

Exploring Approaches for Curve Similarity: A Comprehensive Review

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Abstract: Curve similarity plays a crucial role in various domains where comparing functional or dynamic shapes is essential, including bioassay analysis, trajectory studies, spectroscopy, medical signal interpretation, and functional genomics. Despite its broad impact, research on curve similarity methods remains fragmented across statistical, computational geometry, and signal processing communities, leading to a lack of unified terminology and systematic comparison. To address this gap, this study adopts a structured literature review methodology, in which relevant studies are identified through a comprehensive search of major academic databases and selected based on predefined inclusion criteria, including peer-reviewed publications focusing on similarity measures for curves and time series. The review systematically examines mathematical and statistical approaches to curve similarity, focusing on their theoretical foundations, statistical properties, and practical applications. The selected methods are categorized into five groups: distance-based, alignment-based, topology-oriented, statistical and hypothesis testing, and learning-based approaches. For each category, key aspects such as mathematical formulation, invariance properties, robustness to sampling variability, and computational characteristics are analyzed. In addition, application domains, method comparisons, and common limitations are discussed, along with available software tools that support curve similarity analysis. By providing a structured and methodologically grounded synthesis, this review assists researchers in selecting appropriate techniques and highlights potential directions for developing more robust and scalable similarity assessment frameworks.

Index Terms: Curve Similarity, Functional Data Analysis, Dynamic Time Warping, Topological Data Analysis, Statistical Equivalence Testing.

1. Introduction

Curve similarity is defined as a measure of how closely two curves resemble one another either visually or mathematically, which might indicate patterns, trends, and correlations in data. Curves arise naturally in many scientific and engineering domains, representing relationships such as dose-response shapes, spectral patterns, biomedical waveforms, and movement trajectories. In these contexts, assessing curve similarity is essential for tasks including classification, clustering, alignment, quality control, and statistical inference [1-7]. Table 1 summarizes representative application areas, illustrating the diversity of curve types and the corresponding analytical goals.

Work on curve similarity is distributed across multiple research communities with differing perspectives. In statistics, similarity may be defined via functional distances or hypothesis testing (e.g., assessing parallelism in bioassays) [13,21,22]. In computational geometry, similarity is framed as the geometric distance between parameterized paths (e.g., Fréchet or Hausdorff distances) [14,20]. In time series analysis, similarity is frequently addressed through alignment or warping techniques such as Dynamic Time Warping (DTW) [8,9,24].

Table 1. Representative Application Domains Involving Curve Similarity.

Application Domain	Type of Curves / Data	Purpose of Similarity Measurement	Relevant Method Groups
Speech & Audio Analysis [8, 9]	Time-varying acoustic signals	Matching phonemes/speakers; aligning speech signals	Warping/Alignment (DTW, CTW)
Human Motion & Activity Recognition [10, 11]	Multi-channel motion trajectories	Identifying actions; synchronizing motion sequences	Warping; Manifold-based alignment
Biomedical & Clinical Data [12, 13]	EEG/ECG waveforms, dose-response curves	Diagnosis, classification, assay equivalence	Statistical testing; Topological tools
Shape & Geometry Processing [14, 15]	2D/3D geometric curves	Shape matching; geometric comparison	Distance metrics (Fréchet, Hausdorff); Topology
Computer Vision & Graphics [10, 16]	Object contours, gesture paths	Tracking, object recognition, motion alignment	Warping; Geometric distance models
Spectroscopy [49]	Spectral intensity curves	Pattern matching; clustering; trend assessment	Functional & distance-based metrics
Environmental & Climate Studies [17]	Environmental time series	Trend comparison; similarity of temporal evolution	Functional distances; Alignment-based
Network & Pathway Analysis [18,19]	Biological pathways, graph-derived curves	Structural comparison and canonical representation	Topological (TDA, Reeb graphs)
Finance (stock Market Analysis) [50, 51]	Financial time series (stock prices, indices, returns)	Pattern matching, trend prediction	Alignment; Distance-based metrics
Robotics [9] & Trajectory Planning [20]	Motion paths and spatial trajectories	Path matching; trajectory evaluation	Fréchet; DTW; manifold alignment

To clarify these perspectives, Fig. 1 provides a conceptual illustration of the three dominant methodological viewpoints found in the literature: statistical, geometric, and alignment-based. The overlap between these viewpoints reflects hybrid approaches that combine multiple modeling components (e.g., elastic shape analysis that integrates geometric representations with alignment).

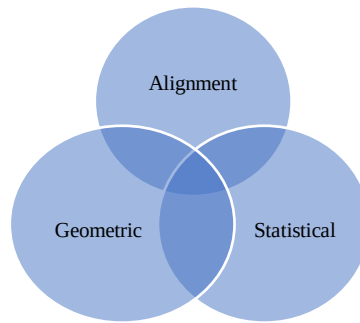


Fig. 1. Conceptual overview of existing methodological perspectives for curve similarity.

The Venn-style diagram (Fig. 1) depicts three major perspectives: (i) statistical methods that compare curves via functional distances or model-based tests, (ii) geometric methods that measure distances between spatial trajectories, and (iii) alignment-based methods that handle temporal or phase variability via warping. Overlapping regions represent hybrid approaches.

To complement the conceptual illustration in Fig. 1, a more formal interpretation of the relationships between statistical, geometric, and alignment-based approaches can be established. Let a curve be represented as a function $f(t)$ defined on a domain $t \in [0,1]$. Statistical methods typically measure similarity using functional distances (e.g., L^2 norms) or model-based inference, focusing on variability across samples. In contrast, geometric approaches define similarity through invariant metrics on shape spaces, where transformations such as translation, scaling, and rotation are removed, and distances such as Fréchet or elastic metrics are computed. Alignment-based methods introduce a warping function $\gamma(t)$, where similarity is defined as an optimization problem of the form $\min_{\gamma} d(f, g \circ \gamma)$, enabling comparison under temporal or phase variability. The overlap between these perspectives arises when alignment is embedded within geometric frameworks (e.g., elastic shape analysis) or when statistical models explicitly incorporate warping functions for registration. Thus, curve similarity can be viewed as a unified problem involving representation, transformation, and distance computation, providing a common mathematical foundation for hybrid approaches.

Curves also exhibit multiple sources of variability, including amplitude differences, phase variation, scaling effects, irregular sampling, and measurement noise [1,25]. Different similarity measures vary considerably in how they handle these sources, and inappropriate choices can lead to misleading conclusions. Moreover, few studies synthesize methods across communities in terms of their statistical properties, invariance characteristics, and computational complexity, which are critical for practical decision-making.

This survey addresses these gaps by providing a structured review of methods for curve similarity, emphasizing theoretical foundations, methodological differences, and inferential perspectives. We propose a taxonomy that organizes

existing approaches into five classes: Distance-Based and Functional Measures, Alignment and Warping-Based Techniques, Topological Similarity, Statistical Equivalence and Hypothesis-Testing frameworks and learning based approaches. We further map methods to relevant application domains, summarize comparative strengths and limitations, and identify open challenges such as uncertainty quantification, scalability, benchmarking, and integration with modern learning-based approaches.

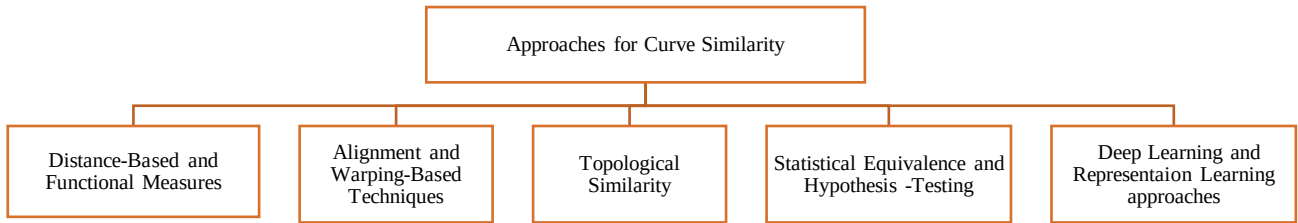


Fig. 2. Taxonomy of Methods and Approaches for Curve Similarity Measurement

1.1. Review methodology

This study follows a structured literature review approach for evaluating existing approaches for curve similarity. A comprehensive literature search was carried out on prominent online journals, including Scopus, Web of Science, IEEE Xplore, Springer Link, and ScienceDirect, covering literature from 1990 to 2025. A list of relevant studies was identified by searching for a combination of keywords, including curve similarity, time series similarity, dynamic time warping, Fréchet distance, Hausdorff distance, functional data analysis, shape analysis, topological data analysis, and deep learning, with Boolean operators applied to refine the search results. The selected literature includes contribution published in well-established journals such as Journal of the American Statistical Association, IEEE Transactions on Pattern Analysis and Machine Intelligence, Expert Systems with Applications, Information Sciences, Statistical Sciences, and journal of Machine Learning Research, as well as leading Conferences venues such as IEEE Conference on Computer Vision and Pattern Recognition, International Conference on Machine Learning, Neural Information Processing Systems, and Symposium on Computational Geometry, ensuring a balanced coverage of theoretical and applied research. The reviewed curve similarity assessment techniques have been categorized in a systematic manner depending on their underlying principles for assessing similarity. Specifically, techniques are categorized into five classes based on their underlying principles: direct pointwise distance computation, temporal alignment, geometric/topological structures, or data representation learning.: distance-based, alignment-based, topology-oriented, statistical/hypothesis testing, and learning-based techniques. Although some techniques may share certain similarities with other techniques in other categories, each method is classified according to its underlying principles for assessing similarity. Such a systematic categorization of the techniques provides an appropriate analytical background for conducting an in-depth review of the techniques in terms of their computational complexity, noise insensitivity, and ability to work with diverse types of data. Qualitative synthesis of the reviewed studies revealed common limitations, which include issues of scalability, lack of standardized benchmarking data sets, insufficient uncertainty modeling, and insufficient integration of classical and learning-based methods. The results serve as a basis for identifying the gaps and motivate the construction of hybrid frameworks to improve similarity in curves.

1.2. Paper layout

This paper is systematically organized to provide a clear and comprehensive exploration of curve similarity assessment methods, ensuring both depth of analysis and ease of understanding. Section 2 introduces the mathematical framework and formally states the curve similarity problem. Section 3 provides a systematic examination of the principal methodological categories for assessing similarity between curves. Section 4 presents a comparative evaluation of these approaches with respect to their invariance properties, computational characteristics, and robustness considerations. Section 5 surveys software environments and libraries that facilitate practical curve similarity analysis. Section 6 outlines the primary challenges and unresolved issues within the field. Section 7 discusses potential directions for future research, and Section 8 offers concluding remarks.

2. Mathematical Preliminaries and Problem Formulation

2.1. Representation of Curve

A curve in R^n is represented as a continuous mapping

$$c: [0,1] \rightarrow R^n, \quad c_1(t), c_2(t) \dots \dots \dots c_n(t)$$

Polygonal curves, widely used in computational geometry, represent curves as ordered sequences of vertices connected by straight segments [14]. Sampled curves arise naturally in time-series and functional data contexts [2].

A reparametrization is a continuous, non-decreasing surjection

$$\phi : [0,1] \rightarrow [0,1]$$

which preserves traversal order. Reparametrizations are central to similarity measures such as the Fréchet distance [7, 14].

2.2. Sources of Variability in Curves

Curve variability arises from multiple sources, including amplitude variation, phase (temporal) misalignment, scaling differences, noise, and irregular sampling. Amplitude variability refers to differences in the vertical magnitude of curves, whereas phase variability captures horizontal misalignment caused by temporal or parametric shifts [23]. These two components are fundamental in functional data analysis, where separating amplitude and phase variation is recognized as a critical challenge for accurate curve comparison [2,25]. In addition to structural variability, real-world curve data are often affected by noise and measurement errors, which introduce random fluctuations and can obscure underlying patterns if not properly addressed. Signal processing and FDA literature emphasize that noise and temporal variability are inherent in observational data and must be accounted for during preprocessing and similarity evaluation [2,60,61]. Furthermore, irregular or sparse sampling presents additional challenges, as observations may not be uniformly distributed over the domain, requiring interpolation, smoothing, or functional reconstruction techniques to ensure meaningful comparison [3,59].

Example: Two ECG waveforms may appear misaligned due to heart rate variability, even though their morphological shapes are nearly identical. Without handling phase variability, a pointwise metric would falsely deem them dissimilar [25].

2.3. Similarity, Distance, and Equivalence

Curve similarity can be formalized through different mathematical notions:

- Metric distances: $D(f, g)$ satisfies non-negativity, identity, symmetry, and the triangle inequality. Geometric distances such as Fréchet and Hausdorff fall into this category [14,20].
- Non-metric similarity scores: Alignment-based measures such as DTW may violate symmetry or the triangle inequality but remain effective for time-warped data [8,9,24].
- Statistical equivalence or parallelism: In bioassay and functional inference, similarity is formalized as a hypothesis:

$$H_0: f \sim g \text{ vs } H_1: f \not\sim g$$

where similarity is defined via tolerances or equivalence regions [13, 21, 22]. These formalisms motivate different methodological classes discussed later in Section 3.

2.4. Mathematical Problem Setup

Given two curves f and g , the similarity problem aims to measure how “close” they are under a chosen representation and transformation class. This involves specifying a representation space F , a similarity functional $S: F \times F \rightarrow \mathbb{R}$ and an optional transformation group G . The general formulation is $S(f, g) = (\min_{\gamma \in G}) D(f, g \circ \gamma)$ where γ may encode time-warping or reparameterization. Different choices of G yield distinct method families: $G = \{\text{identity}\}$ leads to direct pointwise distances, $G = \{\text{translation, scaling}\}$ gives invariant distances, and $G = \Gamma_{\text{diffeomorphism}}$ produces alignment or elastic distances. This unified formulation highlights how various similarity models fit within a common mathematical framework.

2.5. Invariance Properties

Many methods impose invariance to certain transformations:

- Translation invariance: $f(t) \rightarrow f(t) + c$
- Scaling invariance: $f(t) \rightarrow \alpha f(t)$
- Reparameterization invariance: $f(t) \rightarrow f(\gamma(t))$ for $\gamma \in \Gamma$

Reparameterization invariance is central to elastic and geometric frameworks, where Γ is typically defined as:

$$\Gamma = \{\gamma: T \rightarrow T \mid \gamma \text{ is a diffeomorphism} \}$$

2.6. Common Distance and Similarity Measures

Several classical and modern curve similarity methods can be expressed through distance or dissimilarity functionals. The most widely used formulations include the following.

(1) Pointwise L_p Norms (Functional Distances)

For $1 \leq p < \infty$, the L_p distance between two curves f and g is defined as:

$$D_{L_p}(f, g) = \left(\int_T |f(t) - g(t)|^p dt \right)^{1/p}$$

The most common special cases are the L_2 norm and the L_1 norm. The L_2 norm (Euclidean functional distance) measures squared differences and is sensitive to large deviations, while the L_1 norm uses absolute differences and is more robust to outliers. For discrete data, the integral becomes a sum:

$$D_{L_p}(f, g) = \left(\sum_{i=1}^n |f(t_i) - g(t_i)|^p \right)^{1/p}$$

(2) Derivative-Based (Sobolev) Distances:

When derivatives are relevant, distances may include smoothness penalties:

$$D_{H^1}(f, g) = \left(\int_T (f(t) - g(t))^2 + \lambda (f'(t) - g'(t))^2 dt \right)^{1/2}$$

where $\lambda > 0$ controls the influence of derivative differences. This is widely used in FDA and smoothing contexts [1-5].

(3) Area-Based Distance

The area between curves is defined as:

$$D_{area}(f, g) = \int_T |f(t) - g(t)| dt$$

or in normalized form,

$$D_{area}^{norm}(f, g) = \frac{\int_T |f(t) - g(t)| dt}{|T|}$$

Area-based metrics are commonly used in bioassay parallelism testing [9].

(4) Fréchet Distance (Geometric / Trajectory-Based)

$$d_F(C_1, C_2) = \inf_{\alpha, \beta} \max_{t \in [0,1]} \|c_1(\alpha(t)) - c_2(\beta(t))\|$$

where $\alpha, \beta : [0,1] \rightarrow [0,1]$ are continuous, non-decreasing reparametrization with $\alpha(0) = \beta(0) = 0$ and $\alpha(1) = \beta(1) = 1$.

The discrete Fréchet distance is commonly used for trajectories [14, 20, 23].

(5) Dynamic Time Warping (Alignment-Based)

Given discrete sequences $f = (f_1, \dots, f_n)$ and $g = (g_1, \dots, g_m)$, DTW computes:

$$DTW(f, g) = \min_W \left(\sum_{(i,j) \in W} \|f_i - g_j\| \right),$$

where W is a warping path satisfying boundary, monotonicity, and continuity constraints [8, 9, 24]. DTW is not a metric (triangle inequality fails), but it effectively handles phase variability.

3. Analysis of Methods Determining Curve Similarity

Curve similarity methods are categorized based on their underlying principles for measuring similarity. Specifically, the classification considers whether a method relies on direct pointwise distance computation, temporal alignment, geometric or topological structure, or data-driven representation learning. Based on these criteria, the methods are grouped into distance-based, alignment-based, topology-oriented, statistical and hypothesis testing and learning-based approaches, it is important to note that some methods may exhibit overlapping characteristics; however, each method is categorized according to its primary mechanism for similarity assessment. This classification provides a structured framework for comparing different approaches in terms of computational complexity, robustness, and applicability.

We classify existing approaches into five major categories:

- Distance-Based and Functional Measures
- Alignment and Warping-Based Methods
- Topological Similarity
- Statistical Equivalence and Hypothesis-Testing Approaches
- Deep Learning and Representation Learning-Based Approaches

3.1. Distance-Based and Functional Measures

Distance-based and functional similarity measures evaluate curves using mathematical metrics without relying on temporal alignment. Functional metrics commonly employ L_p and Sobolev norms to characterize magnitude and smoothness properties, enabling derivative-aware discrimination and convergence from discrete observations to continuous functions [17, 34]. More advanced variants incorporate first- and second-order derivatives or weighted trend components, enhancing clustering performance and capturing long-term behavioral patterns [17, 33]. In a geometric context, proximity measures like the Hausdorff distance quantify the maximum spatial deviation between two curve sets [35]. Another notable geometric measure is the Fréchet distance, which represents the minimum leash length required to traverse two curves while maintaining traversal order; the continuous form offers precise geometric sensitivity [11], whereas discrete approximations allow efficient computation via sampling [12]. These methods assume common parameterization, making them sensitive to phase variability and nonlinear distortions [25]. Their strengths lie in low computational cost and interpretability, making them suitable for functional regression and clustering in FDA settings [3, 4].

3.2. Alignment and Warping-Based Methods

Alignment-based methods address phase variability, temporal distortion, and structural misalignment by optimizing correspondences between curves (or time series). Classical Dynamic Time Warping (DTW), introduced for speech recognition [8], remains central with extensions to pattern matching [24]. Recent studies provided a comparison of DTW with alternative similarity measures such as the Frechet distance demonstrates the robustness of DTW while also highlighting issue such as noise sensitivity and lack of deep feature representation [9].

3.2.1. Hybrid and Advanced Variants of DTW

In recent years, Dynamic Time Warping (DTW) has evolved beyond its traditional formulation through the development of hybrid and advanced variants designed to improve similarity measurement for complex time series data. Early extensions such as Canonical Time Warping (CTW) [10] and Generalized Time Warping (GTW) [36] enhanced DTW by incorporating feature transformation mechanisms, allowing effective alignment across heterogeneous and multi-modal datasets. These approaches marked a shift from simple point-wise alignment toward combining representation and alignment within a unified framework. This direction was further advanced by Deep Canonical Time Warping (DCTW) [38], which utilizes deep learning models to capture nonlinear relationships in high-dimensional data, thereby improving alignment accuracy in complex scenarios. Similarly, Soft Dynamic Time Warping (Soft-DTW) [37] introduced a differentiable formulation of DTW, enabling its direct integration into modern machine learning models for optimization-based learning tasks.

Alongside these developments, several methods have focused on improving efficiency and representation. Approaches such as Local Extrema DTW (LE-DTW) [53] and Extrema-based Shape DTW (ESDTW) [55] reduce the dimensionality of time series by selecting significant points, thereby lowering computational cost while preserving key structural characteristics. In addition, Continuous DTW (CDTW) [54] extends the concept of alignment from discrete sequences to continuous curves, offering a more precise representation of similarity. More recent hybrid techniques, including enhanced distance-based DTW methods [56] and Angle-distance Penalized Metric DTW (APMDTW) [57], incorporate multiple factors such as nonlinear distance measures and directional information to improve discriminative capability, as summarised in Table 2. Despite these advancements, a common challenge persists, as improvements in accuracy often lead to increased computational complexity, while efficiency-oriented methods may sacrifice important information. This highlights the need for a balanced approach that integrates accuracy, efficiency, and robustness in curve similarity analysis.

Table 2. Analysis of DTW-Based Similarity Measures: Complexity and Performance.

Method	Key Idea	Computational Complexity	Performance	Trade-offs
CTW [10]	CCA + DTW for joint feature transformation and alignment	Higher than quadratic time complexity	High (multi-modal alignment)	Requires training data; increased complexity
GTW [36]	Nonlinear and multi-modal alignment	Significantly higher than quadratic time complexity	Very high accuracy	Very high computational cost; scalability issues
DCTW [38]	Deep learning + DTW alignment	Very high complexity due to deep model training and alignment	Very high (complex data)	Data-intensive; training complexity
Soft-DTW [37]	Differentiable DTW (soft-min)	Quadratic time complexity with higher computational overhead	Excellent for ML tasks	Parameter tuning; increased computation
DTW (Temporal Graphs) [52]	DTW applied to graph sequences	High complexity due to graph processing and alignment	Good for structured data	Feature-dependent; high cost
LE-DTW [53]	Dimensionality reduction via extrema	Reduced complexity (depends on number of extrema points)	Efficient for long series	Information loss
CDTW [54]	Continuous curve alignment	Polynomial time complexity higher than standard DTW	High theoretical accuracy	Computationally expensive
ESDTW [55]	Extrema + shape-based DTW	Moderate complexity based on reduced feature set	Better than LE-DTW	Still depends on feature extraction
Classical DTW (Application Study) [9]	DTW vs Fréchet comparison	Quadratic time complexity	~90% accuracy	Noise sensitivity
Improved Distance DTW [56]	Enhanced distance metric	Slightly higher than quadratic time complexity	Higher classification accuracy	Increased computation
APMDTW [57]	Angle + nonlinear distance DTW	High computational complexity due to multiple factors	Very high accuracy	High complexity; parameter sensitivity

Elastic frameworks such as SRVF models [1, 25, 26, 28] and manifold/geometric warping methods [11] improve alignment by regularizing deformation in Riemannian spaces, while landmark-based registration offers monotone warping control [27]. These methods are widely used in ECG/EEG alignment, handwriting recognition, gait and gesture analysis, and robotic control [9]. Key limitations include computational overhead, non-metric behavior (e.g., DTW violates triangle inequality), and sensitivity to warping path choices, though elastic models partially mitigate these challenges through structured warping constraints [26, 28, 29].

3.3. Topological Similarity

Topology-oriented approaches assess curve similarity by focusing on global structure, connectivity, and shape characteristics rather than pointwise matching or explicit temporal alignment. In one of the first topological curve matching problems, Chambers and Wang [39] introduced a curve matching approach where the curve similarity was defined as the minimum homotopy area that is needed to map one curve into another on a 2-manifold by optimization of admissible homotopy.

A more scalable approach uses persistent homology, which represents the curve by a persistence diagram D , and the distance between two curves is defined as the distance between two diagrams. The bottleneck and Wasserstein distances are commonly used [42], which capture the differences in topological features across scales. In practice, the persistence diagram is often mapped to a vector representation, called the persistence image/landscape, to allow efficient computation of the distance using standard norms, as shown in the work of Conti et al. [41]. This establishes a clear computational pipeline:

$$\text{curve} \rightarrow \text{topological description} \rightarrow \text{vector representation} \rightarrow \text{distance}$$

From a structural point of view, Bauer et al. [18] presented a metric based on the use of Reeb graphs, where each curve is represented by a graph that captures the evolution of level sets. The similarity between two curves is measured by the Reeb graph edit distance, which is the cost of transforming one graph into another. It has been theoretically justified and proved to be a universal metric among stable distances for maximal discriminative power. Instead, recent research has concentrated on enhancing the quality of topological features. Weng and Zhao [40] introduced curvature-adaptive constructions of persistent homology. This allows for the improved detection of important features. This is in line with other advancements in geometry-informed topological data analysis. In this approach, local geometric information is used to improve topological features. In addition to this, Bhattacharya et al. in their work [12] have shown the importance of the type of filtration in the construction of the persistence diagrams in the context of the construction of the similarity. In their work, they have compared the graph-based and Vietoris-Rips filtrations and have shown the importance of the type of filtration in the construction of the similarity.

An alternative to the descriptor-based topologies is to employ topological indices, in which the graph is encoded as a feature vector. In the feature vector, the properties of the graph, such as distances and degrees, are encoded. This feature vector is then compared by using various distance measures [57]. Mathematically, the similarity between two graphs G_1 and G_2 is computed by the p-norm distance of the topological index. This index-based approach establishes another practical pipeline:

$$\text{curve / graph} \rightarrow \text{topological indices} \rightarrow \text{feature vector} \rightarrow \text{distance / similarity}.$$

In addition, the use of several indices allows for increased sensitivity to small variations, making the methods applicable to large-scale problems such as network analysis or similarity evaluation.

Finally, topology-based representation learning methods extend these methods further by learning invariant embeddings from structural descriptors. As demonstrated by Li et al. [19], this approach allows similarity computation via standard distances on canonical representations, facilitating integration with modern machine learning methods.

Overall, these approaches follow a two-stage process: (i) mapping curves to structural descriptors (e.g., persistence diagrams, Reeb graphs), and (ii) computing similarity using distances such as homotopy, Wasserstein, or vector-based norms. Its effectiveness depends on the descriptor and metric, with recent methods improving accuracy and efficiency through geometry-aware and adaptive designs.

3.4. Statistical Equivalence and Hypothesis-Testing Approaches

Statistical equivalence and hypothesis-testing approaches assess curve similarity by evaluating whether two or more functional responses are statistically indistinguishable under predefined criteria. A common use case arises in bioassay and dose–response analysis, where similarity is framed as parallelism testing, i.e., verifying that curves share comparable slopes or shapes across doses. Fast and reliable procedures have been proposed to formally test parallelism and ensure assay validity [21], while more recent curve-based methods quantify similarity directly through statistical measures tailored to bioassay response patterns [13]. These approaches often involve multiple hypothesis testing frameworks to control error rates when evaluating similarity across doses or treatments [22]. Collectively, hypothesis-driven testing enables rigorous statistical decision-making for curve similarity, particularly in regulated settings where equivalence or parallelism must be demonstrated rather than assumed.

3.5. Deep Learning and Representation Learning-Based Approaches

Deep learning and representation learning methods have emerged as powerful approaches for similarity computation by enabling models to learn feature representations directly from data rather than relying on predefined distance measures. Early work by Li et al. [62] demonstrated that trajectories can be embedded into low-dimensional vector spaces, allowing similarity to be computed efficiently while reducing sensitivity to noise and irregular sampling. Building on this idea, Petitjean et al. [63] introduced NeuralWarp, which integrates representation learning with adaptive temporal alignment, thereby capturing both global sequence patterns and local temporal distortions. More recent studies have focused on enhancing representation quality through advanced architectures. For instance, Liu et al. [64] proposed DeepSIM, which combines global and local structural features using a dual-branch network, while Song et al. [65] demonstrated that integrating structural and semantic representations leads to more comprehensive similarity assessment. Furthermore, Siamese-based models have gained prominence in learning similarity functions, as evidenced by SASNet [67], which incorporates attention mechanisms to improve feature discrimination, and TimeSiam [66], which leverages self-supervised learning to generate robust representations without requiring labeled data. In a broader context, Yang et al. [68] highlight that such deep learning approaches, including embedding-based, attention-driven, and hybrid models, are capable of capturing complex nonlinear relationships while improving scalability. Collectively, these studies demonstrate that representation learning, combined with advanced deep architectures, provides an effective and flexible framework for similarity computation across diverse data domains.

Although many deep learning-based similarity methods have been developed for domains such as graphs, trajectories, and structured data, their underlying representation learning mechanisms are highly relevant to curve similarity analysis. These approaches focus on learning discriminative embeddings, capturing both global patterns and local variations, and handling nonlinear relationships properties that are equally important in comparing curves and time-series data. Therefore, despite not being explicitly designed for curve similarity, such methods can be effectively adapted and extended to this domain.

4. Comparative Evaluation of Curve Similarity Methods

Different categories of curve similarity techniques vary notably in terms of invariance properties, computational demands, and sensitivity to noise. Distance-based metrics are relatively straightforward and computationally efficient, yet they generally do not account for temporal or phase variations. Alignment and warping-based strategies achieve temporal and phase invariance, but this advantage comes with higher computational cost. Topological approaches provide strong invariance to structural deformations and tend to handle noise well, although they are typically resource-

intensive. Statistical hypothesis-testing methods evaluate similarity from a distributional perspective rather than through geometric or temporal correspondence, making them appropriate for regulated scientific contexts but less effective for general shape comparison, Table 3 outlines the principal distinctions among these methodological families.

Table 3. Comparative characteristics of major method categories.

Method Category	Key Invariance	Complexity	Noise Robustness	Main Applications
Distance-Based	Low (no phase/time invariance)	Low–Medium	Medium	Functional data analysis, geometric matching
Alignment/Warping	High (phase/time invariance)	Medium–High	Medium–High	Speech recognition, motion analysis, time-series
Topology-Based	High (structural/deformation invariance)	High	High	Geometry processing, biological data, neuroimaging
Statistical Testing	Low (no geometric invariance)	Low–Medium	High (under statistical models)	Bioassay, biostatistics
Learning-Based	High (learned invariance and feature representation)	High (training), Low (inference)	High	Large-scale data analysis, pattern recognition, time-series modeling

From a practical perspective, distance-based methods are suitable for simple geometric comparisons with low computational cost, while alignment-based methods are preferred when temporal or phase variations are significant. Topology-based approaches are particularly effective for data with strong structural variability and noise, whereas learning-based methods are more suitable for large-scale and complex datasets where data-driven representations can capture intricate patterns. Statistical approaches remain useful in controlled experimental settings where model assumptions are valid.

Despite significant progress, existing curve similarity methods face several common challenges. Many approaches involve trade-offs between computational efficiency and accuracy, particularly for large-scale datasets. Alignment-based and topology-based methods may become computationally expensive, while learning-based approaches require substantial training data and lack interpretability. Additionally, handling noisy, irregularly sampled, or high-dimensional curves remains an open challenge across most method categories.

5. Software Libraries Supporting Curve Similarity Assessment

Computational studies on curve similarity increasingly depend on dedicated open-source libraries that implement alignment strategies, functional representations, and metric-based similarity measures. Unlike general-purpose numerical platforms, these libraries incorporate methods such as Dynamic Time Warping (DTW), Soft-DTW, and elastic shape analysis, which enable phase–amplitude decomposition and registration for more reliable comparisons. The choice of library often depends on four methodological considerations: (i) invariance properties (e.g., time-warp or scale invariance), (ii) robustness to noise, (iii) computational complexity, and (iv) suitability to specific application domains.

Table 4. provides an overview of frequently used software libraries for curve similarity analysis, outlining their core techniques, areas of application, robustness characteristics, and typical use cases. These tools facilitate reproducible experimentation, benchmarking, and cross-dataset performance evaluation. For instance, tslearn integrates DTW and Soft-DTW for time-series clustering and classification workflows [46], while fastdtw offers a linear-time approximation of classical DTW for long time-series sequences [44]. Likewise, the fdasrsf package implements elastic shape analysis through the Square-Root Velocity Function (SRVF) representation [45], enabling phase–amplitude separation for geometric shape comparison. The availability of such libraries has notably enhanced reproducibility in curve similarity research and has supported experimental pipelines consistent with modern machine learning and functional data analysis practices.

Table 4. Comparison of Dedicated Libraries for Curve Similarity Assessment.

Library /Toolkit	Core Algorithms	Robustness Characteristics	Complexity	Primary Application Domain
tslearn (Python) [45]	DTW, Soft-DTW, shape-based distances, TS clustering	Time-warping invariance (DTW/Soft-DTW); moderately robust to noise (Soft-DTW smooths alignment)	DTW: $O(n^3)$, Soft-DTW: $O(n^3)$	Machine learning tasks: time-series classification & clustering
fastdtw (Python) [43]	Approximate DTW	Warping invariance; reduced robustness vs exact DTW due to approximation	$O(n)$	Large-scale time-series alignment
Dtaidistance (Python) [42]	DTW, LB_Keogh, Soft-DTW	Warping invariance (DTW); LB_Keogh provides lower-bounding robustness	DTW: $O(n^3)$, LB_Keogh: $O(n)$	Time-series benchmarking & distance evaluation
dtw (R) [46]	DTW with global/local constraints	Warping invariance; sensitive to noise without pre-smoothing	$O(n^3)$	Statistical alignment & clustering

fdasrsf (R/Python) [44]	SRVF elastic metrics & registration	Phase-amplitude separation; robust after smoothing	Registration: $O(n^3)$	Functional/elastic curve shape analysis
TSclust/TSdist (R) [47]	Distance & model-based similarities	Varies by metric; no explicit invariances	Varies	Exploratory statistical distance comparison
similaritymeasures (Python/PyPI) [67]	Fréchet, DTW, PCM, Area	Robust to varying sampling rates.	$O(n^2)$	Material Science, Hysteresis loops.
CurveSimilarities (Python/GitHub) [70]	Continuous/Discrete Fréchet	High precision for continuous curves.	$O(n^2) +$	Scientific Research, Geometry.
curve-matcher (JS/NPM) [71]	Fréchet, Procrustes	Invariant to rotation and scale.	Adjustable	Gestures, Signature matching.
MoveTK (C++/GitHub) [72]	LCSS, Fréchet, Hausdorff	High performance; handles GPS noise.	Optimized	Trajectory Mining, Mobility.
spark-curves (Scala/Spark) [73]	DTW, Fréchet (Distributed)	Highly scalable for massive data.	Distributed	Big Data, Traffic flow.
SimilarityMeasures (R/CRAN) [74]	Fréchet, LCSS, Edit Distance	Robust for trajectory analysis in R.	$O(n^2)$	Ecological Movement, Statistics.

6. Challenges and Open Problems

Despite substantial progress, curve similarity assessment continues to face several fundamental challenges that span distance-based, alignment-based, topological, statistical, and deep learning methods.

6.1. Phase-Amplitude Ambiguity

A central issue arises in alignment-based methods, particularly Dynamic Time Warping (DTW) and its variants such as shapeDTW, where phase (temporal misalignment) and amplitude (shape differences) are not explicitly separated. As a result, these methods may align curves with fundamentally different shapes but similar temporal patterns, leading to ambiguous similarity interpretation. Even shape-aware extensions fail to fully resolve this issue, indicating a persistent limitation of alignment-based similarity frameworks [77].

6.2. Noise and Sampling Variability

Robustness to noise and irregular sampling remains a critical challenge across multiple categories. Distance-based methods, such as Euclidean and Fréchet-type measures, are particularly sensitive to small perturbations, resulting in unstable similarity scores. Meanwhile, alignment-based methods like DTW [9] may overfit noise through excessive warping. Recent multiscale distance-based approaches attempt to mitigate this issue, but consistent robustness across varying noise conditions is still not achieved. [75].

6.3. High-Dimensional and Complex Data

Handling high-dimensional and multivariate curves is a growing challenge. Traditional distance-based and statistical methods often fail to capture interdependencies across dimensions, while topological methods struggle with computational complexity as dimensionality increases. Even recent learning-based approaches face difficulties in representing complex functional relationships efficiently, highlighting the need for scalable and expressive similarity representations [75].

6.4. Lack of Benchmarking and Evaluation Standards

There is a lack of standardized benchmarks for evaluating curve similarity methods across categories. This issue is particularly evident in deep learning-based methods, where different models are evaluated on inconsistent datasets and metrics. Similarly, comparisons between distance-based, alignment-based, and topological approaches are often not conducted under unified experimental settings, making it difficult to assess their relative strengths and limitations objectively [79].

6.5. Absence of Uncertainty Modeling

Most existing approaches, including distance-based, alignment-based, and topological methods, provide deterministic similarity measures without accounting for uncertainty. While statistical methods offer a natural framework for modeling variability, uncertainty quantification is rarely integrated into similarity metrics in a systematic way. This limitation reduces the reliability of similarity assessments in noisy and real-world scenarios [79].

6.6. Scalability and Computational Complexity

Scalability remains a major bottleneck, particularly for alignment-based methods (e.g., DTW and its variants) and topological methods (e.g., homotopy-based or contour-tree approaches). Exact similarity computation becomes computationally expensive for large datasets. Although recent works such as k-DTW [76] and topology-aware similarity measures attempt to improve efficiency, achieving a balance between computational scalability and accuracy remains an open problem [78].

6.7. Interpretability vs. Learning-Based Performance

A key trade-off emerges in deep learning-based methods, where improved performance comes at the cost of interpretability. Unlike distance-based or topological methods, which have clear geometric or structural meaning, learned similarity representations often function as black boxes. This lack of interpretability limits their applicability in scientific and critical decision-making contexts, despite their strong empirical performance [79, 80].

Overall, no single category whether distance-based, alignment-based, topological, statistical, or deep learning methods fully addresses all challenges simultaneously. Developing hybrid frameworks that integrate the strengths of these approaches, while improving robustness, interpretability, and scalability, remains a key direction for future research.

7. Future Research Directions

Future research in curve similarity should prioritize the development of unified, robust, and scalable frameworks that integrate distance-based, alignment-based, topological, statistical, and deep learning paradigms. In particular, hybrid models that combine alignment techniques such as Dynamic Time Warping with multiscale distance measures and topological descriptors offer a promising path toward improved robustness and interpretability. A central challenge is the principled resolution of phase amplitude ambiguity in alignment-based methods, requiring formulations that decouple temporal and geometric variations without over-flexible warping. Equally important is enhancing robustness to noise and irregular sampling through multi-resolution and probabilistic approaches, including Bayesian formulations that enable uncertainty-aware similarity assessment. Scalability remains a critical bottleneck, motivating the design of efficient approximation strategies and learning-accelerated algorithms for computationally intensive similarity measures. The establishment of standardized benchmarking protocols is also essential to ensure consistent and reproducible evaluation across methods. Furthermore, extending similarity analysis to non-Euclidean domains, such as manifolds and graph-structured data, alongside improving the interpretability and generalization of deep learning-based models through integration with geometric and statistical principles, represents an important research frontier. Collectively, these directions emphasize the need for hybrid, uncertainty-aware, and domain-adaptive similarity frameworks that balance theoretical rigor with practical applicability.

8. Conclusion

This survey summarized the principal categories of curve similarity methods, spanning distance-based metrics, temporal alignment techniques, topological frameworks, and statistical testing and deep learning approaches. These method families differ in their treatment of invariance, sensitivity to noise, and computational demands. While distance-based metrics are efficient but phase-sensitive, alignment methods address temporal variation at higher computational cost topological techniques provide deformation invariance yet remain computationally heavy, and statistical tests support functional or distributional equivalence mainly in regulated scientific settings. In practice, no single method achieves robustness, efficiency, and accuracy across all data types and applications. The observed trade-offs highlight the need for improved algorithmic strategies that enable reliable and scalable curve similarity assessment in modern data contexts. As a result, curve similarity continues to be a vibrant research area with significant potential for new methodological contributions.

All the Declarations and Statements

Author Contributions Statement

Shikha Mishra – Conceived the research topic, designed the review methodology, conducted the literature search and selection of relevant studies, analyzed and synthesized the literature, developed the classification framework for curve similarity methods, prepared the tables and figures, and wrote, revised, and formatted the complete manuscript.

Namita Tiwari – Supervised the research throughout the study, provided conceptual and methodological guidance, critically reviewed and edited the manuscript, validated the analysis and interpretations, offered constructive suggestions to improve the quality and clarity of the work, and approved the final version of the manuscript.

All authors have read and agreed to the published version of the manuscript.

Conflict of Interest Statement

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No new data were generated or analyzed in this study. This review is based exclusively on previously published literature, and all cited sources are publicly available.

Ethical Declarations

The research does not involve any human or animal subjects, nor does it use personal data requiring ethical approval. Therefore, ethical approval was not required for this research.

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The authors used AI-assisted tools solely to improve the language and clarity of the manuscript. All content was reviewed and approved by the authors, who take full responsibility for the final manuscript.

Abbreviations

The following abbreviations are used in this manuscript:

DTW - Dynamic Time Warping
TDA - Topological Data Analysis
FDA - Functional Data Analysis
SRVF - Square Root Velocity Function

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