Using the Eq.(3) we have

$$D_E(A_1, A_2) = 0.56632;$$

$$D_E(A_1, A_3) = 0.66899;$$

$$D_E(A_2, A_3) = 0.52468.$$

+ By same way with Eq.(4) we achieve

$$D_H^m(A_1, A_2) = 0.655;$$

$$D_H^m(A_1, A_3) = 0.821;$$

$$D_H^m(A_2, A_3) = 0.711;$$

+ Finally, we using the Eq.(5), we have

$$D_E^m(A_1, A_2) = 0.409;$$

$$D_E^m(A_1, A_3) = 0.5158;$$

$$D_E^m(A_2, A_3) = 0.4533.$$

4. Dissimilarity of Picture Fuzzy Sets

In this section, we introduce the concept of dissimilarity for picture fuzzy sets.

Definition 4.

A function $DM\ : PFS(U) \times PFS(U) \to R$ is a dissimilarity measure between PFS-sets if it satisfies follow properties:

(i) PF-Diss 1: $DM(A, B) = DM(B, A)$
(ii) PF-Diss 2: $DM(A, A) = 0$.
(iii) PF-Diss 3: If $A \subset B \subset C$ then

$$DM(A, C) \geq \max\{DM(A, B), DM(B, C)\}.$$

for all $A, B, C \in PFS(U)$.

Theorem 3.

Given $U = \{u_1, u_2, \dots, u_n\}$ is an universe set. For any $A, B \in PFS(U)$, we define a function$DM_C(A, B)$: $PFS(U) \times PFS(U) \to R$ is defined by:

$$DM_C(A, B) = \frac{1}{3n} \sum_{i=1}^{n} \begin{bmatrix} |S_A(u_i) - S_B(u_i)| \\ +|\eta_A(u_i) - \eta_B(u_i)| \end{bmatrix} \tag{6}$$

Where:

$$S_A(u_i) = |\mu_A(u_i) - \gamma_A(u_i)|, \text{ and}$$

$$S_B(u_i) = |\mu_B(u_i) - \gamma_B(u_i)|$$

Then, $DM_C(A, B)$ is a dissimilarity measure between PFS-sets.

Proof.

We check that DM_C satisfies the conditions of definition 3. Indeed, we have:

PF-Diss 1 and PF-Diss 2 are obviously.
With PF-Diss 3, if $A \subset B \subset C$ we have

$$\begin{cases} \mu_A(u_i) \leq \mu_B(u_i) \leq \mu_C(u_i) \\ \eta_A(u_i) \leq \eta_B(u_i) \leq \eta_C(u_i) \\ \gamma_A(u_i) \geq \gamma_B(u_i) \geq \gamma_C(u_i) \end{cases}$$

for all $u_i \in U$.
So that:

$$S_A(u_i) = |\mu_A(u_i) - \gamma_A(u_i)|$$

$$\geq S_B(u_i) = |\mu_B(u_i) - \gamma_B(u_i)|\,,$$

$$S_B(u_i) = |\mu_B(u_i) - \gamma_B(u_i)|$$

$$\geq S_C(u_i) = |\mu_C(u_i) - \gamma_C(u_i)|$$

and

$$|\eta_A(u_i) - \eta_C(u_i)| \geq \max\{|\eta_A(u_i) - \eta_B(u_i)|, |\,\eta_B(u_i) - \eta_C(u_i)|\}.$$

Hence

$$DM_C(A,C) \geq max\{DM_C(A,B), DM_C(B,C)\}.$$

It means PF-Diss 3 satisfy.
We have some dissimilarity measure in theorem 3, as follows.

Theorem 4.

Given $U = \{u_1, u_2, \dots, u_n\}$ is an universe set. For any $A, B \in PFS(U)$. We have:

(i) $$DM_H(A,B) = \frac{1}{3n}\sum_{i=1}^{n}\begin{bmatrix} |\mu_A(u_i) - \mu_B(u_i)| \\ +|\eta_A(u_i) - \eta_B(u_i)| \\ +|\gamma_A(u_i) - \gamma_B(u_i)| \end{bmatrix} \quad (7)$$

(ii) $$DM_L(A,B) = \frac{1}{5n}\sum_{i=1}^{n}\begin{bmatrix} |S_A(u_i) - S_B(u_i)| \\ +|\mu_A(u_i) - \mu_B(u_i)| \\ +|\eta_A(u_i) - \eta_B(u_i)| \\ +|\gamma_A(u_i) - \gamma_B(u_i)| \end{bmatrix} \quad (8)$$

(iii) $$DM_O(A,B) = \frac{1}{\sqrt{3n}}\sum_{i=1}^{n}\begin{bmatrix} (\mu_A(u_i) - \mu_B(u_i))^2 \\ +(\eta_A(u_i) - \eta_B(u_i))^2 \\ +(\gamma_A(u_i) - \gamma_B(u_i))^2 \end{bmatrix}^{\frac{1}{2}} \quad (9)$$

are the dissimilarity measure between picture fuzzy sets.
The proof of this theorem is similar to the theorem 3.

Example 3. Assume that there are two patterns denoted by picture fuzzy sets on $U = \{u_1, u_2, u_3\}$ as follows

$$A_1 = \begin{Bmatrix} (u_1, 0.609, 0.091, 0.298), \\ (u_2, 0.651, 0.231, 0.059), \\ (u_3, 0.792, 0.095, 0.106) \end{Bmatrix},$$

$$A_2 = \begin{Bmatrix} (u_1, 0.291, 0.365, 0.134), \\ (u_2, 0.438, 0.468, 0.065), \\ (u_3, 0.816, 0.169, 0.006) \end{Bmatrix}.$$

$$A_3 = \begin{Bmatrix} (u_1, 0.679, 0.215, 0.014), \\ (u_2, 0.239, 0.617, 0.086), \\ (u_3, 0.917, 0.045, 0.011) \end{Bmatrix}$$

+ Using Eq.(6) we have:

$$DM_C(A_1, A_2) = 0.360667;$$

$$DM_C(A_1, A_3) = 0.524333;$$

$$DM_C(A_2, A_3) = 0.415667;$$

+ Using Eq.(7) we have

$$DM_H(A_1, A_2) = 0.145556;$$

$$DM_H(A_1, A_3) = 0.139111;$$

$$DM_H(A_2, A_3) = 0.164222;$$

+ Using Eq.(8) we obtain

$$DM_L(A_1, A_2) = 0.193267;$$

$$DM_L(A_1, A_3) = 0.1728;$$

$$DM_L(A_2, A_3) = 0.213467;$$

+ Finally with Eq.(9) we get the results:

$$DM_O(A_1, A_2) = 0.188774;$$

$$DM_O(A_1, A_3) = 0.174893;$$

$$DM_O(A_2, A_3) = 0.222998;$$

5. Numerical Examples for using New Measures in Partern Recognition

In this section, we will give some examples using distance and dissimilarity measure $DM(A, B)$ in decision making. Note that when using similar measure, there are two patterns $A_1, A_2$and a sample B. If $DM(A_1, B) < DM(A_2, B)$ the we consider that sample B belongs to the pattern A_1.

Example 4. Assume that there are two patterns denoted by picture fuzzy sets on $U = \{u_1, u_2, u_3\}$ as follows

$$A_1 = \begin{Bmatrix} (u_1, 0.1, 0.1, 0.1), \\ (u_2, 0.1, 0.4, 0.3), \\ (u_3, 0.1, 0.0, 0.9) \end{Bmatrix},$$

$$A_2 = \begin{Bmatrix} (u_1, 0.7, 0.1, 0.2), \\ (u_2, 0.1, 0.1, 0.8), \\ (u_3, 0.1, 0.1, 0.7) \end{Bmatrix}.$$

Now, there is a sample:

$$B = \begin{Bmatrix} (u_1, 0.4, 0.0, 0.4), \\ (u_2, 0.6, 0.1, 0.2), \\ (u_3, 0.1, 0.1, 0.8) \end{Bmatrix}$$

Question: which pattern does B belong to?

+ Applying the distant measure $D_H(A, B)$ (i.e. Eq. (2)) we have:

$$D_H(A_1, B) = D_H(A_2, B) = 0.2$$

+ Applying the dissimilarity measure $DM_L(A, B)$ in Eq.(8) we have:

$$DM_L(A_1, B) = \frac{2.1}{15} < DM_L(A_2, B) = \frac{2.7}{15}.$$

In this example, we see that using the distant measure $D_H(A, B)$ can not be used to classify the sample B. But, we can see that B belongs to pattern A_1 if we use the dissimilarity measure $DM_L(A, B)$.

Example 5. Assume that there are three patterns denoted by picture fuzzy sets on $U = \{u_1, u_2, u_3\}$ as follows

$$A_1 = \begin{Bmatrix} (u_1, 0.4, 0.5, 0.1), \\ (u_2, 0.7, 0.1, 0.1), \\ (u_3, 0.3, 0.3, 0.2) \end{Bmatrix},$$

$$A_2 = \begin{Bmatrix} (u_1, 0.5, 0.4, 0.0), \\ (u_2, 0.7, 0.2, 0.1), \\ (u_3, 0.4, 0.3, 0.2) \end{Bmatrix}.$$

$$A_3 = \begin{Bmatrix} (u_1, 0.4, 0.4, 0.1), \\ (u_2, 0.6, 0.1, 0.1), \\ (u_3, 0.4, 0.1, 0.4) \end{Bmatrix}$$

Now, there is a sample:

$$B = \begin{Bmatrix} (u_1, 0.1, 0.1, 0.6), \\ (u_2, 0.7, 0.1, 0.2), \\ (u_3, 0.8, 0.1, 0.1) \end{Bmatrix}$$

Question: which pattern does B belong to?

+ Applying the dissimilarity measure $DM_L(A, B)$ in Eq.(8) we have:

$$DM_L(A_1, B) = DM_L(A_3, B) = \frac{2.1}{9};$$

$$DM_L(A_2, B) = \frac{2.2}{9}.$$

+ Applying the distance measure $D_E(A, B)$ in Eq.(3) we have:

$$D_E(A_1, B) = 0.9;$$

$$D_E(A_2, B) = 0.916515139;$$

$$D_E(A_3, B) = 0.8366600265$$

In this example, we see that using the dissimilarity measure $DM_H(A, B)$ can not be used to classify the sample B. But, we can see that B belongs to pattern A_3 if we use the distance measure $D_E(A, B)$.

Example 6. Assume that there are three patterns denoted by picture fuzzy sets on $U = \{u_1, u_2, u_3,\ u_4\}$ as follows:

$$A_1 = \left\{\begin{matrix}(u_1, 0.3, 0.4, 0.1), (u_2, 0.3, 0.4, 0.1), \\ (u_3, 0.6, 0.1, 0.2), (u_4, 0.6, 0.1, 0.2)\end{matrix}\right\}$$

$$A_2 = \left\{\begin{matrix}(u_1, 0.4, 0.4, 0.1), (u_2, 0.3, 0.2, 0.4), \\ (u_3, 0.6, 0.1, 0.3), (u_4, 0.5, 0.2, 0.2)\end{matrix}\right\}$$

$$A_3 = \left\{\begin{matrix}(u_1, 0.4, 0.4, 0.1), (u_2, 0.3, 0.1, 0.3), \\ (u_3, 0.6, 0.1, 0.2), (u_4, 0.5, 0.2, 0.1)\end{matrix}\right\}$$

Now, there is a sample:

$$B = \left\{\begin{matrix}(u_1, 0.35, 0.65, 0), (u_2, 0.55, 0.35, 0.1), \\ (u_3, 0.65, 0.1, 0.1), (u_4, 0.6, 0.15, 0.2)\end{matrix}\right\}$$

Question: *which pattern does B belong to?*

+ Applying the distance measure $D_H^m(A, B)$ in Eq.(4) we have:

$$D_H^m(A_1, B) = D_H^m(A_3, B) = 0.7;$$

$$D_H^m(A_2, B) = 0.85.$$

+ Applying the dissimilarity measure $DM_C(A, B)$ in Eq.(6) we have:

$$DM_C(A_1, B) = 0.0875;$$

$$DM_C(A_2, B) = DM_C(A_3, B) = 0.1$$

In this example, we see that using the distance measure $D_H^m(A,B)$ can not be used to classify the sample B. But, we can see that B belongs to pattern A_1 if we use the dissimilarity measure $DM_C(A,B)$.

Example 7. Assume that there are two patterns denoted by picture fuzzy sets on $U=\{u_1,u_2\}$ as follows

$$A_1=\begin{Bmatrix}(u_1,0.4,0.5,0.1),\\(u_2,0.3,0.4,0.2)\end{Bmatrix},$$

$$A_2=\begin{Bmatrix}(u_1,0.5,0.4,0.1),\\(u_2,0.4,0.3,0.1)\end{Bmatrix}.$$

Now, there is a sample:

$$B=\begin{Bmatrix}(u_1,0.1,0.1,0.1),\\(u_2,0.5,0.5,0.0)\end{Bmatrix}$$

Question: *which pattern does B belong to?*

+ Applying the distant measure $D_E^m(A,B)$ (i.e. Eq.(5)) we have:

$$D_E^m(A_1,B)=D_E^m(A_2,B)=0.44721$$

+ Applying the dissimilarity measure $DM_O(A,B)$ (i.e. Eq.(9)) we have:

$$DM_O(A_1,B)=0.3265;$$

$$DM_O(A_2,B)=0.3041241.$$

In this example, we see that using the distant measure $D_E^m(A,B)$ can not be used to classify the sample B. But, we can see that B belongs to pattern A_2 if we use the dissimilarity measure $DM_O(A,B)$.

6. Appliction in Multiple Attribute Decision Making

In modern decision science, the multiple attribute decision making is an important research and has been widely in many fields, such as economy, management, medical and so on. In this section, we will use our dissimilarity measure to apply in a multiple attribute decision making. It present in Example 6.

Example 8. This example is developed from the example that presented by Xu (in 2007). This example is to evaluate the university faculty for tenure and promotion. There are six faculty candidates (alternatives), A_i, $(i=1,2,\dots,6)$, to be evaluated, the used attributes are G_1: teaching, G_2: research and G_3: service. In which, the weights of each attribute G_j is $w_j, j=1,2,3$; $\sum_{j=1}^3 w_j=1$ the used weight vector is $w=(0.55,0.25,0.2)$.

The picture fuzzy decision matrix $R=(r_{ij})_{6\times 3}$, where $r_{ij}=(\mu_{ij},\eta_{ij},\gamma_{ij}),(i=1,2,\dots,6; j=1,2,3)$ is shown in Table 1.

We denote the alternatives A_i, $(i=1,2,\dots,6)$ and $A_b(j)=(1,0,0); j=1,2,3)$. Then, we have $A_b=(A_b(1),A_b(2),A_b(3))$ be the largest picture fuzzy number.

Table 1. Picture Fuzzy Decision Matrix R

	G_1	G_2	G_3
A_1	(0.61, 0.08, 0.11)	(0.59, 0.04, 0.26)	(0.41, 0.36, 0.08)
A_2	(0.54, 0.45, 0.01)	(0.5, 0.15, 0.25)	(0.66, 0.12, 0.01)
A_3	(0.72, 0.23, 0.03)	(0.53, 0.05, 0.35)	(0.5, 0.03, 0.08)
A_4	(0.63, 0.3, 0.06)	(0.59, 0.29, 0.08)	(0.58, 0.07, 0.16)
A_5	(0.59, 0.3, 0.09)	(0.41, 0.57, 0.01)	(0.59, 0.19, 0.1)
A_6	(0.73, 0.1, 0.15)	(0.54, 0.37, 0.06)	(0.46, 0.24, 0.06)

The dissimilarity measure between alternatives A_i and the possible ideal alternative A_b is defined as follows:

$$DM(A_i, A_b) = \sum_{j=1}^{3} w_j DM(r_{ij}, A_b(j)) \tag{10}$$

where $DM(r_{ij}, A_b(j))$ is the dissimilarity measure of r_{ij} and $A_b(j)$, $j = 1,2,3;\ i = 1,2,\dots,6$.

So that, the smaller of the dissimilarity measure is the better alternatives.

We consider the calculated results according to the measurements in the section 4.

+ Apply the Eq.(10), with the dissimilarity measure $DM_H(A, B)$ in the Eq.(7) we have:

$$DM_H(A_1, A_b) = 0.23417$$

$$DM_H(A_2, A_b) = 0.275$$

$$DM_H(A_3, A_b) = 0.21217$$

$$DM_H(A_4, A_b) = 0.24217$$

$$DM_H(A_5, A_b) = 0.29083$$

$$DM_H(A_6, A_b) = 0.2255$$

Here, we have ranking:

$$A_3 \succ A_6 \succ A_1 \succ A_4 \succ A_2 \succ A_5,$$

where the symbol " $\succ$ " mean that order relation “superior”. Hence, the most faculty (alternative) is A_3.

+ Apply the Eq.(10), with the dissimilarity measure $DM_O(A, B)$ in the Eq.(9) we have:

$$DM_O(A_1, A_b) = 0.1627$$

$$DM_O(A_2, A_b) = 0.19027$$

$$DM_O(A_3, A_b) = 0.14949$$

$$DM_O(A_4, A_b) = 0.16072$$

$$DM_O(A_5, A_b) = 0.19381$$

$$DM_O(A_6, A_b) = 0.14857$$

Here, we have ranking

$$A_6 \succ A_3 \succ A_4 \succ A_1 \succ A_2 \succ A_5,$$

Hence, the most faculty (alternative) is A_6.

+ Apply the Eq.(10), with the dissimilarity measure $DM_L(A, B)$ in the Eq.(8) we have:

$$DM_L(A_1, A_b) = 0.21033$$

$$DM_L(A_2, A_b) = 0.22635$$

$$DM_L(A_3, A_b) = 0.18529$$

$$DM_L(A_4, A_b) = 0.19651$$

$$DM_L(A_5, A_b) = 0.23562$$

$$DM_L(A_6, A_b) = 0.18512$$

Here, we have ranking:

$$A_6 \succ A_3 \succ A_4 \succ A_1 \succ A_2 \succ A_5,$$

where the symbol " $\succ$ " mean that order relation "superior". Hence, the most faculty (alternative) is A_6.

+ Apply the Eq.(10), with the dissimilarity measure $DM_C(A, B)$ in the Eq.(6) we have:

$$DM_C(A_1, A_b) = 0.22877$$

$$DM_C(A_2, A_b) = 0.22411$$

$$DM_C(A_3, A_b) = 0.19316$$

$$DM_C(A_4, A_b) = 0.20167$$

$$DM_C(A_5, A_b) = 0.23985$$

$$DM_C(A_6, A_b) = 0.19934$$

Here, we have ranking:

$$A_3 \succ A_6 \succ A_4 \succ A_2 \succ A_1 \succ A_5,$$

hence the most faculty (alternative) is A_3.

Example 9. Now, we use the distance measure to rank the alternative base on attribute in example 8.

Table 2. Picture Fuzzy Decision Matrix R

	G_1	G_2	G_3
A_1	(0.61, 0.08, 0.11)	(0.59, 0.04, 0.26)	(0.41, 0.36, 0.08)
A_2	(0.54, 0.45, 0.01)	(0.5, 0.15, 0.25)	(0.66, 0.12, 0.01)
A_3	(0.72, 0.23, 0.03)	(0.53, 0.05, 0.35)	(0.5, 0.03, 0.08)
A_4	(0.63, 0.3, 0.06)	(0.59, 0.29, 0.08)	(0.58, 0.07, 0.16)
A_5	(0.59, 0.3, 0.09)	(0.41, 0.57, 0.01)	(0.59, 0.19, 0.1)
A_6	(0.73, 0.1, 0.15)	(0.54, 0.37, 0.06)	(0.46, 0.24, 0.06)

The distance measure between alternatives A_i and the possible ideal alternative A_b is defined as follows:

$$D(A_i, A_b) = \sum_{j=1}^{3} w_j D(r_{ij}, A_b(j)) \qquad (11)$$

where $D\big(r_{ij}, A_b(j)\big)$ is the distance measure of r_{ij} and $A_b(j)$, $j = 1,2,3;\ i = 1,2,\ldots,6$.
So that, the smaller of the distance measure is the better alternatives.
We consider the calculated results according to the measurements in the section 3.

+ Apply the Eq.(11), with the distance measure $D_H(A, B)$ in the Eq.(2) we have:

$$D_H(A_1, A_b) = 0.23417$$

$$D_H(A_2, A_b) = 0.275$$

$$D_H(A_3, A_b) = 0.21217$$

$$D_H(A_4, A_b) = 0.24217$$

$$D_H(A_5, A_b) = 0.29083$$

$$D_H(A_6, A_b) = 0.2255$$

Here, we have ranking:

$$A_3 \succ A_6 \succ A_1 \succ A_4 \succ A_2 \succ A_5,$$

where the symbol " $\succ$ " mean that order relation “superior”, hence the most faculty (alternative) is A_3.

+ Apply the Eq.(11), with the dissimilarity measure $D_E(A, B)$ in the Eq.(3) we have:

$$D_E(A_1, A_b) = 0.48811$$

$$D_E(A_2, A_b) = 0.57081$$

$$D_E(A_3, A_b) = 0.44846$$

$$D_E(A_4, A_b) = 0.48216$$

$$D_E(A_5, A_b) = 0.58144$$

$$D_E(A_6, A_b) = 0.4457$$

Here, we have ranking:

$$A_6 \succ A_3 \succ A_4 \succ A_1 \succ A_2 \succ A_5,$$

Hence, the most faculty (alternative) is A_6.

+ Apply the Eq.(11), with the distance measure $D_H^m(A, B)$ in the Eq.(4) we have:

$$D_H^m(A_1, A_b) = 0.435$$

$$D_H^m(A_2, A_b) = 0.446$$

$$D_H^m(A_3, A_b) = 0.3715$$

$$D_H^m(A_4, A_b) = 0.39$$

$$D_H^m(A_5, A_b) = 0.455$$

$$D_H^m(A_6, A_b) = 0.3715$$

Here, we have ranking:

$$A_3 = A_6 \succ A_4 \succ A_1 \succ A_2 \succ A_5,$$

Hence, the most faculty (alternative) is A_3, A_6.

This case to see that the distance measure $D_H^m(A, B)$ is not well as dissimilarity measures, which we use before, be cause It does not know which alternative is better between A_3, A_6.

7. Conclusion

In this paper, we introduce the concepts of the difference between PFS-sets, distance measure and dissimilarity between picture fuzzy sets. We give some distant measure and dissimilarity measure of picture fuzzy sets. Beside, we Illustrate with numerical examples the above measures in decision making. In the future, we will study the properties of these measure and applications of them in practical problems.

Finally, we applied the similarity measures in multiple attribute decision making. Since, we see that dissimilarity is a useful way to deal with realistic problems and can be extended in other application fields.

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