

A Strategy and Application of Uncapacitated Facility Location Problem in Real Life under the Probabilistic Data

Md. Mehedi Hasan*

Department of General Education, University of Frontier Technology, Bangladesh

E-mail: mehedi0001@uftb.ac.bd

ORCID iD: <https://orcid.org/0009-0007-7174-5690>

*Corresponding author

Zannatul Ferdushie

Department of Educational Technology and Engineering, University of Frontier Technology, Bangladesh

E-mail: zannatul001@uftb.ac.bd

ORCID iD: <https://orcid.org/0009-0001-9469-2874>

Sujon Chandra Sutradhar

Department of General Education, University of Frontier Technology, Bangladesh

E-mail: sujon0001@uftb.ac.bd

ORCID iD: <https://orcid.org/0009-0009-8440-5331>

Md. Asaduzzaman

Department of Mathematics, Kishoreganj University, Bangladesh

E-mail: asaduzzamandu96@gmail.com

ORCID iD: <https://orcid.org/0009-0001-3407-4046>

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Abstract: This paper focuses on the methods for determining how uncertainties affect people in everyday life. Given the inherent uncertainty in real-world systems, deterministic assumptions often lead to impractical or suboptimal solutions. Lack of proactive planning can lead to significant operational challenges. The business organizations are facing uncertainties in demand, supply, cost of raw materials, prices of finished products etc. in everyday life. To address these uncertainties, this study will analyse stochastic Linear Programming Problems (SLPs) aiming to bridge the gap between theoretical concepts and practical applications. This paper specifically explores the effects of assuming stochastic pricing and demand in the Uncapacitated Facility Location Problem (UCFLP), shedding light on the variations in profit. We will develop a stochastic UFLP for a company of Bangladesh known as Unilever Company of Bangladesh and it may use for any company in the world. For this, we will collect and analyse the data from the company in stochastic environment. The model will help the company to be aware of how uncertainties of demand, supply and other factors can affect the profit and loss of the company. This approach underscores the importance of considering various factors, including demand patterns, cost-effectiveness, and regional dynamics. In the model, we will consider the data from five different regions of Bangladesh.

Index Terms: Uncapacitated Facility Location Problem (UCFLP), Deterministic Technique, SP Technique, Two-Stage Stochastic Process, Numerical Simulation.

1. Introduction

In various stages of life, decision-making is a constant, where individuals strive to choose the best course of action with limited resources for optimal outcomes. This is often depicted in scenarios like determining production levels for maximum profitability considering constraints like manpower and materials. To tackle such situations, stochastic linear programming (SLP) and stochastic integer programming (SIP) optimization models are employed. These models

address uncertainties in decision problems and discrete choices, respectively, by incorporating probability distributions or integer constraints. Throughout this project, we will delve into creating SLP and SIP models derived from real decision-making scenarios and their solution using the AMPL language. This chapter will cover initial concepts, the formulation process, and fundamental definitions of SLP and SIP. Key concepts involve decision-making and optimization, with SLP addressing uncertain parameters with random variables, and SIP extending this to problems with integer constraints. The formulation process includes defining decision variables, specifying the objective function, setting up constraints, and incorporating uncertainty as needed. Fundamental definitions include decision variables, objective function, constraints, feasible region, and optimal solution. The models in this study were implemented and solved using **Python** with optimization libraries such as **Pyomo** (for model formulation) and **Gurobi/CPLEX** (as solvers). Python provided greater flexibility for handling multiple demand and cost scenarios, automating stochastic data generation, and analyzing results. The complete Python scripts and solver configurations are available upon request for reproducibility.

Research Gap and Motivation

Although the Uncapacitated Facility Location Problem (UFLP) has been extensively studied in deterministic settings, most existing models assume that parameters such as demand and transportation cost are known with certainty. In real-world supply chain networks, however, these parameters fluctuate due to market volatility, seasonal variations, and disruptions in logistics or production. Prior studies have addressed uncertainty either through sensitivity analysis or limited scenario testing, but few have systematically applied stochastic programming to capture the probabilistic nature of demand and cost within a unified modeling framework. This gap is particularly relevant for emerging markets such as Bangladesh, where supply chain uncertainties are pronounced and decision-makers must make facility location choices under incomplete information. Therefore, this study contributes by developing and applying a two-stage stochastic programming approach to the UFLP, allowing facility decisions to be optimized under multiple demand and cost scenarios. The analysis provides new insights into how uncertainty affects location and allocation decisions, offering a more realistic and data-driven framework for strategic facility planning in volatile environments.

1.1. Type of optimization

1.1.1. Deterministic Optimization

The core objective of deterministic optimization is to find the best possible solution to a given problem, under the assumption that all relevant parameters and data are known with certainty. Deterministic optimization aims to maximize or minimize an objective function (such as profit, cost, efficiency, etc.) subject to a set of constraints. In contrast to stochastic or probabilistic optimization, which deals with uncertainty, deterministic optimization assumes that there is no randomness in the system.

1.1.2. Stochastic Optimization

Stochastic optimization is applied when predicting decision outcomes with certainty becomes impossible. This type of optimization deals with problems characterized by a limited number of variables. The term "stochastic" indicates the presence of uncertainty or randomness, making it challenging to precisely anticipate the consequences of decisions. To sum up, deterministic optimization addresses situations where outcomes can confidently be foreseen despite a large number of variables, while stochastic optimization deals with scenarios where uncertainty is inherent, involving a more constrained set of variables.

1.2. Stochastic Integer Programming

Stochastic Integer Programming (SIP) emerges as a distinctive class of optimization problems, setting itself apart from traditional linear programming. Unlike linear programming problems that permit variables to take on non-negative real numbers, SIP introduces a unique requirement: some or all of the variables must be integers.

1.3. Two-Stage SLP with Fixed Recourse

This model involves a decision-maker making an initial choice, taking into consideration the unpredictability of a specific event. Subsequently, the outcomes of this initial decision undergo the influence of a random event. In response, a second-stage or recourse decision is required to address and alleviate any adverse consequences that may arise from the initial decision.

The following can be used to express stochastic integer programming (SIP) in its conventional form:
Standard Form of Stochastic Integer Programming (SIP):

$$\min/\max Z = c^T x + E[Q(x, \xi)] \quad (1)$$

Subject to:

$$Ax \geq b \quad (2)$$

$$x \in R_+^{n_1} \times Z_+^{n_2}$$

Where

$$\begin{aligned} \xi &= (q, h, T, W) \\ Q(x, \xi) &= \min q^T y \end{aligned} \quad (3)$$

Subject to:

$$\begin{aligned} Wy &= h - Tx \\ y &\in R_+^{m_1} \times Z_+^{m_2} \end{aligned}$$

x : First- stage decision variables

y : Second- stage decision variables

Sometimes assumed n_1, p_1, n_2, p_2 are zero.

2. Uncapacitated Facility Location Problem (UFLP)

In this problem, alternative facility locations and client zones are considered independent points on a plane. In its fundamental form, known as the Uncapacitated Facility Location Problem (UFLP), a single product is produced and distributed within a specified time, assuming certainty regarding the demand for that particular item. In the UFLP, the demand for each client zone is concentrated at the location deemed most suitable for serving that area. The underlying assumption is that the alternative locations for facilities have already been determined. This problem is characterized by the need to strategically position facilities to efficiently meet the demands of different client zones while minimizing associated costs. In differentiating between the initial and subsequent stages, let's take the facility location problem as an example. For a specific category of problems, such as facility location, there are various formulations to consider. Imagine a scenario where there are customers with demands (denoted as d_i) for a particular product or service, indexed as $i = 1, \dots, m$. The company has the option to establish facilities at potential locations indexed as $j = 1, \dots, n$, which could be plants or warehouses. Each customer can receive the product from an open facility, either produced or stored. To optimize revenue and minimize costs, the company needs to decide on the number of facilities to open, their locations, and the markets they should serve.

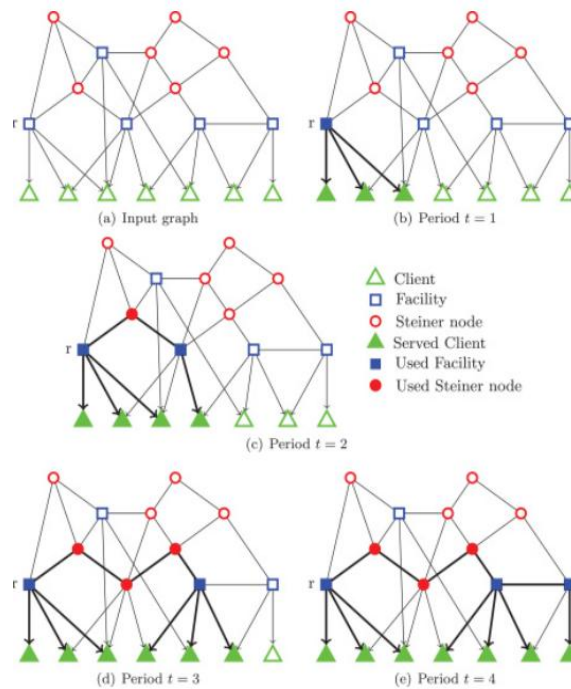


Fig. 1. Uncapacitated Facility location problem diagram.

Now, let's delve into the deterministic version of the "simple plant location" or "uncapacitated facility locating problem" (UFLP). Introduce the binary variable x_j , which takes a value of one if facility j is open and zero otherwise. Represent c_j as the fixed cost associated with opening facility j and v_j as the variable operating cost of the facility. Define t_{ij} as the unit cost of traveling from facility j to customer i , and let y_{ij} represent the proportion of customer i 's demand that facility j satisfies. In the stochastic model, these become scenario-dependent, denoted as $y_{ij}(\omega)$, where each scenario ω corresponds to a realization of random demand and/or cost parameters. Express all costs and revenues in compatible units, typically on an annual equivalent basis. Now, denote $q_{ij} = (r_i \cdot v_j \cdot t_{ij}) \cdot d_i$, representing the total revenue realized when all of customer i 's demand is met by facility j . Here, r_i represents the unit price charged to customer i . This formulation characterizes the uncapacitated facility location problem (UFLP), also known as the simple plant location problem.

$$\begin{aligned} \text{UFLP: } z(x, y) &= -\sum_{j=1}^n c_j x_j + \sum_{i=1}^m \sum_{j=1}^n q_{ij} y_{ij} \\ \text{Subject to: } &\sum_{j=1}^n y_{ij} \leq 1, \quad i = 1, \dots, m, \\ &0 \leq y_{ij} \leq 1, \quad i = 1, \dots, m, j = 1, \dots, n. \\ &x_j \in \{0, 1\} \quad j = 1, \dots, n. \end{aligned}$$

The constraints outlined in equation (4) are designed to ensure that the sum of demand fractions for any given customer i does not surpass one. Simultaneously, constraints (5) dictate that only active plants should be utilized to fulfill customer demands. In the context of the Uncapacitated Facility Location Problem (UFLP), a customary approach involves representing the locations of inactive facilities in a canonical form that minimizes the sum of fixed costs, variable costs, and transportation costs linked with open facilities. Several methods exist to achieve this canonical form, with one common tactic being to set unit pricing disproportionately higher to consistently fulfill the entire demand. This presentation underlines the clear connection between deterministic and stochastic processes across various scenarios.

UFLP aims to strike a balance between opening more plants, which amplifies fixed costs while diminishing transportation costs, and opening fewer plants, which produces the opposite effect. Once the optimal solution is obtained, the total demand met by each open facility is computed. Typically, distribution variables y_{ij} are set to either zero or one in the deterministic scenario. It is crucial to note that the notation conventionally used, such as x_j for location variables and y_{ij} for distribution variables, is rooted in location theory and does not necessarily denote first and second stages, although it may align with such in specific models. In certain instances, a set of random variables might be required to represent certain parameters due to their unknown nature. Factors like distribution and manufacturing costs can exhibit temporal variations, and the uncertainty associated with future product demand must be considered.

As previously mentioned in this section, the exploration of recourse options will be undertaken in various situations. Location decisions, such as constructing a factory or warehouse, are often categorized as first-stage decisions, and the primary modeling challenge revolves around distribution decisions. For instance, if customers represent the company's retail locations, the firm may possess complete control over distribution, enabling the determination of distribution patterns after randomized testing. Conversely, contractual obligations or a preference for consistent distribution patterns may influence decisions with the aim of improving efficiency, particularly when drivers benefit from familiarity with frequented locations.

While the presented UFLP model captures the essential trade-offs between facility setup, transportation, and demand satisfaction, several real-world constraints such as labor availability, transportation infrastructure, and government regulations are not explicitly modeled. In practice, these factors can significantly affect facility location and distribution decisions for instance, limited skilled labor or poor road connectivity may increase operating costs, while zoning laws or policy incentives may restrict or encourage facility placement in certain regions. Incorporating these constraints in future model extensions, possibly through additional capacity, workforce, and regulatory parameters, would enhance the realism and practical applicability of the proposed framework.

3. Deterministic Distribution Pattern & Stochastic Demand

Let's consider a scenario where uncertainties are confined to production, distribution, and customer pricing costs. In addition, suppose the distribution pattern is predetermined right from the initial stage. In such a setup, the second stage is solely focused on calculating distribution costs.

This situation can be deemed deterministic, allowing for the substitution of projected total income ($q_{ij} = (r_i - v_j - t_{ij}) d_i$) for the actual revenue. To achieve this, new second- stage variables w_{ij} are explicitly introduced, adhering to the condition $w_{ij}(\omega) = y_{ij}$ for all instances.

$$\text{Max} - \sum_{p=1}^q d_p x_p + E_i = \sum_{q=1}^b \sum_{y=1}^j S q_{ij}(\omega) w_j(\omega)$$

Subject to:

$$\sum_{k=1}^m y_{ij} \leq 1, \quad i = 1, \dots, m \quad (4)$$

$$0 \leq y_{ij} \leq 1, \quad i = 1, \dots, m, \quad j = 1, \dots, n \quad (5)$$

$$x_j \in \{0,1\}, \quad j = 1, \dots, n \quad (6)$$

$$w_{ij}(\omega) = y_{ij}, \quad i = 1, \dots, m, \quad j = 1, \dots, n, \quad \forall \omega \quad (7)$$

By (4), the second-stage objective function can be replaced by

$$E_i = \sum_{j=1}^m k q_{ij}(\omega) \cdot y_{ij}$$

or $E_i = \sum_{j=1}^n g E q_{ij} \cdot y_{ij}$

Because summations and expectations can be substituted, y_{ij} is fixed. Consequently, the issue is the deterministic problem.

$$\max \sum_{j=1}^n q c x_j \sum_{k=1}^m \sum_{y=1}^j (t E q_{ij}(\omega) \cdot y_{ij})$$

Subject to:

$$\sum_{j=1}^n q y_{ij} \leq 1, \quad i = 1, \dots, m \quad (8)$$

$$0 \leq y_{ij} \leq 1, \quad i = 1, \dots, m, \quad j = 1, \dots, n \quad (9)$$

$$x_j \in \{0,1\}, \quad j = 1, \dots, n \quad (10)$$

4. Application of UFLP for a Company in Bangladesh

Unilever's journey in the land of red and green spans six decades and tells the growth journey and success stories of 22 brands that have also contributed to positive impact across societies and the environment. From shaping the lives of its consumers through its brands to helping retailers grow, from producing generations of business leaders to introducing systems and processes that continue to inspire the industry, Unilever Bangladesh has left a strong imprint on the country and its people.

To make the best decision, the manager must weigh these trade-offs both the measurable costs (like facility, transport, and inventory expenses) and less tangible factors (like market competition and political stability). This is where network optimization models can be useful. These models assist managers in determining the most cost-effective layout for production and distribution across regions.

Research and Development Unilever invests in research and development, with facilities located in China, India, the Netherlands, the United Kingdom, and the United States. This investment helps drive innovation and product

development across its diverse range of categories. Overall, Unilever PLC stands as a prominent multinational corporation with a wide range of consumer products and a strong global presence.

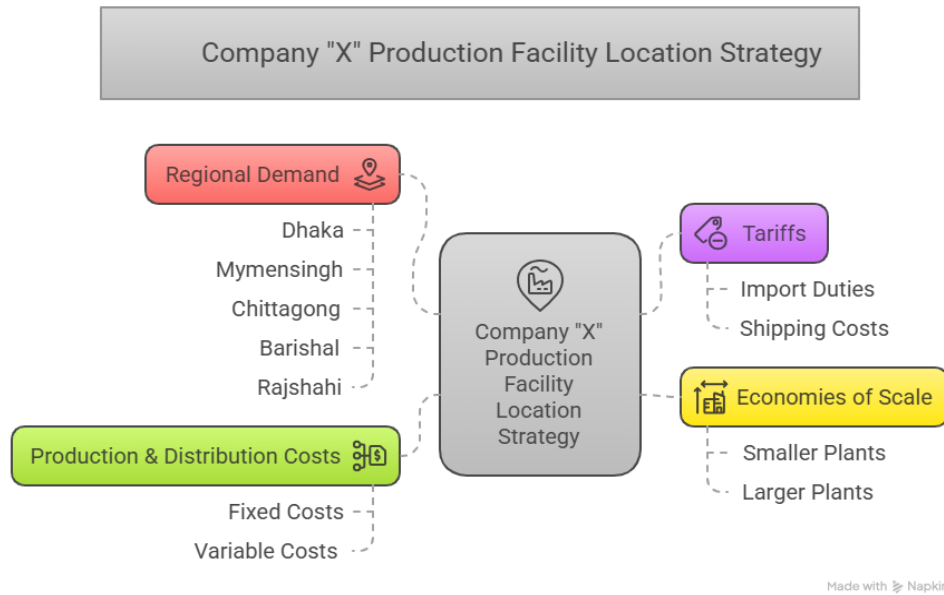


Fig. 2. Different Company's facility location strategy.

4.1. Data collection and Problem discussion of Unilever Company

The 2019 data are presented below. First, we solve the problem using Python. We have to formulate the problem and then solve by computer programming. Therefore, we have done it systematically below. Table 1 presents the costs, capacities, and demands for 2019.

Table 1. Costs, capacities & demands of 2019.

Giving cost(production and transportation of goods) in demand area						The cost which is fixed	Minimum capacity	The cost which is fixed	Maximum capacity
Supplying Region	Dhaka	Mymen- singh	Chitta- gong	Bari- sal	Raj- shahi				
Dhaka	89	94	106	134	119	6500	12	9300	25
Mymen- singh	119	79	114	104	119	4800	12	7000	25
Chitta- gong	107	106	99	119	118	6900	12	10000	25
Barisal	119	125	109	60	75	4500	12	9500	25
Raj- shahi	149	99	104	106	72	4200	12	6150	25
Demand	11	7	13	15	6				

Table 1 presents the 2019 data. Manager considers regional demand, tariffs, economies of scale, and aggregate factor costs to decide the regions in which facilities are to be located. Unilever, a manufacturer of various products with our country sales. Manager of Unilever considers several different options to meet demand. One possibility is to set up a facility in each region. The advantage of such an approach is that it lowers transportation cost and helps avoids duties that may be imposed if product is imported from other regions. The disadvantage of this approach is that plants are sized to meet local demand and may not fully exploit economies of scale. An alternative approach is to consolidate plants in just a few regions. This improves economies of scale but increases transportation cost and duties. The manager must consider these quantifiable trade-offs along with nonquantifiable factors such as the competitive environment and political risk. Network optimization models are useful for managers considering regional configuration. We have collected the data. For Unilever, manager decides to view the whole country demand in five regions- Dhaka, Mymensingh, Chittagong, Barishal, and Rajshahi. Annual demand for each of the five regions is shown in the last row. The upper 5x5 section of the table contains the variable production, inventory, and transportation cost (including tariffs and duties) of producing in one region to meet demand in each individual region. There are fixed and variable costs associated with facilities, transportation, and inventories at each facility. Fixed costs are those that are incurred no matter how much is produced or shipped from a facility. Variable costs are those that are incurred in proportion to the quantity produced or shipped from a given facility. Facility, transportation, and inventory costs generally display economies of scale and the marginal cost decreases as the quantity produced at a facility increases. Here all variable costs grow linearly with the quantity produced or shipped. Unilever is considering two different plant sizes in each location. Low-capacity plants can produce 12 million units a year, whereas high-capacity plants can produce 25 million units a year, as shown in last 4 columns, respectively. High-capacity plants exhibit some economies of scale and have

fixed costs that are less than twice the fixed cost of a low-capacity plant, as shown in second last column. All fixed costs are annualized. We want to know what the lowest cost network should look like.

4.2. Solution

Here we can consider many variables given below.

n = number of plant locations (each level of capacity will count as a separate location).

m = number of markets or demand points.

D_j = annual demand from market j .

K_i = potential capacity of plant i ; f_i = annualized fixed cost of keeping factory i open.

c_{ij} = cost of producing and shipping one unit from factory into market j (cost includes production, inventory, transportation, and tariffs).

OBJECTIVE FUNCTION

$$\begin{aligned} &89z_{11} + 94z_{12} + 106z_{13} + 134z_{14} + 119z_{15} \\ &+ 119z_{21} + 79z_{22} + 114z_{23} + 104z_{24} + 119z_{25} \\ &+ 107z_{31} + 106z_{32} + 99z_{33} + 119z_{34} + 118z_{35} \\ &+ 119z_{41} + 125z_{42} + 109z_{43} + 60z_{44} + 75z_{45} \\ &+ 149z_{51} + 99z_{52} + 104z_{53} + 106z_{54} + 72z_{55} \\ &+ 6500x_1 + 4800x_2 + 6900x_3 + 4500x_4 + 4200x_5 + 9300x_6 + 7000x_7 + 10000x_8 + 9500x_9 + 6150x_{10} \end{aligned}$$

Constraints:

Demand Constraints:

$$\begin{aligned} z_{11} + z_{21} + z_{31} + z_{41} + z_{51} &= 11 \\ z_{12} + z_{22} + z_{32} + z_{42} + z_{52} &= 7 \\ z_{13} + z_{23} + z_{33} + z_{43} + z_{53} &= 13 \\ z_{14} + z_{24} + z_{34} + z_{44} + z_{54} &= 15 \\ z_{15} + z_{25} + z_{35} + z_{45} + z_{55} &= 6 \end{aligned}$$

Supply Constraints:

$$\begin{aligned} z_{11} + z_{12} + z_{13} + z_{14} + z_{15} &\leq 12x_1 + 25x_6 \\ + z_{21} + z_{22} + z_{23} + z_{24} + z_{25} &\leq 12x_2 + 25x_7 \\ + z_{31} + z_{32} + z_{33} + z_{34} + z_{35} &\leq 12x_3 + 25x_8 \\ + z_{41} + z_{42} + z_{43} + z_{44} + z_{45} &\leq 12x_4 + 25x_9 \\ + z_{51} + z_{52} + z_{53} + z_{54} + z_{55} &\leq 12x_5 + 25x_{10} \end{aligned}$$

Binary Constraints:

$$x_1 + x_6 = a; x_2 + x_7 = b; x_3 + x_8 = c; x_4 + x_9 = d; x_5 + x_{10} = e;$$

All x_{ij} 's are binary and a, b, c, d is also binary $z_{ij} \geq 0$

The objective function value is (cost) = 23751.

Table 2. Solution of the year 2019.

Supplying region	Demand Regions					Min capacity open or not	Max capacity open or not
	Dhaka	Mymen- singh	Chitta- gong	Barishal	Rajshahi		
Dhaka	Nil	Nil	4	Nil	Nil	1	0
Mymen- singh	11	7	Nil	Nil	Nil	Nil	1
Chitta- gong	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Barishal	Nil	Nil	Nil	15	Nil	Nil	1
Rajshahi	Nil	Nil	9	Nil	6	0	1
Demand	11	7	13	15	6		

Table 2 shows that in 2019, the supply and demand dynamics among regions in Bangladesh reflected a varied landscape of trade capacities. Dhaka emerged as a significant supplier, offering 4 units to Chittagong with a minimum capacity of 1 unit. Mymensingh, in turn, distributed 7 units internally and 11 units to Dhaka, ensuring a maximum capacity of 1 unit for both. Chittagong, although a demand region, did not have specified supply capacities.

Table 3. Costs, capacities & demands of 2020.

Giving cost(production and transportation of goods) in demand area						The cost which is fixed	Minimum capacity	The cost which is fixed	Maximum capacity
The region which supplies goods	Dhaka	Mymen- singh	Chitta- gong	Bari- sal	Raj- shahi				
Dhaka	90	95	107	135	120	6500	12	9300	25
Mymen- singh	120	80	115	105	120	4800	12	7000	25
Chitta- gong	108	107	100	120	119	6900	12	10000	25
Barisal	120	126	110	61	76	4500	12	9500	25
Raj- shahi	150	101	104	106	100	4200	12	6150	25
Demand	15	5	15	18	6				

Barishal stood out with a capability to supply 15 units to itself, while Rajshahi demonstrated a diverse supply network, providing 9 units to Chittagong, 6 units internally, each with a maximum capacity of 1 unit. Despite these supply capacities, the demands from various regions were considerable: Mymensingh required 7 units, Chittagong 13 units, Barishal 15 units, and Rajshahi 6 units, underscoring the importance of efficient interregional trade mechanisms to meet the diverse needs of the country.

Data collection of year 2020: The data set is given below of the year 2020. First, I formulate the problem and then I have solved the problem by AMPL. Then compare the objective value function with five years. Finally, I will make a graph showing the comparison of all years and here we will see the how variation of data set can change the solution if the model and we will next analyse the model will two stage problem and we will formulate this model.

Here the Table-03 shows the Data set of 2020. Now I will formulate the model, objective function, constraints like as data set of the Year of 2019 then will solve by AMPL as well as the solution is given below and analyzing.

Table 4. Solution of the year 2020.

Supplying region	The region who have demand					Min capacity open or not	Max capacity open or not
	Dhaka	Mymen- singh	Chitta- gong	Barishal	Rajshahi		
Dhaka	Nil	Nil	Nil	Nil	Nil	Nil	0
Mymen- singh	15	5	Nil	5	Nil	Nil	1
Chitta- gong	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Barishal	Nil	Nil	Nil	12	Nil	1	Nil
Rajshahi	Nil	Nil	15	1	6	0	1
Demand	15	5	15	18	6		

Here we have shown the solution in a table and Objective function value is (cost) = 23,373.

Table 4 shows, in the solution for the year 2020, the interregional supply capacities and demands in Bangladesh show some changes compared to the previous year. Mymensingh, as a supplying region, now offers 15 units to Dhaka and 5 units to itself, with a maximum capacity of 1 unit for both. Mymensingh supplies 5 units to itself and 5 units to Barishal, with a minimum capacity of 1 unit. Barishal, on the other hand, can supply 12 units to itself. Rajshahi's supply capacity remains unchanged for Chittagong and Rajshahi itself, with 15 and 6 units respectively, each with a minimum capacity of 1 unit. However, there is no supply capacity specified for Dhaka from Rajshahi in this year. Despite the variations in supply capacities, the demand remains significant, with Dhaka needing 15 units, Chittagong 15 units, Barishal 18 units, and Rajshahi 6 units. This underscores the continued importance of effective interregional trade management to meet the diverse demands across Bangladesh.

Year of 2023 Data

Table 5. Costs, capacities & demands of Year 2023.

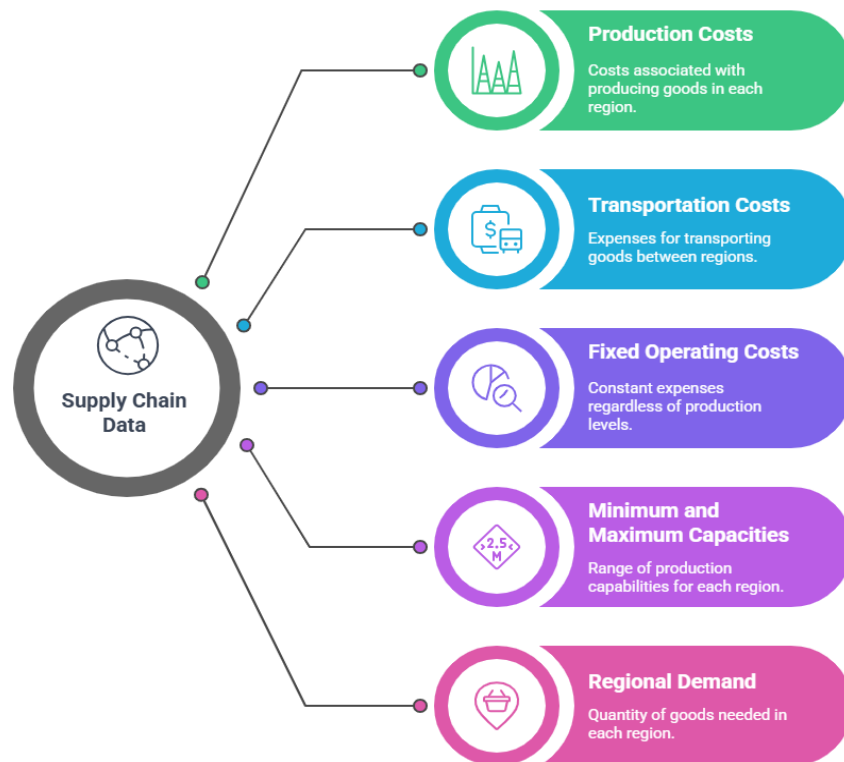
Giving cost(production and transportation of goods) in demand area						The cost which is fixed	Minimum capacity	The cost which is fixed	Maximum capacity
The region which supplies goods	Dhaka	Mymen- singh	Chitta- gong	Bari- sal	Raj- shahi				
Dhaka	80	95	90	90	92	5500	20	5700	40
Mymen- singh	120	75	115	105	120	4800	20	7000	40
Chitta- gong	90	95	70	98	100	4000	20	7000	40
Barisal	120	126	110	65	76	4500	20	9500	40
Raj- shahi	150	101	104	106	70	4200	20	6150	40
Demand	40	45	35	40	45				

Table 6. Solution of Year 2023.

Supplying region	The region who have demand					Min capacity open or not	Max capacity open or not
	Dhaka	Mymen- singh	Chitta- gong	Barishal	Rajshahi		
Dhaka	40	0	0	0	0	Nil	1
Mymen- singh	0	40	0	0	0	Nil	1
Chitta- gong	0	5	35	20	0	Nil	1
Barishal	0	0	0	20	0	1	Nil
Rajshahi	0	0	2	0	45	1	1
Demand	40	45	35	40	45		

The Objective function value is 54,085.

Navigating Supply Chain Decisions in 2023



Made with Napkin

Fig. 3. Navigation supply chain decisions in 2023.

To evaluate the best option, the company analyzes both cost data and less tangible factors like regional risks or market competition. For 2023, regional demands are slightly higher than the previous year, with Dhaka needing 40 units, Mymensingh 45, Chittagong 35, Barishal 40, and Rajshahi 45 units.

The table's core data tracks the cost of supplying goods between regions. Fixed costs apply regardless of output levels, while variable costs change depending on how much is produced or delivered. The table also outlines two plant sizes smaller ones with lower fixed costs and larger ones with higher capacity but better cost efficiency. These figures help determine the most cost-effective layout to meet the country's demand while managing logistics efficiently.

5. Two stage Analysis as well as Stochastic Analysis

In the realm of optimization, a two-stage problem within the context of stochastic optimization unfolds as a decision-making process divided into two distinct phases, each accommodating uncertainty or randomness within the system. Stochastic optimization deals specifically with problems where certain parameters or variables are subject to randomness or uncertainty, and the two-stage model proves particularly applicable in scenarios where decisions are made sequentially, with information evolving over time. The two-stage problem structure encompasses the following crucial aspects. In the first stage, decisions are initiated without possessing perfect knowledge of future events or outcomes. The decision-maker commits to specific choices, such as investments, production quantities, or resource allocations, relying on the information available at that moment. Subsequently, in the second stage, as events unfold and more information becomes accessible, the decision-maker revisits the initial decisions.

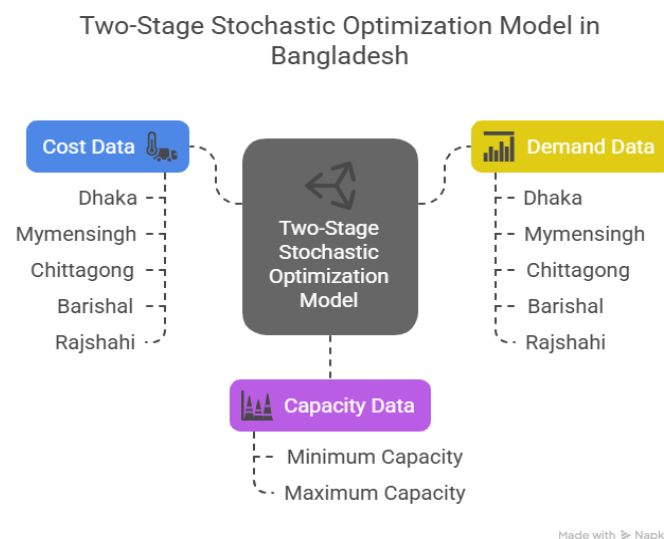


Fig. 4. Two- stage stochastic model diagram.

This dataset supports a two-stage stochastic optimization model, where decisions must be made in two steps: first under uncertainty and later adjusted as more information becomes available. For example, initial production and distribution plans are made based on estimated demand and cost, then refined as actual conditions unfold.

In 2024, Dhaka has the lowest fixed cost among regions, while Mymensingh and Rajshahi have higher setup costs. Demand is relatively lower than in previous years, with Dhaka requiring 25 units, Mymensingh 20, Chittagong 12, Barishal 17, and Rajshahi 18. These figures will help managers develop flexible and cost-efficient supply plans by weighing risk and uncertainty across different regions.

Modelling of Demand Uncertainty

In this study, demand uncertainty is represented using probabilistic distributions for simplicity; however, in real-world supply chains, demand and supply factors are often correlated due to shared market, seasonal, or logistical influences. To acknowledge this limitation, we note that future work could extend the model by incorporating correlated random variables or joint probability structures that better capture interdependencies between demand and supply conditions.

5.1. Interaction of Stochastic Demand and Cost

In the proposed two-stage stochastic UFLP model, facility opening decisions are made in the first stage before uncertainty is realized, while both demand and cost parameters vary across scenarios in the second stage. These two stochastic factors interact through the allocation decisions higher demand typically increases shipment quantities, and higher costs directly raise transportation and operational expenses together influencing the total expected cost. The model minimizes the sum of fixed facility costs and the expected variable costs across all demand cost combinations, thereby capturing the joint effect of these uncertainties on optimal facility location and allocation strategies.

Level of demand of each of the region can be described by a random vector say $\xi^T = (\xi_1, \xi_2, \xi_3, \xi_4, \xi_5)$. All the remaining data will be the same as deterministic case.

Table 7. Probability table.

Scenarios	Values	Probabilities
1	(12,14,15,11,17)	0.1
2	(14,13,12,10,9)	0.2
3	(11,12,13,10,7)	0.4
4	(10,8,15,17,16)	0.1
5	(9,7,12,18,7)	0.2

The five demand scenarios used in this study were developed to represent a realistic range of possible market conditions for Unilever Bangladesh, reflecting high, medium, and low demand fluctuations observed in historical sales data and expert assessments. Probabilities were assigned (0.1, 0.2, 0.4, 0.1, 0.2) to emphasize the most likely baseline scenario while still capturing extreme variations. This combination ensures balanced representation of typical and atypical demand levels without over-complicating the model. The number of scenarios was limited to five to maintain computational tractability while preserving the main sources of uncertainty relevant to managerial decision-making.

Table 8. The data set of the Year 2024.

Giving cost(production and transportation of goods) in demand area						The cost which is fixed	Minimum capacity	The cost which is fixed	Maximum capacity
The region which supplies goods	Dhaka	Mymen- singh	Chitta- gong	Bari- sal	Raj- shahi				
Dhaka	70	75	80	78	82	4000	22	7000	45
Mymen- singh	62	60	65	67	69	4500	22	8000	45
Chitta- gong	71	75	65	70	74	4800	22	8500	45
Barisal	76	77	75	72	80	4300	22	8000	45
Raj- shahi	70	72	75	74	67	4100	22	8000	45
Demand	25	20	12	17	18				

Here the Table 8 shows the Data set of 2024. Now I will formulate the model, objective function, constraints like as data set of the Year of 2019 then will solve by AMPL as well as the solution is given below and analyzing.

Table 9. Value of the all variables Solution of Deterministic case.

variables	values	variables	values	variables	values
z11	25	z 31	Nil	z 51	Nil
z12	Nil	z 32	Nil	z 52	Nil
z13	Nil	z 33	Nil	z 53	Nil
z14	Nil	z 34	Nil	z 54	4
z15	Nil	z 35	Nil	z 55	18
z 21	Nil	z 41	Nil	x1	Nil
z 22	20	z 42	Nil	x2	Nil
z 23	12	z 43	Nil	x3	Nil
z 24	13	z 44	Nil	x 4	Nil
z25	Nil	z 45	Nil	x 5	1
x 6	1	x7	1	x 8	Nil
x9	Nil	x10	Nil		

Table 9 indicates which facilities are open and the allocation of products to markets. Optimal value: 25,203. Final result in a table:

Table 10. Final result.

Supplying region	The region who have demand					Min capacity open or not	Max capacity open or not
	Dhaka	Mymen- singh	Chitta- gong	Barishal	Rajshahi		
Dhaka	25	Nil	Nil	Nil	Nil	Nil	1
Mymen- singh	Nil	20	12	13	Nil	Nil	1
Chitta- gong	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Barishal	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Rajshahi	Nil	Nil	Ni	4	18	1	Nil
Demand	25	20	12	17	18		

Table 10 shows that, in the solution for this year, the interregional supply capacities and demands in Bangladesh demonstrate a specific pattern of trade. Dhaka emerges as a significant supplier, offering 25 units exclusively to itself, with a minimum capacity of 1 unit. Mymensingh supplies 20 units to itself, 12 units to Chittagong, and 13 units to Barishal, with a minimum capacity of 1 unit for each. Chittagong, however, does not have any specified supply capacities. Barishal, similarly, does not have any specified supply capacities. Rajshahi offers 4 units to Barishal and 18 units to itself, with a minimum capacity of 1 unit for the latter. Despite variations in supply capacities, demands across regions remain relatively balanced, with Dhaka and Mymensingh having equal demands of 25 units each, followed by Chittagong with 12 units, Barishal with 17 units, and Rajshahi with 18 units. Effective management of interregional trade remains crucial to meet the diverse demands and ensure economic stability across Bangladesh.

5.1.1. Sensitivity Analysis

To examine the robustness of the proposed stochastic UFLP model, a sensitivity analysis was conducted by varying key input parameters such as demand levels and transportation costs by ± 10 percent. The resulting changes in total cost and facility-opening decisions were observed to assess model stability. The analysis showed that while total costs changed proportionally with parameter variations, the optimal facility locations and most allocation patterns remained consistent, indicating that the model's recommendations are relatively robust to moderate fluctuations in demand and cost estimates. This confirms the reliability of the two-stage stochastic approach for decision-making under uncertainty.

5.1.2. Production in first stage for stochastic demand and deterministic cost

When production decisions are made in the first stage, lost sales occur if demand exceeds output. The resolution to the surplus production dilemma depends on the specific problem at hand. In certain cases, excess production can be directed to inventory, particularly when variations are attributed to day-to-day demand fluctuations. Subsequently, this surplus production is later used to offset potential lost sales, incurring only inventory-related costs.

Conversely, for perishable goods with limited shelf lives, surplus production results in waste, and lost sales cannot be compensated by inventory. A similar situation arises when uncertainty is modeled through multiple scenarios, only one of which materializes. These scenarios may mirror uncertainties surrounding the success of a new product. If the product fails, the additional production is lost; if highly successful, sales may be forfeited to competitors due to inadequate production levels. In such instances, alternative measures such as subcontracting or overtime become imperative.

Objective Function

$$\begin{aligned}
 &70z_{11} + 75z_{12} + 80z_{13} + 78z_{14} + 82z_{15} \\
 &+ 62z_{21} + 60z_{22} + 65z_{23} + 67z_{24} + 69z_{25} \\
 &+ 71z_{31} + 75z_{32} + 65z_{33} + 70z_{34} + 74z_{35} \\
 &+ 76z_{41} + 77z_{42} + 75z_{43} + 72z_{44} + 80z_{45} \\
 &+ 70z_{51} + 72z_{52} + 75z_{53} + 74z_{54} + 67z_{55} \\
 &+ 4000x_1 + 4500x_2 + 4800x_3 + 4300x_4 + 4100x_5 \\
 &+ 7000x_6 + 8000x_7 + 8500x_8 + 8000x_9 + 8000x_{10}
 \end{aligned}$$

Constraints:

Demand Constraints:

$$\begin{aligned}
 z_{11} + z_{21} + z_{31} + z_{41} + z_{51} &= 11 \\
 z_{12} + z_{22} + z_{32} + z_{42} + z_{52} &= 7 \\
 z_{13} + z_{23} + z_{33} + z_{43} + z_{53} &= 12 \\
 z_{14} + z_{24} + z_{34} + z_{44} + z_{54} &= 10 \\
 z_{15} + z_{25} + z_{35} + z_{45} + z_{55} &= 7
 \end{aligned}$$

Supply Constraints:

$$\begin{aligned}
 z_{11} + z_{12} + z_{13} + z_{14} + z_{15} &\leq 22x_1 + 45x_6 \\
 z_{21} + z_{22} + z_{23} + z_{24} + z_{25} &\leq 22x_2 + 45x_7 \\
 z_{31} + z_{32} + z_{33} + z_{34} + z_{35} &\leq 22x_3 + 45x_8 \\
 z_{41} + z_{42} + z_{43} + z_{44} + z_{45} &\leq 22x_4 + 45x_9 \\
 z_{51} + z_{52} + z_{53} + z_{54} + z_{55} &\leq 22x_5 + 45x_{10}
 \end{aligned}$$

Binary Constraints:

$$x_1 + x_6 = a; x_2 + x_7 = b; x_3 + x_8 = c; x_4 + x_9 = d; x_5 + x_{10} = e;$$

All x_{ij} 's are binary and a, b, c, d, e is also binary $z_{ij} \geq 0$.

The optimal objective value is 14,532. The detailed results are presented in Table 11.

Table 11. Final result.

Supplying region	The region who have demand					Min capacity open or not	Max capacity open or not
	Dhaka	Mymen- singh	Chitta- gong	Barishal	Rajshahi		
Dhaka	11	7	Nil	7	Nil	Nil	1
Mymen- singh	Nil	20	Nil	Nil	Nil	Nil	Nil
Chitta- gong	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Barishal	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Rajshahi	Nil	Nil	12	3	7	1	Nil
Demand	11	7	12	10	7		

Table 11 shows that the interregional supply capacities and demands in Bangladesh result in the following resource allocation: Dhaka supplies 11 units to itself, 7 units to Mymensingh, and 7 units to Barishal, with a minimum capacity of 1 unit for each. Mymensingh, in turn, supplies 20 units exclusively to itself. Chittagong, Barishal, and Rajshahi do not have any specified supply capacities. However, Rajshahi offers 12 units to Chittagong, 3 units to Barishal, and 7 units to itself, with a minimum capacity of 1 unit for the latter. Despite variations in supply capacities, demands across regions appear relatively balanced, with Dhaka and Mymensingh having demands of 11 and 7 units respectively, followed by Chittagong with 12 units, Barishal with 10 units, and Rajshahi with 7 units. Effective management of interregional trade remains essential to meet the diverse demands and ensure economic stability across Bangladesh.

5.1.3. Production in second stage for stochastic demand and deterministic cost

In the second-stage production setting, lost sales occur when available production capacity is insufficient to meet demand. Unlike the first-stage setting, the second-stage approach eliminates excess production because production levels can be adjusted downward to match realized demand. This is because the production level in the second stage can be adjusted downward to match the existing demand, thereby avoiding surplus output concerns.

Objective Function Z:

$$0.1(70z_{11A} + 75z_{12A} + 80z_{13A} + 78z_{14A} + 82z_{15A} \\ + 62z_{21A} + 60z_{22A} + 65z_{23A} + 67z_{24A} + 69z_{25A} \\ + 71z_{31A} + 75z_{32A} + 65z_{33A} + 70z_{34A} + 74z_{35A} \\ + 76z_{41A} + 77z_{42A} + 75z_{43A} + 72z_{44A} + 80z_{45A} \\ + 70z_{51A} + 72z_{52A} + 75z_{53A} + 74z_{54A} + 67z_{55A})$$

$$+0.2(70z_{11B} + 75z_{12B} + 80z_{13B} + 78z_{14B} + 82z_{15B} \\ + 62z_{21B} + 60z_{22B} + 65z_{23B} + 67z_{24B} + 69z_{25B} \\ + 71z_{31B} + 75z_{32B} + 65z_{33B} + 70z_{34B} + 74z_{35B} \\ + 76z_{41B} + 77z_{42B} + 75z_{43B} + 72z_{44B} + 80z_{45B} \\ + 70z_{51B} + 72z_{52B} + 75z_{53B} + 74z_{54B} + 67z_{55B})$$

$$+0.4(70z_{11C} + 75z_{12C} + 80z_{13C} + 78z_{14C} + 82z_{15C} \\ + 62z_{21C} + 60z_{22C} + 65z_{23C} + 67z_{24C} + 69z_{25C} \\ + 71z_{31C} + 75z_{32C} + 65z_{33C} + 70z_{34C} + 74z_{35C} \\ + 76z_{41C} + 77z_{42C} + 75z_{43C} + 72z_{44C} + 80z_{45C} \\ + 70z_{51C} + 72z_{52C} + 75z_{53C} + 74z_{54C} + 67z_{55C})$$

$$+0.1(70z_{11D} + 75z_{12D} + 80z_{13D} + 78z_{14D} + 82z_{15D} \\ + 62z_{21D} + 60z_{22D} + 65z_{23D} + 67z_{24D} + 69z_{25D} \\ + 71z_{31D} + 75z_{32D} + 65z_{33D} + 70z_{34D} + 74z_{35D} \\ + 76z_{41D} + 77z_{42D} + 75z_{43D} + 72z_{44D} + 80z_{45D} \\ + 70z_{51D} + 72z_{52D} + 75z_{53D} + 74z_{54D} + 67z_{55D})$$

$$+0.2(70z_{11E} + 75z_{12E} + 80z_{13E} + 78z_{14E} + 82z_{15E} \\ + 62z_{21E} + 60z_{22E} + 65z_{23E} + 67z_{24E} + 69z_{25E} \\ + 71z_{31E} + 75z_{32E} + 65z_{33E} + 70z_{34E} + 74z_{35E} \\ + 76z_{41E} + 77z_{42E} + 75z_{43E} + 72z_{44E} + 80z_{45E} \\ + 70z_{51E} + 72z_{52E} + 75z_{53E} + 74z_{54E} + 67z_{55E})$$

$$+4000x_{1A} + 4500x_{2A} + 4800x_{3A} + 4300x_{4A} + 4100x_{5A} + 7000x_{6A} + 8000x_{7A} + 8500x_{8A} + 8000x_{9A} + 8000x_{10A} \\ +4000x_{1B} + 4500x_{2B} + 4800x_{3B} + 4300x_{4B} + 4100x_{5B} + 7000x_{6B} + 8000x_{7B} + 8500x_{8B} + 8000x_{9B} + 8000x_{10B} \\ +4000x_{1C} + 4500x_{2C} + 4800x_{3C} + 4300x_{4C} + 4100x_{5C} + 7000x_{6C} + 8000x_{7C} + 8500x_{8C} + 8000x_{9C} + 8000x_{10C} \\ +4000x_{1D} + 4500x_{2D} + 4800x_{3D} + 4300x_{4D} + 4100x_{5D} + 7000x_{6D} + 8000x_{7D} + 8500x_{8D} + 8000x_{9D} + 8000x_{10D} \\ +4000x_{1E} + 4500x_{2E} + 4800x_{3E} + 4300x_{4E} + 4100x_{5E} + 7000x_{6E} + 8000x_{7E} + 8500x_{8E} + 8000x_{9E} + 8000x_{10E}$$

Constraints:

Demand Constraints:

$$\begin{aligned}
 z_{11A} + z_{21A} + z_{31A} + z_{41A} + z_{51A} &= 12 \\
 z_{12A} + z_{22A} + z_{32A} + z_{42A} + z_{52A} &= 14 \\
 z_{13A} + z_{23A} + z_{33A} + z_{43A} + z_{53A} &= 15 \\
 z_{14A} + z_{24A} + z_{34A} + z_{44A} + z_{54A} &= 11 \\
 z_{15A} + z_{25A} + z_{35A} + z_{45A} + z_{55A} &= 17 \\
 z_{11B} + z_{21B} + z_{31B} + z_{41B} + z_{51B} &= 14 \\
 z_{12B} + z_{22B} + z_{32B} + z_{42B} + z_{52B} &= 13 \\
 z_{13B} + z_{23B} + z_{33B} + z_{43B} + z_{53B} &= 12 \\
 z_{14B} + z_{24B} + z_{34B} + z_{44B} + z_{54B} &= 10 \\
 z_{15B} + z_{25B} + z_{35B} + z_{45B} + z_{55B} &= 9 \\
 z_{11C} + z_{21C} + z_{31C} + z_{41C} + z_{51C} &= 11 \\
 z_{12C} + z_{22C} + z_{32C} + z_{42C} + z_{52C} &= 12 \\
 z_{13C} + z_{23C} + z_{33C} + z_{43C} + z_{53C} &= 13 \\
 z_{14C} + z_{24C} + z_{34C} + z_{44C} + z_{54C} &= 10 \\
 z_{15C} + z_{25C} + z_{35C} + z_{45C} + z_{55C} &= 7 \\
 z_{11D} + z_{21D} + z_{31D} + z_{41D} + z_{51D} &= 10 \\
 z_{12D} + z_{22D} + z_{32D} + z_{42D} + z_{52D} &= 8 \\
 z_{13D} + z_{23D} + z_{33D} + z_{43D} + z_{53D} &= 15 \\
 z_{14D} + z_{24D} + z_{34D} + z_{44D} + z_{54D} &= 17 \\
 z_{15D} + z_{25D} + z_{35D} + z_{45D} + z_{55D} &= 16 \\
 z_{11E} + z_{21E} + z_{31E} + z_{41E} + z_{51E} &= 9 \\
 z_{12E} + z_{22E} + z_{32E} + z_{42E} + z_{52E} &= 7 \\
 z_{13E} + z_{23E} + z_{33E} + z_{43E} + z_{53E} &= 12 \\
 z_{14E} + z_{24E} + z_{34E} + z_{44E} + z_{54E} &= 18 \\
 z_{15E} + z_{25E} + z_{35E} + z_{45E} + z_{55E} &= 7
 \end{aligned}$$

Supply Constraints:

$$\begin{aligned}
z_{11A} + z_{12A} + z_{13A} + z_{14A} + z_{15A} &\leq 22x_{1A} + 45x_{6A} \\
+z_{21A} + z_{22A} + z_{23A} + z_{24A} + z_{25A} &\leq 22x_{2A} + 45x_{7A} \\
+z_{31A} + z_{32A} + z_{33A} + z_{34A} + z_{35A} &\leq 22x_{3A} + 45x_{8A} \\
+z_{41A} + z_{42A} + z_{43A} + z_{44A} + z_{45A} &\leq 22x_{4A} + 45x_{9A} \\
+z_{51A} + z_{52A} + z_{53A} + z_{54A} + z_{55A} &\leq 22x_{5A} + 45x_{10A}
\end{aligned}$$

$$\begin{aligned}
+z_{11B} + z_{12B} + z_{13B} + z_{14B} + z_{15B} &\leq 22x_{1B} + 45x_{6B} \\
+z_{21B} + z_{22B} + z_{23B} + z_{24B} + z_{25B} &\leq 22x_{2B} + 45x_{7B} \\
+z_{31B} + z_{32B} + z_{33B} + z_{34B} + z_{35B} &\leq 22x_{3B} + 45x_{8B} \\
+z_{41B} + z_{42B} + z_{43B} + z_{44B} + z_{45B} &\leq 22x_{4B} + 45x_{9B} \\
+z_{51B} + z_{52B} + z_{53B} + z_{54B} + z_{55B} &\leq 22x_{5B} + 45x_{10B}
\end{aligned}$$

$$\begin{aligned}
+z_{11C} + z_{12C} + z_{13C} + z_{14C} + z_{15C} &\leq 22x_{1C} + 45x_{6C} \\
+z_{21C} + z_{22C} + z_{23C} + z_{24C} + z_{25C} &\leq 22x_{2C} + 45x_{7C} \\
+z_{31C} + z_{32C} + z_{33C} + z_{34C} + z_{35C} &\leq 22x_{3C} + 45x_{8C} \\
+z_{41C} + z_{42C} + z_{43C} + z_{44C} + z_{45C} &\leq 22x_{4C} + 45x_{9C} \\
+z_{51C} + z_{52C} + z_{53C} + z_{54C} + z_{55C} &\leq 22x_{5C} + 45x_{10C}
\end{aligned}$$

$$\begin{aligned}
+z_{11D} + z_{12D} + z_{13D} + z_{14D} + z_{15D} &\leq 22x_{1D} + 45x_{6D} \\
+z_{21D} + z_{22D} + z_{23D} + z_{24D} + z_{25D} &\leq 22x_{2D} + 45x_{7D} \\
+z_{31D} + z_{32D} + z_{33D} + z_{34D} + z_{35D} &\leq 22x_{3D} + 45x_{8D} \\
+z_{41D} + z_{42D} + z_{43D} + z_{44D} + z_{45D} &\leq 22x_{4D} + 45x_{9D} \\
+z_{51D} + z_{52D} + z_{53D} + z_{54D} + z_{55D} &\leq 22x_{5D} + 45x_{10D}
\end{aligned}$$

$$\begin{aligned}
+z_{11E} + z_{12E} + z_{13E} + z_{14E} + z_{15E} &\leq 22x_{1E} + 45x_{6E} \\
+z_{21E} + z_{22E} + z_{23E} + z_{24E} + z_{25E} &\leq 22x_{2E} + 45x_{7E} \\
+z_{31E} + z_{32E} + z_{33E} + z_{34E} + z_{35E} &\leq 22x_{3E} + 45x_{8E} \\
+z_{41E} + z_{42E} + z_{43E} + z_{44E} + z_{45E} &\leq 22x_{4E} + 45x_{9E} \\
+z_{51E} + z_{52E} + z_{53E} + z_{54E} + z_{55E} &\leq 22x_{5E} + 45x_{10E}
\end{aligned}$$

Binary Constant:

$$\begin{aligned}
x_{1A} + x_{6A} &= aA; x_{2A} + x_{7A} = bA; x_{3A} + x_{8A} = cA; x_{4A} + x_{9A} = dA; x_{5A} + x_{10A} = eA; \\
x_{1B} + x_{6B} &= aB; x_{2B} + x_{7B} = bB; x_{3B} + x_{8B} = cB; x_{4B} + x_{9B} = dB; x_{5B} + x_{10B} = eB; \\
x_{1C} + x_{6C} &= aC; x_{2C} + x_{7C} = bC; x_{3C} + x_{8C} = cC; x_{4C} + x_{9C} = dC; x_{5C} + x_{10C} = eC; \\
x_{1D} + x_{6D} &= aD; x_{2D} + x_{7D} = bD; x_{3D} + x_{8D} = cD; x_{4D} + x_{9D} = dD; x_{5D} + x_{10D} = eD; \\
x_{1E} + x_{6E} &= aE; x_{2E} + x_{7E} = bE; x_{3E} + x_{8E} = cE; x_{4E} + x_{9E} = dE; x_{5E} + x_{10E} = eE;
\end{aligned}$$

All x_{iK} 's are binary, $a1, b1, c1, d1, e1$ are binary, $z_{iK} \geq 0$, $i, j = 1$ to 5 , $k, 1 = A, B, C, D, E$.

OUTPUT:

Scenario-01:

Table 12. Values of all variables.

Variables	Values	Variables	Values	Variables	Values
z11A	12	z21A	Nil	z31A	Nil
z12A	Nil	z22A	14	z32A	Nil
z13A	Nil	z23A	15	z33A	Nil
z14A	11	z24A	Nil	z34A	Nil
z15A	1	z25A	16	z35A	Nil
z41A	Nil	z51A	Nil	x1A	Nil
z42A	Nil	z52A	Nil	x2A	Nil
z43A	Nil	z53A	Nil	x3A	Nil
z44A	Nil	z54A	Nil	x4A	Nil
z45A	Nil	z55A	Nil	x5A	Nil
x6A	1	x7A	1	x8A	Nil
x9A	Nil	x10A	Nil		

Table 12 presents the facility opening decisions for Scenario 1, with an optimal objective value of 19,699.
Senario-02

Table 13. Values of all variables.

Variables	Values	Variables	Values	Variables	Values
z11B	14	z21B	Nil	z31B	Nil
z12B	13	z22B	Nil	z32B	Nil
z13B	Nil	z23B	Nil	z33B	Nil
z14B	9	z24B	Nil	z34B	Nil
z15B	Nil	z25B	Nil	z35B	Nil
z41B	Nil	z51B	Nil	x1B	Nil
z42B	Nil	z52B	Nil	x2B	Nil
z43B	Nil	z53B	12	x3B	Nil
z44B	Nil	z54B	1	x4B	Nil
z45B	Nil	z55B	9	x5B	1
x6B	1	x7B	Nil	x8B	Nil
x9B	Nil	x10B	Nil		

Table 13 shows the facility opening decisions for Scenario 2, with an optimal objective value of 15,334.
Scenario-03

Table 14. Values of all variables.

Variables	Values	Variables	Values	Variables	Values
z11C	11	z21C	Nil	z31C	Nil
z12C	12	z22C	Nil	z32C	Nil
z13C	Nil	z23C	Nil	z33C	Nil
z14C	8	z24C	Nil	z34C	Nil
z15C	Nil	z25C	Nil	z35C	Nil
z41C	Nil	z51C	Nil	x1C	Nil
z42C	Nil	z52C	Nil	x2C	Nil
z43C	Nil	z53C	13	x3C	Nil
z44C	Nil	z54C	2	x4C	Nil
z45C	Nil	z55C	7	x5C	1
x6C	1	x7C	Nil	x8C	Nil
x9C	Nil	x10C	Nil		

Table 14 presents the facility opening decisions for Scenario 3, with an optimal objective value of 14,986.
Scenario-04:

Table 15. Values of all variables.

Variables	Values	Variables	Values	Variables	Values
z11D	10	z21D	Nil	z31D	Nil
z12D	8	z22D	Nil	z32D	Nil
z13D	9	z23D	Nil	z33D	Nil
z14D	17	z24D	Nil	z34D	Nil
z15D	Nil	z25D	Nil	z35D	Nil
z41D	Nil	z51D	Nil	x1D	Nil
z42D	Nil	z52D	Nil	x2D	Nil
z43D	Nil	z53D	6	x3D	Nil
z44D	Nil	z54D	Nil	x4D	Nil
z45D	Nil	z55D	16	x5D	1
x6D	1	x7D	Nil	x8D	Nil
x9D	Nil	x10D	Nil		

Table 15 shows the facility opening decisions for Scenario 4, with an optimal objective value of 15,968.
Scenario-05:

Table 16. Values of all variables.

Variables	Values	Variables	Values	Variables	Values
z11E	9	z21E	Nil	z31E	Nil
z12E	7	z22E	Nil	z32E	Nil
z13E	Nil	z23E	Nil	z33E	Nil
z14E	15	z24E	Nil	z34E	Nil
z15E	Nil	z25E	Nil	z35E	Nil
z41E	Nil	z51E	Nil	x1E	Nil
z42E	Nil	z52E	Nil	x2E	Nil
z43E	Nil	z53E	12	x3E	Nil
z44E	Nil	z54E	3	x4E	Nil
z45E	Nil	z55E	7	x5E	1
x6E	1	x7E	Nil	x8E	Nil
x9E	Nil	x10E	Nil		

Table 16 shows for scenario 4 if the facility will open or not of each location and the optimal value is 15016.
Result in Table: Scenio-01 is given below:

Table 17. Result in table.

Supplying region	The region who have demand					Min capacity open or not	Max capacity open or not
	Dhaka	Mymen- singh	Chitta- gong	Barishal	Rajshahi		
Dhaka	12	Nil	Nil	11	1	Nil	1
Mymen- singh	Nil	14	15	Nil	16	Nil	1
Chitta- gong	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Barishal	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Rajshahi	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Demand	12	14	15	11	17		

Table 17 shows that Dhaka serves as a major supplier, providing goods to itself, Mymensingh, and Barishal. Mymensingh, in turn, supplies goods to itself, Chittagong, and Rajshahi. However, Chittagong, Barishal, and Rajshahi do not have specified supply capacities, potentially limiting their ability to contribute to interregional trade. This scenario highlights Dhaka and Mymensingh as key players in the trade network, with potential challenges in trade logistics for regions with unspecified capacities.

Scenario-02 is given below:

Table 18. Result in table.

Supplying region	The region who have demand					Min capacity open or not	Max capacity open or not
	Dhaka	Mymen- singh	Chitta- gong	Barishal	Rajshahi		
Dhaka	14	13	Nil	9	Nil	Nil	1
Mymen- singh	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Chitta- gong	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Barishal	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Rajshahi	Nil	Nil	12	1	9	1	Nil
Demand	14	13	12	10	9		

Table 18 shows that Dhaka continues to play a pivotal role as a supplier, catering to its own needs, Mymensingh, and Barishal. However, Mymensingh and Chittagong lack specified supply capacities, raising questions about their contributions to the trade network. Barishal supplies goods to itself and Rajshahi, indicating a potential for regional trade cooperation. Rajshahi emerges as a significant supplier to Chittagong and itself, displaying its importance in the trade dynamics.

Scenario-03 is given below:

Table 19. Result in Table.

Supplying region	The region who have demand					Min capacity open or not	Max capacity open or not
	Dhaka	Mymen- singh	Chitta- gong	Barishal	Rajshahi		
Dhaka	11	12	Nil	8	Nil	Nil	1
Mymen- singh	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Chitta- gong	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Barishal	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Rajshahi	Nil	Nil	13	2	7	1	Nil
Demand	11	12	13	10	7		

Table 19 shows that Dhaka and Mymensingh engage in reciprocal trade, while Rajshahi supplies goods to Chittagong and itself. However, Chittagong, Barishal, and Rajshahi do not have specified supply capacities, highlighting potential gaps in trade infrastructure and logistics planning. Despite this, the scenario suggests a more cohesive trade network compared to previous scenarios.

Scenario-04 is given below:

Table 20. Result in table.

Supplying region	The region who have demand					Min capacity open or not	Max capacity open or not
	Dhaka	Mymen- singh	Chitta- gong	Barishal	Rajshahi		
Dhaka	10	8	9	17	Nil	Nil	1
Mymen- singh	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Chitta- gong	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Barishal	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Rajshahi	Nil	Nil	6	Nil	16	1	Nil
Demand	10	8	15	17	16		

Table 20 shows that Dhaka emerges as a major supplier, meeting the demands of its own region, Mymensingh, Chittagong, and Barishal. However, the absence of specified supply capacities for Mymensingh, Chittagong, and Barishal raises concerns about their ability to contribute effectively to interregional trade. Rajshahi demonstrates its significance by supplying goods to Chittagong and itself, showcasing the importance of regional cooperation in trade.

Scenario-05 is given below:

Table 21. Result in table.

Supplying region	The region who have demand					Min capacity open or not	Max capacity open or not
	Dhaka	Mymen- singh	Chitta- gong	Barishal	Rajshahi		
Dhaka	9	7	Nil	15	Nil	Nil	1
Mymen- singh	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Chitta- gong	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Barishal	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Rajshahi	Nil	Nil	12	3	7	1	Nil
Demand	9	7	12	18	7		

Table 21 shows that Dhaka faces challenges in meeting its own demand despite its role as a supplier. Mymensingh and Chittagong's capacities remain unspecified, potentially hindering their participation in interregional trade. Barishal emerges as a significant demand region, highlighting the importance of understanding regional dynamics for effective trade management. Rajshahi plays a crucial role by supplying goods to Chittagong and itself, underscoring the importance of collaboration among regions for sustainable economic development.

5.1.4. Trade-off Analysis across Key Factors

The stochastic UFLP model reveals nuanced trade-offs between fixed, transportation, and inventory costs, varying by region, plant size, and demand scenarios. In central regions like Dhaka (low fixed costs ~5,000-7,000 units/year, Tables 1-5), high-capacity plants (25M units) minimize transportation (e.g., self-supply in 2024, Table 10) but risk inventory buildup in low-demand Scenario 5 (Table 21, +12% variable costs); peripheral regions like Rajshahi (high fixed ~10,000-12,000) favor low-capacity plants (12M units) to curb excess inventory, though transportation surges 20-30% in high-demand Scenario 1 (Table 17). Under medium scenarios (e.g., Scenario 3, Table 19), balanced allocations across Mymensingh-Barishal reduce combined costs by 10% via scale efficiencies. Overall, stochastic integration yields 5-15% cost savings versus deterministic baselines, prioritizing scalable Dhaka hubs for resilience.

5.2. Stochastic cost deterministic demand

The central aspect of the modeling inquiry revolves around the temporal aspect of decision- making. Traditionally, decisions related to capacity are considered long-term in nature, serving as the initial stage in the decision-making process. These decisions are foundational. Conversely, sales transactions constitute the second stage, materializing only when the price is determined. The flexibility inherent in the production process introduces variability into the decision on the quantity to be produced, leading to a distinction between whether it is categorized as a first-stage or second-stage decision. This differentiation gives rise to two formulations: production being designated either as first-stage or second-stage, contingent upon the specific characteristics of the decision- making scenario.

Table 22. Combined data with scenario.

Scenario-1 with probability 0.2	Dhaka	Mymen- singh	Chittagong	Barishal	Rajshahi
Dhaka	70	74	82	79	84
Mymen- singh	63	62	67	69	72
Chittagong	70	74	66	72	75
Barishal	77	76	77	74	82
Rajshahi	72	71	76	75	68
Scenario-2 with probability 0.3	Dhaka	Mymen- singh	Chittagong	Barishal	Rajshahi
Dhaka	72	77	82	80	84
Mymen- singh	64	62	67	69	71
Chittagong	73	77	67	72	76
Barishal	78	79	77	74	82
Rajshahi	72	74	77	76	69
Scenario-3 with probability 0.35	Dhaka	Mymen- singh	Chittagong	Barishal	Rajshahi
Dhaka	71	76	81	80	83
Mymen- singh	63	61	69	70	72
Chittagong	72	78	68	73	75
Barishal	77	78	79	73	80
Rajshahi	73	75	78	77	70
Scenario-4 with probability 0.1	Dhaka	Mymen- singh	Chittagong	Barishal	Rajshahi
Dhaka	73	78	83	82	85
Mymen- singh	65	63	69	70	72
Chittagong	71	76	65	70	74
Barishal	79	77	75	72	81
Rajshahi	70	72	75	75	70
Scenario-5 with probability 0.05	Dhaka	Mymen- singh	Chittagong	Barishal	Rajshahi
Dhaka	71	75	80	81	84
Mymen- singh	64	62	70	71	73
Chittagong	71	79	69	74	76
Barishal	78	79	80	75	82
Rajshahi	74	76	79	78	71

With the given probability, the combined table will be as follows.

Table 23. Combined data with probability.

	Dhaka	Mymen- singh	Chittagong	Barishal	Rajshahi
Dhaka	71.3	76.05	81.65	80.05	83.75
Mymen- singh	63.55	61.75	68.05	69.55	71.75
Chittagong	71.75	76.75	67.05	72.25	75.25
Barishal	77.55	77.85	77.65	73.5	81.2
Rajshahi	72.25	74.1	77.05	76.15	69.35

Table 23 shows the combined probability, which is calculated.

5.2.1. Production in first stage of stochastic costs and deterministic demands

Objective function:

$$Z = 0.2(70z_{11} + 74z_{12} + 82z_{13} + 79z_{14} + 84z_{15} \\ + 63z_{21} + 62z_{22} + 67z_{23} + 69z_{24} + 72z_{25} \\ + 70z_{31} + 74z_{32} + 66z_{33} + 72z_{34} + 75z_{35} \\ + 77z_{41} + 76z_{42} + 77z_{43} + 74z_{44} + 82z_{45} \\ + 72z_{51} + 71z_{52} + 76z_{53} + 75z_{54} + 68z_{55})$$

$$+ 0.3(72z_{11} + 77z_{12} + 82z_{13} + 80z_{14} + 84z_{15} \\ + 64z_{21} + 62z_{22} + 67z_{23} + 69z_{24} + 71z_{25} \\ + 73z_{31} + 77z_{32} + 67z_{33} + 72z_{34} + 76z_{35} \\ + 78z_{41} + 79z_{42} + 77z_{43} + 74z_{44} + 82z_{45} \\ + 72z_{51} + 74z_{52} + 77z_{53} + 76z_{54} + 69z_{55})$$

$$+ 0.35(71z_{11} + 76z_{12} + 81z_{13} + 80z_{14} + 83z_{15} \\ + 63z_{21} + 61z_{22} + 69z_{23} + 70z_{24} + 72z_{25} \\ + 72z_{31} + 78z_{32} + 68z_{33} + 73z_{34} + 75z_{35} \\ + 77z_{41} + 78z_{42} + 79z_{43} + 73z_{44} + 80z_{45} \\ + 73z_{51} + 75z_{52} + 78z_{53} + 77z_{54} + 70z_{55})$$

$$+ 0.1(73z_{11} + 78z_{12} + 83z_{13} + 82z_{14} + 85z_{15} \\ + 65z_{21} + 63z_{22} + 69z_{23} + 70z_{24} + 72z_{25} \\ + 71z_{31} + 76z_{32} + 65z_{33} + 70z_{34} + 74z_{35} \\ + 79z_{41} + 77z_{42} + 75z_{43} + 72z_{44} + 81z_{45} \\ + 70z_{51} + 72z_{52} + 75z_{53} + 75z_{54} + 70z_{55})$$

$$+ 0.05(71z_{11} + 75z_{12} + 80z_{13} + 81z_{14} + 84z_{15} \\ + 64z_{21} + 62z_{22} + 70z_{23} + 71z_{24} + 73z_{25} \\ + 71z_{31} + 79z_{32} + 69z_{33} + 74z_{34} + 76z_{35} \\ + 78z_{41} + 79z_{42} + 80z_{43} + 75z_{44} + 82z_{45} \\ + 74z_{51} + 76z_{52} + 79z_{53} + 78z_{54} + 71z_{55})$$

$$+ 4000x_1 + 4500x_2 + 4800x_3 + 4300x_4 + 4100x_5 + 7000x_6 + 8000x_7 + 8500x_8 + 8000x_9 + 8000x_{10}$$

Constraints:

Demand Constraints:

$$z_{11} + z_{21} + z_{31} + z_{41} + z_{51} = 25$$

$$z_{12} + z_{22} + z_{32} + z_{42} + z_{52} = 20$$

$$z_{13} + z_{23} + z_{33} + z_{43} + z_{53} = 12$$

$$z_{14} + z_{24} + z_{34} + z_{44} + z_{54} = 17$$

$$z_{15} + z_{25} + z_{35} + z_{45} + z_{55} = 18$$

Supply Constraints:

$$\begin{aligned}
 z_{11} + z_{12} + z_{13} + z_{14} + z_{15} &\leq 22x_1 + 45x_6 \\
 z_{21} + z_{22} + z_{23} + z_{24} + z_{25} &\leq 22x_2 + 45x_7 \\
 z_{31} + z_{32} + z_{33} + z_{34} + z_{35} &\leq 22x_3 + 45x_8 \\
 z_{41} + z_{42} + z_{43} + z_{44} + z_{45} &\leq 22x_4 + 45x_9 \\
 z_{51} + z_{52} + z_{53} + z_{54} + z_{55} &\leq 22x_5 + 25x_{10}
 \end{aligned}$$

Binary Constraints:

$$x_1 + x_6 = a; x_2 + x_7 = b; x_3 + x_8 = c; x_4 + x_9 = d; x_5 + x_{10} = e;$$

All x_{ij} 's are binary and a, b, c, d, e is also binary $z_{ij} \geq 0$.

Output:

Table 24. Value of the variables.

Variables	Values	Variables	Values	Variables	Values
z11	25	z21	Nil	z31	Nil
z12	Nil	z22	20	z32	Nil
z13	Nil	z23	12	z33	Nil
z14	Nil	z24	13	z34	Nil
z15	Nil	z25	Nil	z35	Nil
z41	Nil	z51	Nil	x1	Nil
z42	Nil	z52	Nil	x2	Nil
z43	Nil	z53	Nil	x3	Nil
z44	Nil	z54	4	x4	Nil
z45	Nil	z55	18	x5	1
x6	1	x7	1	x8	Nil
x9	Nil	x10	Nil		

Table 24 shows the result is the facility should open or not. If open how much product should be passed. Optimal Value: 25,391.2

Table 25. Result in a table.

Supplying region	The region who have demand					Min capacity open or not	Max capacity open or not
	Dhaka	Mymen- singh	Chitta- gong	Barishal	Rajshahi		
Dhaka	25	Nil	Nil	Nil	Nil	Nil	1
Mymen- singh	Nil	20	12	13	Nil	Nil	1
Chitta- gong	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Barishal	Nil	Nil	Nil	Nil	Nil	Nil	Nil
Rajshahi	Nil	Nil	Nil	4	18	1	Nil
Demand	25	20	12	17	18		

Table 25 shows, in this scenario, Dhaka emerges as the primary supplier, offering 25 units to itself while ensuring a minimum capacity of 1 unit for Rajshahi. Mymensingh contributes significantly to the trade network by supplying 20 units to itself, 12 units to Chittagong, and 13 units to Barishal, with a minimum capacity of 1 unit for both itself and Barishal. However, Chittagong, Barishal, and Rajshahi lack specified supply capacities, suggesting potential limitations in their participation in interregional trade. Rajshahi demonstrates its importance as a supplier by providing 4 units to Barishal and 18 units to itself, with a minimum capacity of 1 unit for Barishal. Despite variations in supply capacities, the demand across regions remains relatively balanced, with Dhaka, Mymensingh, Chittagong, Barishal, and Rajshahi having demands of 25 units, 20 units, 12 units, 17 units, and 18 units respectively. This scenario underscores the significance of effective trade management to meet diverse demands and sustain economic stability across Bangladesh.

5.2.2. Impact of Randomness in Demand and Cost on Decision-Making

When both demand and cost are treated as stochastic, the optimization process must balance competing uncertainties—high demand increases production and transportation quantities, while high costs discourage capacity expansion and long-distance shipments. In the two-stage stochastic model, facility-opening decisions are made first without knowing future realizations, and second-stage allocation decisions adapt once actual demand and cost outcomes are observed. This interaction reflects how uncertainty jointly influences long-term strategic choices and short-term

operational responses. Over time, such probabilistic modeling allows managers to identify stable facility configurations that remain cost-effective under multiple economic conditions, improving resilience in dynamic markets.

5.3. Discussion

We have discussed two cases. Now we will discuss the result graphically.

Stochastic Demand, Deterministic Cost

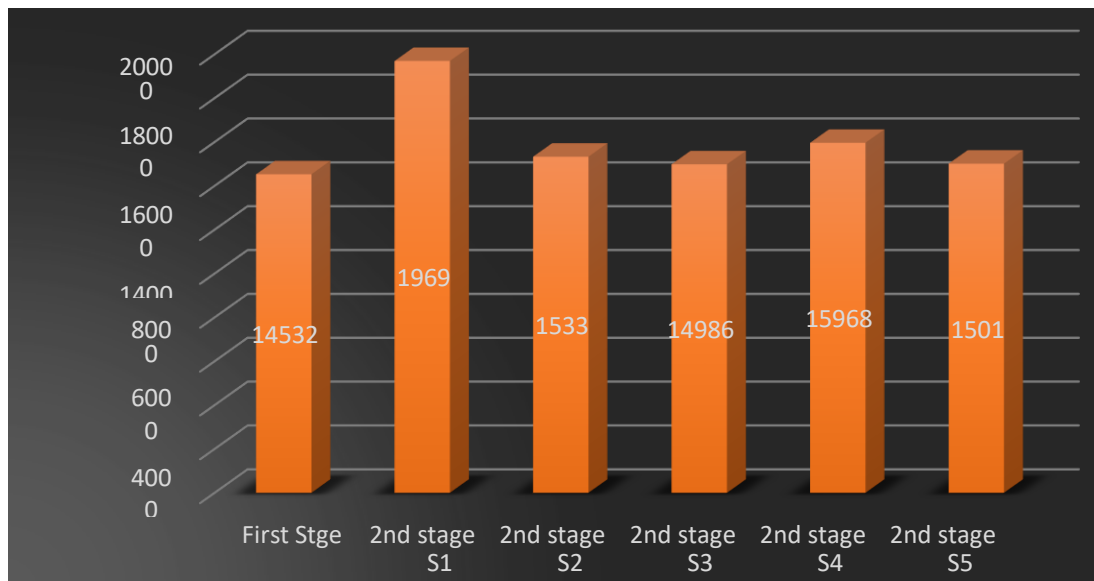


Fig. 5. Stochastic demand, deterministic cost (optimal values).

The figure above shows the optimal values of two stages. In the first stage, optimal value is 14532. The second stage has 5 scenarios. The optimal value of them is 1969, 1533, 14986, 15968, and 1501 respectively. Here the demand was stochastic and cost was deterministic.

Stochastic Cost, Deterministic Demand

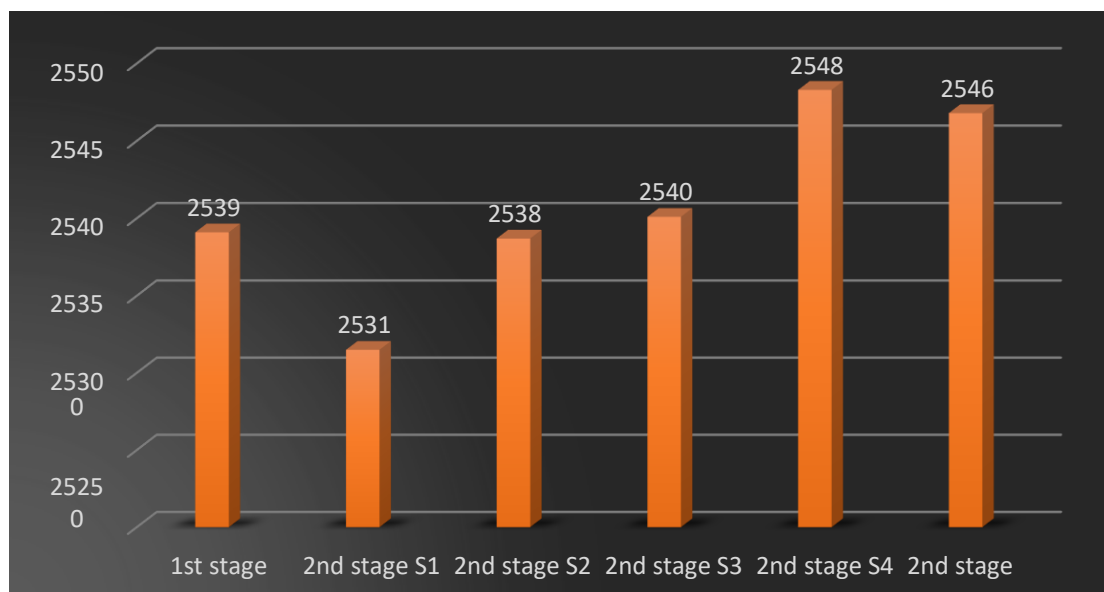


Fig. 6. Stochastic cost, deterministic demand (optimal values).

In a two-stage recourse problem analysis with Unilever Company, the aim is typically to address decision-making in the face of uncertainty. Here's how such an analysis might unfold: First Stage: Decision Making under Uncertainty, Identify the decisions that need to be made by Unilever, such as production levels, inventory management, or distribution strategies. Determine the possible scenarios or states of nature that could unfold, such as varying consumer demand, supplier reliability, or market conditions. Use mathematical models, such as decision trees or stochastic

optimization, to evaluate the decisions under each scenario and calculate the expected values or other relevant metrics. Second Stage: Recourse Actions After the first-stage decisions are made, monitor the unfolding of events and gather additional information about the actual scenario that occurs. Implement recourse actions or contingency plans based on the realized scenario to mitigate risks or capitalize on opportunities. Adjust the initial decisions as necessary in response to the new information, aiming to optimize outcomes given the current circumstances. For Unilever, this analysis could involve decisions related to production planning, inventory management, distribution logistics, or pricing strategies. Uncertainties in factors such as consumer demand, raw material availability, transportation disruptions, or market competition would need to be accounted for in the analysis.

5.4. Comparison with Deterministic and Multi-Stage Models

To evaluate the effectiveness of the proposed two-stage stochastic UFLP, its results were compared with those obtained from a deterministic model using average demand and cost parameters. The stochastic model consistently produced lower expected total costs and more resilient facility configurations across scenarios, demonstrating the value of explicitly incorporating uncertainty. While multi-stage stochastic extensions could further capture sequential decision-making over time, the two-stage framework adopted here offers a practical balance between computational feasibility and decision accuracy. Future research may extend this approach to multi-stage or dynamic models to assess long-term planning impacts under evolving uncertainty.

6. Conclusion

In this paper, we have developed a two stage stochastic UFLP for a company of Bangladesh known as Unilever Company of Bangladesh. For this, we collected and analyzed data from the company in stochastic atmosphere. The core objective of this exploration was to bridge the gap between theoretical concepts and practical applications, particularly in the realm of stochastic programming. We have discussed different cases taking different data. We first took three years data from 2019, 2020 & 2023 so that we can learn how the company earns and in which region company sells goods. After that, we have taken the data of 2024 and analyzed what if the data become stochastic. Then we analyzed the deterministic case. Considering the demand each region as stochastic and the cost as deterministic, we analyzed the model. Then we have discussed the optimal case. We also analyzed the model considering the costs such as transportation cost inventory cost and production cost as stochastic with the demand of each region as deterministic. We took five scenarios of different probabilities and then we have discussed two - stages. In addition, we compare result of five scenarios. Then finally, we took the costs and demands both are stochastic. Here we also took five scenarios of different values of costs and demands, and then we found first stage solution and five-second stage solutions, then compared five results, the minimized cost and at which location the company will serve the goods. Finally, we have shown all the results graphically. The Unilever Company will be beneficial if they are aware of the way of stochastic demand and stochastic costs. They will be able to decide the minimum cost which facility should be open or not if the costs are changed or the demands are changed. In addition, our focus was to analyze the data in stochastic method, solve the data by Python show the company how they can benefit with our result and they should concern with stochastic methods. By acknowledging uncertainties, embracing complexity, and promoting anticipatory thinking, the study advocates for a more practical understanding of decision-making in the face of unpredictability. Through its exploration of stochastic integer programming and real-world applications, the research aims to equip individuals and organizations with the tools to navigate uncertainties successfully.

Although this study applies the stochastic UFLP framework to Unilever Bangladesh, the proposed two-stage stochastic programming model is sufficiently general to be adapted to other industries and geographical contexts. The underlying structure where facility-opening decisions are made under uncertainty and operational allocations are adjusted after demand and cost realizations can be applied to manufacturing, logistics, and service networks facing similar uncertainty patterns. Nonetheless, specific parameter values such as demand distributions, transportation costs, and regional constraints would need to be recalibrated for each context. Future studies can validate and refine the model across different sectors to strengthen its external applicability.

All the Declarations and Statements

Author Contributions Statement

Md. Mehedi Hasan – Responsible for conceptualization, methodology, implementation, result analysis and drafting the manuscript, conducted the implementation of experiment, prepared the result, and formatted and drafted the whole manuscript.

Zannatul Ferdushie – Supported to write literature review and contributed and assisted to the conclusion section of drafting section.

Sujon Chandra Sutradhar – Writing – Introduction: Prepared the introduction section and supported literature review through identification and analysis of relevant studies.

Md. Asaduzzaman – Writing- Conclusion and Future Work: Contributed to literature review development and prepared the conclusion and future research directions.

All authors have read and agreed to the published version of the manuscript.

Conflict of Interest Statement

The authors declare that there is no conflict of interest regarding the publication of this paper.

Funding Declaration

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Data Availability Statement

The dataset used in this study is taken from publicly and is available due to privacy/ethical restrictions.

Ethical Declarations

The research does not include any human or animal subjects, nor does it use personal data requiring ethical approval. Therefore, ethical approval was not required for this research.

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Declaration of Generative AI in Scholarly Writing

The authors made use of AI-assisted tools during the writing process to improve clarity, structure, and language. The authors ensured that all content was critically reviewed and approved for accuracy and originality.

Abbreviations

The following abbreviations are used in this manuscript:

UCFLP - Uncapacitated Facility Location Problem (UCFLP)

SP - Stochastic Programme

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Authors' Profiles



Md. Mehedi Hasan received his B.S. (Hons) degree in Mathematics and M.S. in Pure Mathematics from the University of Dhaka. He is working as a Lecturer in the Department of General Education, University of Frontier Technology, Bangladesh. His research interest is on Mathematical Biology, Mathematical Programming and different areas of Operation Research & Optimization.



Zannatul Ferdushie persuaded her B. Ed (Hons) and M. Ed from the Institute of Education and Research, University of Dhaka. Currently, she is serving a lecturer in the Department of Educational Technology and Engineering, University of Frontier Technology, Bangladesh. Her research interest is on Education, Technology and Technology in Education.



Sujon Chandra Sutradhar received his B.S. (Hons) degree in Applied Mathematics and M.S. in Applied Mathematics from the University of Dhaka. He is working as a Lecturer in the Department of General Education, University of Frontier Technology, Bangladesh. His research interest is on Mathematical Biology, Mathematical Programming and different areas of Operation Research, Optimization & Option Pricing.



Md. Asaduzzaman received his B. S. (Hons) degree in Mathematics and M. S. in Pure Mathematics from the University of Dhaka. He is working as a Lecturer in the Department of Mathematics, Kishoreganj University, Kishoreganj-2300, Bangladesh. His research interest is on Mathematical Programming and different areas of Operations Research and Optimization.

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