I.J. Mathematical Sciences and Computing, **2016, 3, 49-57**
Published Online July 2016 in MECS (http://www.mecs-press.net)
DOI: 10.5815/ijmsc.2016.03.05

***Available online at* http://www.mecs-press.net/ijmsc**

A Note on Quasi-coincidence for Fuzzy Points of Fuzzy Topology on the Basis of Reference Function

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Abstract

In this article our main aim is to revisit the definition of fuzzy point and fuzzy quasi-coincident of fuzzy topology which is accepted in the literature of fuzzy set theory. We analyse some results and also prove some proposition with extended definition of complementation of fuzzy sets on the basis of reference function and some new definitions have also been introduced whenever possible. In this work the main efforts have been made to show that the existing definition of complement of fuzzy point and definition of fuzzy quasi-coincident are not acceptable.

Index Terms: Fuzzy Point, Fuzzy Quasi-Coincident, Fuzzy Topology.

1. Introduction

Fuzzy set theory was discovered by Zadeh [1] in 1965. The theory of fuzzy sets actually has been a generalization of the classical theory of sets in the sense that the theory of sets should have been a special case of the theory of fuzzy sets. But unfortunately it has been accepted that for fuzzy set A and its complement A^C, neither $A \cap A^C$ is empty set nor $A \cup A^C$ is the universal set. Whereas the operations of union and intersection of crisp sets are indeed special cases of the corresponding operation of two fuzzy sets, they end up giving peculiar results while defining $A \cap A^C$ and $A \cup A^C$. In this regard Baruah [2, 3] has forwarded an extended definition of complement of fuzzy sets which enable us to define complement of fuzzy sets in a way that give us $A \cap A^C$ is empty and $A \cup A^C$ is universal set.

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Chang [4] introduced fuzzy topology. After the introduction of fuzzy sets and fuzzy topology, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. In fuzzy topology also many results are not same as general topology. It is seen that when we used the existence definition of fuzzy set in fuzzy boundary then closure of any fuzzy set is not equal to union of a fuzzy set and its boundary. The main reason behind it is expression of complement of existing definition of fuzzy set. Present author [5] has expressed nicely in this regard with extended definition of fuzzy set and showed that closure of a fuzzy set is equal to union of a fuzzy set and its boundary.

In this article we apply the extended definition of complementation of fuzzy sets on the basis of reference function to give definition of fuzzy point and quasi-coincidence of fuzzy topology and try to prove some results on quasi-coincidence for fuzzy point so that we can get result more accuracy than before.

2. Related Work

To avoid difficulty of complement of fuzzy set many new ideas were developed. The concept of intuitionistic fuzzy set was introduced by Atnassov [6] as a generalization of fuzzy set. By observing this idea in 1997 Coker [7] introduced the concept of intuitionistic fuzzy topology. Pu and Liu [8] were discussed on Fuzzy topology I neighbourhood structure of a fuzzy point and Moore-Smith convergence. Coker and Demirci [9] were explained very nicely on intuitionistic fuzzy points. In this article we would discuss fuzzy point and fuzzy quasi coincident on the basis of extended definition of fuzzy set.

3. Baruah’s Definition of Complementation of Fuzzy Sets

Baruah [2, 3] gave an extended definition of complementation of fuzzy set. According to Baruah [2, 3] to define a fuzzy set, two functions namely fuzzy membership function and fuzzy reference function are necessary. Fuzzy membership value is the difference between fuzzy membership function and fuzzy reference function.

Let $\mu_1(x)$ and $\mu_2(x)$ be two functions such that $0\leq \mu_2(x) \leq \mu_1(x) \leq 1$. For fuzzy number denoted by $\{x, \mu_1(x), \mu_2(x); x\epsilon U\}$, we call $\mu_1(x)$ as fuzzy membership function and $\mu_2(x)$ a reference function such that $(\mu_1(x) - \mu_2(x))$ is the fuzzy membership value.

4. Basic Operations

Let $A=\{x, \mu_1(x), \mu_2(x); x\epsilon U\}$ and $B=\{x, \mu_3(x), \mu_4(x); x\epsilon U\}$ be two fuzzy sets defined over the same universe U.

1. $A \subseteq B$ iff $\mu_1(x)\leq\mu_3(x)$ and $\mu_4(x)\leq \mu_2(x)$ for all $x\epsilon U$.
2. $A\bigcup B =\{x, \max(\mu_1(x), \mu_3(x)), \min(\mu_2(x), \mu_4(x))\}$ for all $x\epsilon U$.
3. $A\bigcap B =\{x, \min(\mu_1(x), \mu_3(x)), \max(\mu_2(x), \mu_4(x))\}$ for all $x\epsilon U$.

If for some $x\epsilon U$, $\min(\mu_1(x), \mu_3(x)) \leq \max(\mu_2(x), \mu_4(x))\}$, then our conclusion will be $A\bigcap B=\phi$

4. $A^C=\{x, \mu_1(x), \mu_2(x); x\epsilon U\}^C=\{x, \mu_2(x), 0; x\epsilon U\}\bigcup \{x, 1, \mu_1(x); x\epsilon U\}$
5. If $D = \{x, \mu(x), 0; x\epsilon U\}$ then $D^C=\{x, 1, \mu(x); x\epsilon U\}$ for all $x\epsilon U$.

Now we shall discuss some propositions regarding fuzzy topology considering the concepts of reference function, which are as follows:

4.1. Proposition

For fuzzy sets A, B, C over the same universe X, we have the following proposition

1.1 $A \subseteq B$, $B \subseteq C \Rightarrow A \subseteq C$
1.2 $A \cap B \subseteq A$, $A \cap B \subseteq B$
1.3 $A \subseteq A \cup B$, $B \subseteq A \cup B$
1.4 $A \subseteq B \Rightarrow A \cap B = A$
1.5 $A \subseteq B \Rightarrow A \cup B = B$

4.2. Proposition

Let $\tau=\{A_i: i \epsilon I\}$ be a collection of fuzzy sets over the same universe U. Then

3.1. $\bigcup_i A_i=\{x, \max(\mu_{1i}), \min(\mu_{2i}); x \epsilon U\}$
3.2. $\bigcap_i A_i=\{x, \min(\mu_{1i}), \max(\mu_{2i}); x \epsilon U\}$

4.3. Proposition

Let $\tau=\{A_i: i \epsilon I\}$ be a collection of fuzzy sets over the same universe U. Then

3.1. $\{\bigcup_i A_i\}^C=\bigcap_i A_i^C$
3.2. $\{\bigcap_i A_i\}^C=\bigcup_i \{A_i\}^C$

4.4. Proposition

For a fuzzy set $A=\{x, \mu(x), \gamma(x); x \epsilon U\}$.
$(A^C)^C=A$.

4.5. Proposition

For a fuzzy set A

1. $A \cap A^C=\phi$
2. $A \cup A^C=U$.

4.6. Definition:

Let X and Y be two non empty sets and f: $X \rightarrow Y$ be a function

Let B={y, μ_B(y), γ_B(y): yϵY} be fuzzy set on Y, then preimage of B under f denoted by f^{-1}(B), is fuzzy set in X defined by
f^{-1}(B) ={x, f^{-1}(μ_B)(x), f^{-1}(γ_B)(x): xϵX}, where f^{-1}(μ_B)(x)=μ_B(f(x)) and f^{-1}(γ_B)(x)=γ_B(f(x)).

If A={x, μ_A(x), γ_A(x): xϵX} be fuzzy set in X, then image of A under f is denoted by f(A) and defined as f(A)(y)= $\bigcup$ { xϵX, f(x)=y, μ_A(x), γ_A(x)}.

4.7. Theorem

Let f be a function from X to Y. Then

1. $B_1 \subseteq B_2 \Rightarrow f^{-1}[B_1] \subseteq f^{-1}[B_2]$, B_1 and B_2 are fuzzy sets in Y.
2. $A_1 \subseteq A_2 \Rightarrow f[A_1] \subseteq f[A_2]$, A_1 and A_2 are fuzzy sets in X.
3. $B \supseteq f[f^{-1}[B]]$ for any fuzzy subset B in Y.
4. $A \subseteq f^{-1}[f[A]]$ for any fuzzy subset A in X.
5. $f^{-1}[\bigcup B]= \bigcup f^{-1}[B]$
6. $f^{-1}[\bigcap B]= \bigcap f^{-1}[B]$
7. $f[\bigcup A]= \bigcup f[A]$
8. $f[\bigcap B]= \bigcap f[B]$

4.8. Theorems

Let f be a function from X to Y. Then

1. $f^{-1}[1_U]=1_U$.
2. $f^{-1}[0_U]=0_U$.
3. $f^{-1}[B^C]=\{f^{-1}[B]\}^C$ for any fuzzy set B in Y.
4. $\{f[A]\}^C \subseteq f[A^C]$ for any fuzzy set A in X.

It is clearly seen that the above theorems and propositions are also true when we use our extended definition of fuzzy set.

Now using our extended definition of fuzzy set we would like to discussed on fuzzy point and fuzzy quasi-coincident of fuzzy topology.

4.9. Definition

A fuzzy topology on a nonempty set X is a family τ of fuzzy set in X satisfying the following axioms

(T1) $0_X, 1_X \in \tau$

(T2) $G_1 \bigcap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(T3) $\bigcup G_i \in \tau$, for any arbitrary family $\{G_i: G_i \in \tau, i \in I\}$.

In this case the pair (X,τ) is called a fuzzy topological space and any fuzzy set in τ is known as fuzzy open set in X and clearly every element of τ^C is said to be fuzzy closed set.

Example: Let X={a, b}.

Let M={(a, 0.4, 0), (b, 0.5, 0)}, N={(a, 0.6, 0), (b, 0.8, 0)}
Then the family $\delta=\{0_X, 1_X, M, N\}$ is a fuzzy topology.

4.10. Definition

Let X be a non empty set and p be a fixed element of X. Let $r\in(0, 1)$ and $s\in[0, 1)$ such that $r\text{-}s \langle 1$, then the fuzzy set $p^r_s(y)=\{ x, p_r(x), p_s(x); x\in X\}$ is called fuzzy point in X, where $p_r(x)=r$, when x=y, otherwise zero, denotes the membership function and $p_s=r$, when x=y, otherwise zero, denotes the reference function.

Note: Let $A=\{x, \mu_A(x), \gamma_A(x); x\in X\}$. The fuzzy point $p^r_s=\{ x, p_r(x), p_s(x); x\in X\}$ is contained in A if and only if $\mu_A(x)\geq p_r(x)$ and) $\gamma_A(x)\leq p_s(x)$.

4.11. Definition

1. Let A and B are two fuzzy sets in X then A and B are said to be intersecting to each other if and only if there exists a point $x\in X$ such that $A\bigcap B\neq \phi$.
2. Also, two fuzzy sets A and B are said to be equal if and only if $p\in A \Leftrightarrow p\in B$, for fuzzy point p in X.

4.12. Properties

Let us consider the family of fuzzy sets$\{A_i: i\in I\}$ in X and P be fuzzy point on X. Then

1. If $P\in\bigcap \{A_i: i\in I\}$, then for If $i\in I$ we have If $P\in A_i$.
2. $f(P^C)=(f(P))^C$

It is seen that these properties are easily verified if the complementation is defined on the basis of reference function.

4.13. Definition

A fuzzy point p is said to be quasi-coincident with the fuzzy set A if $p \supseteq A^C$, denoted by pqA.

4.14. Proposition

Let (X, δ) be fuzzy topology. Let A and B be two fuzzy sets then AqB at x $\Leftrightarrow$ BqA at x.

Proof

Case 1 when reference function is zero.
Let $A=\{x, \mu_1(x), 0; x\in X\}$ and $B=\{x, \mu_2(x), 0; x\in X\}$
Let AqB at x $\Rightarrow A\supseteq B^C$ at x
$\Rightarrow (B^C)^C \supseteq A^C$ at x
$\Rightarrow B\supseteq A^C$ at x
Hence, BqA at x
Conversely let BqA at x
Now BqA at x $\Rightarrow B\supseteq A^C$ at x
$\Rightarrow (A^C)^C\supseteq B^C$ at x
$\Rightarrow A\supseteq B^C$ at x

Thus AqB at x

Case 2 when reference function is not zero.
Let $A=\{x,\ \mu_1(x), \gamma_1(x); x\epsilon X\}$ and $B=\{x,\ \mu_2(x), \gamma_2(x); x\epsilon X\}$
And $B^C=\{x,1,\ \ \mu_2(x); x\epsilon X\} \bigcup \{x,\ \gamma_2(x), 0; x\epsilon X\}$.
Now
Let AqB at x $\Rightarrow$ $A \supseteq B^C$ at x
$\Rightarrow$ Membership value of A $\geq$ Membership value of B^C, at x
$\Rightarrow (\ \mu_1(x)- \gamma_1(x)\ )\geq (1-\ \mu_2(x)) + \gamma_2(x)$
$\Rightarrow 1-(\ \mu_1(x)- \gamma_1(x)\ )\leq 1- (1-\ \mu_2(x)) + \gamma_2(x)$
$\Rightarrow 1-(\ \mu_1(x)- \gamma_1(x)\ )\leq (\mu_2(x)) - \gamma_2(x)$
$\Rightarrow$ Membership value of $A^C \leq$ Membership value of B
$\Rightarrow$ BqA at x
Conversely let BqA then $A^C=\{x,1,\ \mu_1(x); x\epsilon X\} \bigcup \{x,\ \gamma_1(x), 0; x\epsilon X\}$.
Now BqA at x $\Rightarrow$ $B \supseteq A^C$ at x
$\Rightarrow$ Membership value of B $\geq$ Membership value of A^C at x
$\Rightarrow (\mu_2(x) - \gamma_2(x)) \geq (1-\ \mu_1(x)) + \gamma_1(x)$
$\Rightarrow 1-(\mu_2(x) - \gamma_2(x)) \leq 1- (1-\ \mu_1(x)) + \gamma_1(x)$
$\Rightarrow 1-(\mu_2(x) - \gamma_2(x)) \leq (\mu_1(x)) - \gamma_1(x)$
$\Rightarrow$ Membership value of $B^C \leq$ Membership value of A, at x
$\Rightarrow$ AqB, at x
Hence AqB at x $\Leftrightarrow$ BqA at x.

4.15. Proposition

Let (X, δ) be fuzzy topology. Let A and B be two fuzzy sets then AqB $\Leftrightarrow$ BqA.
Proof we can prove this proposition by following prove of the proposition1.

4.16. Proposition

Let (X, δ) be fuzzy topology. Let A, B and C be fuzzy sets, if $A \subseteq B$ then CqA $\Rightarrow$ CqB.

Proof

Case 1 when reference function is zero.
Let $A=\{x,\ \mu_1(x), 0; x\epsilon X\}$, $B=\{x,\ \mu_2(x), 0; x\epsilon X\}$ and $C=\{x,\ \mu_3(x), 0; x\epsilon X\}$.
We have $A \subseteq B$ so clearly $\mu_1(x) \leq \mu_2(x)$.
Now
CqA $\Rightarrow C \supseteq A^C$
Since $A \subseteq B \Rightarrow\ B^C \subseteq A^C$.
So CqA $\Rightarrow C \supseteq A^C \supseteq B^C$
$\Rightarrow C \supseteq B^C$
$\Rightarrow$ CqB

Case 2 When reference function is not zero.

Let A={x, $\mu_1(x)$, $\gamma_1(x)$; x$\in$X}, B={x, $\mu_2(x)$, $\gamma_2(x)$; x$\in$X} and C={x, $\mu_3(x)$, $\gamma_3(x)$; x$\in$X}.
Also, A^C={x, 1, $\mu_1(x)$; x$\in$X} $\bigcup$ {x, $\gamma_1(x)$, 0; x$\in$X}, B^C={x,1, $\mu_2(x)$; x$\in$X} $\bigcup$ {x, $\gamma_2(x)$, 0; x$\in$X}.
Now
CqA $\Rightarrow$ $C \supseteq A^C$
$\Rightarrow$ Membership value of C $\geq$ Membership value of A^C
$\Rightarrow (\mu_3(x) - \gamma_3(x)) \geq (1- \mu_1(x)) + \gamma_1(x)$
Again as
$A \subseteq B \Rightarrow B^C \subseteq A^C$
$\Rightarrow$ Membership value of $B^C \leq$ Membership value of A^C
$\Rightarrow (1- \mu_2(x)) + \gamma_2(x) \leq (1- \mu_1(x)) + \gamma_1(x)$.
Hence
CqA $\Rightarrow (\mu_3(x) - \gamma_3(x)) \geq (1- \mu_1(x)) + \gamma_1(x) \geq (1- \mu_2(x)) + \gamma_2(x)$
$\Rightarrow (\mu_3(x) - \gamma_3(x)) \geq (1- \mu_2(x)) + \gamma_2(x)$
$\Rightarrow C \supseteq B^C$
$\Rightarrow$ CqB
Therefore when $A \subseteq B$ then CqA $\Rightarrow$ CqB.

4.17. Proposition

Let (X, δ) be fuzzy topology. Let A, B fuzzy sets, if $A \subseteq B$ then pqA $\Rightarrow$ pqB.
Proof Prove is straightforward.

4.18. Proposition

Let (X, δ) and (Y, Γ) be two fuzzy topological spaces and let A and B be fuzzy sets. Let f be a function from X to Y then

i. $Aqf^{-1}(B) \Leftrightarrow f(A)qB$
ii. $AqB \Rightarrow f(A)q\, f(B)$
iii. $f^{-1}(A)\, q\, f^{-1}(B) \Rightarrow AqB$

(i) Proof

We have $Aqf^{-1}(B) \Rightarrow A \supseteq (f^{-1}(B))^C$
$\Rightarrow A \supseteq (f^{-1}(B^C))$
$\Rightarrow f(A) \supseteq f(f^{-1}(B^C))$
$\Rightarrow f(A) \supseteq B^C$
$\Rightarrow f(A)qB$
Conversely let f(A)qB.
Now
$f(A)qB \Rightarrow f(A) \supseteq B^C$
$\Rightarrow f^{-1}(f(A)) \supseteq f^{-1}(B^C)$
$\Rightarrow A \supseteq (f^{`1}(B))^C$
$\Rightarrow Aqf^{-1}(B)$

Hence $Aqf^{-1}(B) \Leftrightarrow f(A)qB$.

(ii) Proof

Let $AqB \Rightarrow A \supseteq B^C$
Now
$f(A)(y)=\bigcup \{A(x); x\epsilon X: f(x)=y\}$
$\supseteq \bigcap \{B^C(x); x\epsilon X: f(x)=y\}$, as $A \supseteq B^C$
$=(\bigcup \{B(x); x\epsilon X: f(x)=y\})^C$
$=(f(B))^C$
$\Rightarrow f(A)q\, f(B)$
Hence $AqB \Rightarrow f(A)q\, f(B)$

(iii) Proof

The prove is straightforward following prove of i and ii.

5. Conclusion

In this article we attempted extended definition of complementation of fuzzy set on the basis of reference function to give definition of fuzzy point and quasi-coincidence for fuzzy point because there are some drawbacks in the existing definition of complement of fuzzy sets. In this article the definition of complement of fuzzy set proposed by Baruah [2, 3] can be seen as a particular case of what we are giving. We give value our definition of complement of an extended definition of fuzzy set with example which was discussed very nicely by Baruah [2, 3] and show that indeed our definition satisfies all those properties that complement of a set really does in classical sense. By observing this idea we used extended definition of fuzzy set to define fuzzy point and quasi coincident for fuzzy point because complement of fuzzy set plays important role to define fuzzy point and quasi coincident for fuzzy point. The main purpose of this article is to revisit and comment on some results associated with the existing definition of complementation of fuzzy sets. These results which are associated with existing definition of fuzzy sets are discussed from the standpoints of the new definition of complementation of fuzzy sets on the basis of reference function also some proposition are proved. It is accepted that these new definitions would be able to remove the drawbacks that exist. We have seen that if we used the extended definition of fuzzy set and complement of an extended definition of fuzzy set then we arrive at the conclusion that the fuzzy sets too follow the set theoretic axioms. We hope that this work will help for further work of fuzzy topology.

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How to cite this paper: Kangujam Priyokumar Singh, Bhimraj Basumatary,"A Note on Quasi-coincidence for Fuzzy Points of Fuzzy Topology on the Basis of Reference Function", International Journal of Mathematical Sciences and Computing(IJMSC), Vol.2, No.3, pp.49-57, 2016.DOI: 10.5815/ijmsc.2016.03.05