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A Simple Recourse Strategy for Efficient Allocation of Aircrafts for Satisfying Uncertain Passenger Demands

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Abstract: This paper explores the use of stochastic optimization techniques to address the aircraft allocation problem under uncertain passenger demand. The proposed stochastic allocation model successfully meets the study's objectives by demonstrating how uncertainty in passenger demand can be effectively incorporated into aircraft assignment decisions through a two-stage stochastic programming framework. Simulation results across multiple demand scenarios show that the model provides stable and adaptive allocations that minimize total cost while maintaining service quality, even under high variability. Incorporating the simple recourse approach enables post-decision flexibility, reducing penalties for unmet demand, and the use of Geometric Brownian Motion (GBM) offers a realistic representation of continuous demand fluctuations over time. These outcomes confirm the model's practical value in bridging deterministic planning and real-time decision environments. While future research will focus on extending the model to a Markov Decision Process (MDP) framework and integrating real-time data streams, the current results establish a solid foundation by quantifying how uncertainty directly impacts fleet utilization, cost efficiency, and service reliability.

Index Terms: SP Technique; Multi-stage; Scenario Tree Method; Recourse Problem; Numerical Simulation

1. Introduction

The "aircraft allocation problem" refers to the task of efficiently assigning available aircraft to various tasks or missions. The aviation industry operates in an inherently dynamic environment, where factors like passenger demand fluctuate unpredictably due to various external influences. Efficiently allocating the resources of aircraft to meet these uncertain demands is a critical challenge faced by airlines worldwide. In this study, we delve into the complex realm of stochastic optimization for aircraft allocation with a specific focus on incorporating simple recourse strategies to ensure passenger satisfaction in real-life scenarios. First, we examined a few articles regarding the straightforward recourse problem for allocating airplanes, and we modified the Cleefs test model in certain ways.

Salazar-Gonzales (2014) worked on the integrated aircraft routing, fleet assignment, and crew-pairing problem [1]. They developed a heuristic algorithm, taking an integer-programming model as a base. Cadarso and Marin (2013) proposed a robust model for the integrated fleet assignment and flight-scheduling problem to minimize the number of misconnected passengers [2].

A fleet assignment model introduced by Abara (1989) uses a connection-based network structure that encompasses coverage and flow balance constraints and considers the number of available aircrafts [2]. Hane et al. (1995) developed a large-scale integer program that uses a time-space network structure for the fleet assignment [3]. Since this method eliminates the use of connection decision variables, it becomes a widely accepted and adopted approach utilized for the fleet assignment problem formulation [4].

The demands for air travel is subject to rapid changes, influenced by factors ranging from seasonal variations and economic shifts to unforeseen events like pandemics. In this context, the ability to allocate aircraft resources optimally becomes paramount. Inefficient allocation not only results in increased operational costs for airlines but can also lead to passenger dissatisfaction due to flight delays, cancellations or overbookings. We will develop with many concepts for the aircraft allocation problems. It is a logistical challenge faced by airlines, military organizations and other aviation entities that operate multiple aircraft and need to optimize their utilization. There are many issues for aircraft allocation problem, which involves factors such as flight schedules, passenger or cargo demands, maintenance requirements, crew availability and operational constraints. The main target is to maximize the utilization of aircraft resources while minimizing costs, meeting operational objectives, and ensuring safety. The problem becomes more complex when considering factors like varying aircraft capabilities, different mission requirements, route planning, fuel efficiency and considering potential disruptions or uncertainties such as weather conditions or unexpected events. Aiming to allocate the right aircraft to the right flights, aircraft allocation takes into account a number of variables and constraint, including flight demand, aircraft availability, maintenance schedules, crew availability and operational limitations that makes a mathematical model to handle the stochastic element. The goal is to increase the airline's total operational effectiveness and profitability while satisfying consumer demand and guaranteeing safety. Consider efficiency, cost-effectiveness, viability and other pertinent factors when evaluating the allocation strategy. If the preliminary findings are unsatisfactory, improve the mathematical model or change how the problem is posed. Continue the optimization procedure until a successful outcome is attained. Our research sets out to address this pressing issue by proposing a comprehensive approach to stochastic optimization for aircraft allocation. We aim to bridge existing gaps in the literature by considering real-life passenger demand uncertainty and simple recourse strategies, making our model not only practical but also adaptable to a variety of airline operations.

While stochastic programming has been extensively applied to aircraft allocation and related operational problems, the present study contributes to the literature by introducing a simple recourse mechanism integrated with a continuous stochastic demand model based on Geometric Brownian Motion (GBM). Traditional approaches generally rely on discrete scenario trees or static probability distributions that capture uncertainty only at fixed intervals. In contrast, our formulation incorporates a continuous-time stochastic process that more realistically represents the random and time-varying nature of passenger demand. The integration of the simple recourse framework with GBM provides a computationally tractable yet dynamically adaptive decision-making model, enabling airlines to adjust allocation strategies in response to evolving demand conditions. This combined methodology enhances the robustness and realism of stochastic fleet assignment and offers a distinctive advancement over conventional discrete-probability or single-stage stochastic optimization models.

The rest of the paper is organized as follows. In Section-2, we discuss the relevant theory and mathematical formulation. In Section-3, we have presented the Cleefs aircraft allocation model. In Section-4, we developed our model and analyzed different characteristics. We also developed a computer technique by implementing (Python). Finally, we conclude our paper in Section-5.

2. Theory and Mathematical Formulation

In this section, we present the mathematical formulation of the aircraft allocation problem under both deterministic and stochastic conditions. The deterministic model assumes fixed parameters, while the stochastic version captures demand uncertainty using random variables [5].

In deterministic optimization, outcomes are fully predictable because all parameters are known. In contrast, stochastic optimization explicitly models random variations in parameters or constraints to capture operational uncertainty such as fluctuating passenger demand in airline networks.

The following formulation is deterministic and the next part is stochastic formulation.

2.1 Deterministic Model:

The deterministic model assumes that all problem parameters—such as passenger demand, aircraft capacity, and operating cost—are precisely known in advance. In this case, the optimization objective can be expressed as a standard linear or integer programming problem:

$$Max \,/\, Min \quad z = c^T x \tag{1}$$

Subject to:

$$Ax(<,=,>)\ b \tag{2}$$

This model provides a benchmark for comparison but does not capture real-world fluctuations in passenger demand or operational constraints.

In the following figure, we write the coefficient matrix, decision vectors, and cost coefficient in matrix form. We can easily find out the characteristics of the solution using matrix form.

Where $\boldsymbol{x} \geq 0$

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \quad \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_m \end{bmatrix} \quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \quad \boldsymbol{c} = \begin{bmatrix} c_1 \\ c_2 \\ \cdot \\ \cdot \\ c_n \end{bmatrix}$$

Stochastic Optimization Program:

To account for uncertainty, the deterministic model is extended into a stochastic optimization framework in which certain parameters such as passenger demand are treated as random variables. The corresponding formulation is expressed as:

$$Max/Min \quad z = \boldsymbol{c^T x} \tag{3}$$

Subject to:

$$A\boldsymbol{x}(<,=,>)\ \boldsymbol{b}$$
$$T(\omega)\boldsymbol{x} \leq H(\omega) \ \text{ Where, } \boldsymbol{x} \geq 0$$

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \quad \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_m \end{bmatrix} \quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \quad \boldsymbol{c} = \begin{bmatrix} c_1 \\ c_2 \\ \cdot \\ \cdot \\ c_n \end{bmatrix}$$

The following matrix formulation depends on scene-based problems. The stochastic terms are associated with this matrix. They are the following:

$$T(\omega) = \begin{bmatrix} a_{11}(\omega) & a_{12}(\omega) & \dots & a_{1n}(\omega) \\ a_{21}(\omega) & a_{22}(\omega) & \dots & a_{2n}(\omega) \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{m1}(\omega) & a_{m2}(\omega) & \dots & a_{mn}(\omega) \end{bmatrix}, \quad H(\omega) = \begin{bmatrix} h_1(\omega) \\ h_2(\omega) \\ \cdot \\ \cdot \\ h_l(\omega) \end{bmatrix}$$

As a result, this challenge considers the uncertainty of its constraints, which also indicates the unpredictability of the limited company resources. The problem is best expressed as follows:

$$Max/Min \quad z = \ c^T x \ + Q(x) \tag{4}$$

Subject to:

$$Ax <= b \ \ \& \ \ T(\omega\ )x <= H(\omega) \ \text{ Where x≥0} \tag{5}$$

Here, $Q(x) = E_\omega Q(x,\omega) = \sum_\omega p(\omega) Q(x,\omega)$

$Q(x,\omega) = \ min(abs(q(\omega)^T)y(\omega))$,

Where $w(\omega)y(\omega) \geq H(\omega) - T(\omega)x$ and ω is scenario.

In the aircraft allocation context, the stochastic formulation models demand uncertainty across different routes and time periods. The inclusion of random variables enables airlines to anticipate fluctuations and adjust capacity accordingly, rather than relying solely on deterministic forecasts. The model thus serves as a foundational step toward the more advanced two-stage and multi-stage stochastic programs presented in the following sections, which incorporate recourse decisions for real-time adaptation.

2.2 Two-Stage Stochastic Mixed Integer Program (SMIP)

An integer variable is a decision variable that only accepts integer values. Both integer and real numeric values can be entered into the mixed integer variable [12].

The following can be used to express mixed integer programming (MIP) in its conventional form:
Standard Form of Mixed Integer Programming (MIP):

$$Min/Max \qquad z = c^T x + \mathrm{E}[\mathrm{Q}(x, \xi)] \tag{6}$$

Subject to:

$$Ax \geq b \tag{7}$$

Subject to:

$$\begin{gathered} x \in {R_+}^{n1} \times {\mathbb{Z}_+}^{p1} \\ \xi = (q, h, T, W) \\ Q(x, \xi) = min \ q^T y \\ Wy = h - Tx \\ y \in {R_+}^{n2} \times {\mathbb{Z}_+}^{p2} \end{gathered} \tag{8}$$

x: First- stage decision variables
y: Second- stage decision variables
Sometimes assumed n_1, p_1, n_2, p_2 are zero.

The formulation of aircraft allocation issues as mathematical models using MIP approaches allows the application of strong optimization solvers to discover optimal or nearly optimal solutions. As a result, airlines and aviation operators are able to maximize fleet utilization, cut expenses, boost operational effectiveness, and make wise judgments when allocating aircraft to different jobs or routes.

2.3 A Multi-Stage Stochastic Recourse Program

In this section, we will focus on resource problems in aircraft allocation in the airline industry. In the next chapter, we are developing some models using this method. The recourse problem in stochastic optimization concerns the decision-making process that follows the observation of ambiguous or unpredictable events. Decision variables are selected in stochastic optimization before the realization of the uncertain parameters is known. When judgments must be made that depend on the realization of uncertain parameters and the purpose is to optimize an objective function. Then the recourse problem occurs finding the most effective decision-making approach while taking the uncertainties. These decisions are all referred to as first-stage decisions and the period during which they are made is referred to as "the initial phase. A vector x is typically used to represent these choices. Several choices can be made following the experiment. Second-stage choices are what they are known as.

The standard Form of Multi-stage Stochastic Optimization is as follows:

$$Max/Min \quad z = {C_T}^T x_T \tag{9}$$

Subject to:

$$\begin{gathered} A_1 x_1 = b_1 \\ B_2 x_1 + A_2 x_2 = b_2 \\ B_3 x_2 + A_3 x_3 = b_3 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots.. \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots.. \\ A_T x_T + B_T x_{T-1} = b_T \\ x_T \geq 0 \end{gathered} \tag{10}$$

Let x_1,x_2… x_T be the decisions vectors corresponding to time periods (0,1,2… T). c_T, x_T, b_T, A_T are also time period parameters. B_T is the random parameters with associated times. In the above model, the multi-stage is known time by time. It is applied only knowing its previous time. The event or mathematical result is organized by known times. The model is not suitable if the previous data is not available. Greater time is required, which is why, compared to single-stage difficulties, multi-stage problems frequently take more time to solve. The amount of time needed increases with each stage and the process may involve waiting for feedback or findings from one stage before moving on to the next.

2.4 A Stochastic Program with Binary Decision Variables

In this section, we will focus on binary decision variables. Most of the variables are binary, such as allocated roots, number of flights and a passenger going on a selective aircraft fleet. The fleet assignment problem is associated with binary decision variables.

Let us consider a general stochastic optimization problem with binary decision variables. Suppose we aim to minimize an objective function $f(x,\xi)$ Subject to constraints, where x represents the decision variables and ξ denotes the stochastic parameters.

The general form of a stochastic optimization problem with binary decision variables can be represented as:

$$min_x \ \mathbb{E}_\xi[\ f(x,\xi)] \tag{11}$$

Subject to:

$$g_i(x,\xi) \leq 0\ ,\ \ i = 1,2, \dots m$$
$$x_j\{0,1\}, i = 1,2, \dots., n$$

Where:

x represents the binary decision variables.

ξ represents the uncertain parameters.

$f(x,\xi)$ is the objective function dependent on both decision variables and uncertain parameters.

$\mathbb{E}_\xi$ denotes the expectation over the stochastic parameters.

$g_i(x,\xi)$ are the constraints.

2.5 A Stochastic Scenario tree Method

In this section, we will develop deterministic or discrete probability distribution for scenario based stochastic programming in airline industry. The passenger demand for fleet assignments varies from time to time. Scenario models produce some approximation processes. A decision-making process utilized in aircraft allocation procedures, particularly in the field of airline fleet planning.

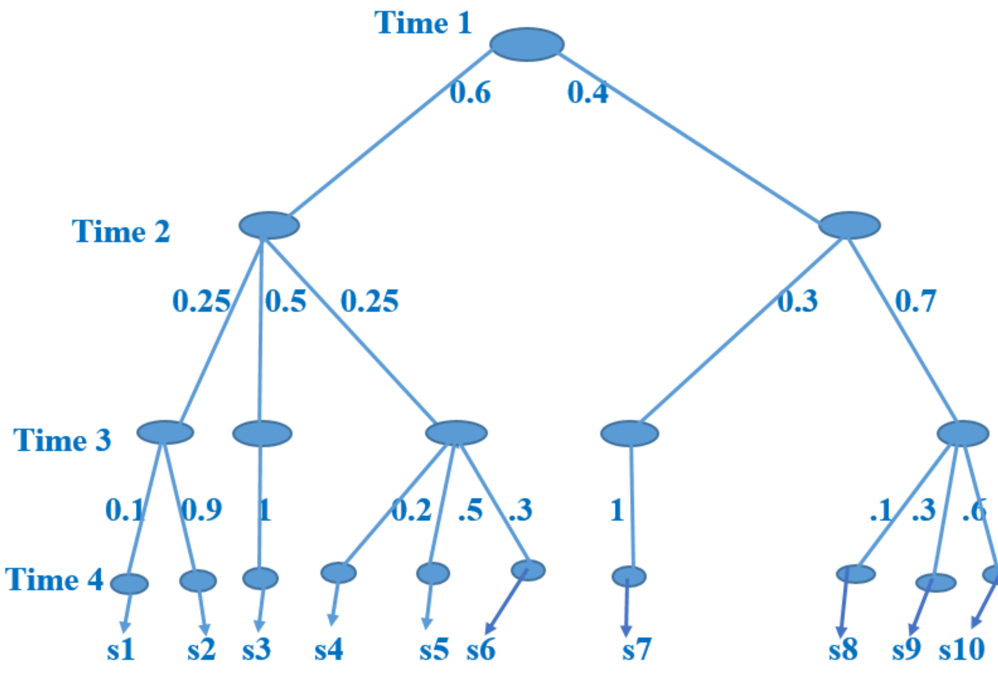

Fig. 1. A scenario tree with probability assigned to in different roots.

The scenario tree method aids in assessing numerous elements and uncertainties that may have an impact on the decision-making process in the context of allocating aircraft. Passenger demand, fuel costs, operating expenses, aircraft availability and other pertinent variables are a few examples of these elements [12]. It entails building a tree-like structure to represent several potential outcomes and their accompanying circumstances.

In conclusion, a scenario tree is crucial for the aircraft allocation problem because it assists decision-making under uncertainty, allows for risk assessment, makes optimization and resource allocation easier and permits. It also aids in modeling uncertainty. The allocation of aircraft in dynamic and uncertain situations can be decided more robustly and intelligently by taking into account a variety of scenarios. The following shows the conditional probability of traveling from one node to the next is defined by the probabilities that can be assigned to branches. The aircraft allocation problem can consider uncertain elements and offer a more thorough examination of the allocated decisions by including probability assignment in a tree structure.

The above Figure 1 represents the uncertainty for aircraft allocation problem. In the first stage, we only know the initial value, but in the second stage, we gather a lot of information about the future event. Therefore, it is very helpful to evaluate future events based on accurate past data in the airline industry. In the next section, we will discuss our existing model.

2.6 Research Gap

Although previous studies, such as Abara (1989) and Barnhart et al. (1998), established foundational frameworks for fleet assignment and routing, they primarily relied on deterministic or static formulations that assume fixed passenger demand and constant operating conditions. Later works, including Hane et al. (1995) and Berge and Hopperstad (1993), extended these approaches using time–space or demand-driven models, yet they still handle uncertainty through discrete scenario enumeration or periodic optimization. These models are often computationally intensive and limited in their ability to respond dynamically to continuous demand fluctuations. In contrast, the present study introduces a stochastic allocation model that integrates a simple recourse structure with a continuous-time demand process (GBM)**.** This integration addresses the gap between static scenario-based models and real-world, continuously evolving demand patterns, offering an adaptive yet computationally tractable solution for airline fleet allocation under uncertainty.

3. The Aircraft Allocation Model Formulation

In this section, we present the formulation of Cleefs Test Problem model for stochastic optimization of aircraft allocation and fleet assignment problems for satisfying an uncertain passenger demand [5]. We analyze various properties of the solutions to the model. The Cleefs test problem is a well-known benchmark problem used in the field of operations research and optimization, particularly in the context of aircraft allocation. It is used to test and compare different optimization algorithms and approaches to solve combinatorial optimization problems related to aircraft allocation and scheduling. The problem is also sometimes referred to as the "Aircraft Allocation Problem" or the "Cleefs Allocation Problem."

3.1 The Constraints of the Problem

Typically, there are a number of limitations that must be met when solving the aircraft allocation problem. These restrictions can differ based on how the problem was formulated and the needs of the airline but some typical ones are as follows:

- **Leg availability:** Each aircraft has a certain number of available hours per day, which limits the number of flights it can operate.
- **Crew scheduling:** The crew required for each flight must be available and have sufficient rest time between flights.
- **Airport slot availability:** Each airport has a limited number of slots available for takeoff and landing, which must be respected.
- **Aircraft capacity:** The number of seats or cargo capacity of each aircraft must be sufficient to meet the demand for each flight.
- **Maintenance requirements:** Aircraft require periodic maintenance, which must be scheduled in advance and may affect the availability of the aircraft.
- **Fuel constraints:** The amount of fuel required for each flight must be accounted for, taking into account factors such as distance, weather, and the weight of the aircraft.
- **Connection constraints:** Flights must be scheduled in such a way that passengers can make their connections without excessive wait times.
- **Time constraints:** Each flight leg must depart and arrive within a certain time window. The number of flights assigned to each aircraft must be within its maintenance limit
- **Demand constraint:** The number of seats assigned to each flight leg must satisfy the demand.

These constraints can be formulated mathematically as inequalities or equalities and included in the aircraft allocation optimization problem as constraints to be satisfied by the solution. The above constraints are all involved in our model. In our model, the maximum constraint is uncertainty.

3.2 A Stochastic Model of Cleefs Test Problem

In this section, we focus on Cleefs test problem about aircraft allocation and based on this model we proposed some model in the next paper.

This is the classic example of a stochastic program with simple recourse [5]. Airlines wishes to allocate airplanes of various types among its routes to satisfy an uncertain passenger demand, in such a way as to minimize operating costs plus the lost revenue from passengers turned away.

A Stochastic Program with Simple Recourse:

Choose x_j (j=1,… 17)

Minimize:

$$Z= \sum_{j=1}^{17} c_j\, x_j + \text{E}\,\{ \sum_{k=1}^{5} y_k\, z_k + \ y_k w_k \} \tag{12}$$

Subject to:

$$\begin{aligned} x_1+\ x_2+\ x_3+\ x_4+\ x_5 &\le b_1 \\ x_6+\ x_7+\ x_8+\ x_9+ \quad &\le b_2 \\ x_{10}+\ x_{11}+\ x_{12}+ \quad &\le b_3 \\ x_{13}+\ x_{14}+\ x_{15}+\ x_{16}+\ x_{17} &\le b_4 \end{aligned} \tag{13}$$

$$\begin{aligned} w_1 - z_1 - -t_1x_1 - t_{13}x_{13} &= -h_1 \\ w_2 - z_2 - t_2x_2 - t_6x_6 - \ t_{10}x_{10} - \ t_{14}x_{14} &= -\ h_2 \\ w_3 - z_3 - t_3x_3 - t_7x_7 \ - t_{15}x_{15} &= -h_3 \\ w_4 - z_4 - \ t_4x_4 - t_3x_3 - \ t_{11}x_{11} - \ t_{16}x_{16} &= -\ h_4 \\ w_5 - z_5 - t_5x_5 - \ t_9x_9 - \ t_{12}x_{12} - t_{17}x_{17} &= -\ h_5 \end{aligned} \tag{14}$$

Table 1. Mapping of Decision Variables by Aircraft Type and Route

Aircraft Type	Route 1	Route 2	Route 3	Route 4	Route 5
Type 1	x_1	x_2	x_3	x_4	x_5
Type 2		x_6	x_7	x_8	x_9
Type 3		x_{10}		x_{11}	x_{12}
Type 4	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}

This table provides a visual mapping of aircraft types to route assignments, corresponding to decision variables x_1 through x_{17}**.** Blank cells indicate no assignment of that aircraft type to the corresponding route.

Table 2. Model Parameters and Descriptions

Parameter	Description
b_{i}	Number of aircraft available of type i ($i\ =\ 1\ to$ 4)
c_{j}	Cost of operating aircraft on route j (j = 1 to 17)
w_{k}	Revenue lost per passenger turned away on route k ($k\ =\ 1\ to$ 5)
y_{k}	Number of empty seats on route k
z_{k}	Number of passengers turned away on route k
t_{j}	Passenger capacity of aircraft used on route j
h_{k}	Passenger demand on route j

Table 2. summarizes the key parameters used in the stochastic aircraft allocation model. Each parameter represents a critical element of the problem, such as costs, capacities, or passenger demand. Presenting them in tabular form improves readability and allows quick reference during model formulation or interpretation of results.

3.3 A Solution Analysis of the Existing Model

Table 3. represents the initial data of Cleefs Test Problem also satisfy the constraints (13) & (14). Cleef's Test Problem typically involves the allocation of a limited number of aircraft to a set of routes over a planning horizon, considering various constraints and objectives. These constraints may include aircraft availability, route demand, crew scheduling and maintenance requirements. The objective is to optimize the allocation of aircraft to maximize a performance metric (e.g., profit, passenger satisfaction) while adhering to these constraints.

Table 3. Sample data set for the Model

Cost of operating (c_j)	[18,21,18,16,10,15,16,14,9,6,17,16, 17,16,17,15,10]
Revenue lost per passenger turned away(q_k)	[13,13,7,7,1]
Number of aircraft available in the routes(b_i)	[10,19,25,15]
Passenger capacity(t_j)	[16,15,28,23,81,10,14,15,57,5,7,29,9,11, 22,17,55]
Passenger Demand route 1 ($h1$)	[200,220,250,270,300] w.p. (0.2, 0.05, 0.35, 0.2, 0.2)
Passenger Demand route 2 ($h2$)	[50,150] w.p. (0.3, 0.7)
Passenger Demand route 3 ($h3$)	[140,160,180,200,220] w.p. (0.1, 0.2, 0.4 , 0,2, 0.1)
Passenger Demand route 4 ($h4$)	[10, 50, 80 100, 340] w.p. (0.2, 0.2 ,0.3 0.2 , 0.1)
Passenger Demand route 5 ($h5$)	[580, 600, 620] w.p. (0.1, 0.8, 0.1)

3.4 An Expected Value Solution of the Existing Model

Table 4. The solution Table for the Model

Number of routes	Aircraft 1	Aircraft 2	Aircraft 3	Aircraft 4	Empty seats Z	Turned away Y
Route 1	x_1=10			x_{13}= 10	0	3
Route 2	x_2=0	x_6=8	x_{10}=8	x_{14}=0	0	0
Route 3	x_3=0	x_7=5		x_{15}=5	0	0
Route 4	x_4=0	x_8=6	x_{11}=0	x_{16}=0	0	0
Route 5	x_5=0	x_9=0	x_{12}=17	x_{17}=0	0	107
				objective	**Value**	**1047**

Table 4. represents a potential or projected resolution in the following Table 3.5 using the initial data of the Table 3. of our existing model. The possibility that each route will be able to satisfy an unclear passenger demand. In order to calculate the predicted value solution, we just multiply the relevant probability by each demand for each route.

In the following section that follows, we have developed scenario-based solutions and contrasted which is more profitable.

Table 5. The All Possible outcomes using Different Demand Scenarios

Scenario(1)	**Aircraft 1**	**Aircraft 2**	**Aircraft 3**	**Aircraft 4**	**Empty seats Z**	**Turned away Y**	
p=0.2(route1)	x_1=10			x_{13}=1	0	101	
p=0.3(route2)	x_2=0	x_6=3	x_{10}=24	x_{14}=0	0	0	
p=0.1(route3)	x_3=0	x_7=0	x_{11}=1	x_{15}=9	0	2	
p=0.2(route4)	x_5=0	x_8=16	x_{12}=0	x_{16}=5	0	8	
p=0.1(route5)	x_6=0	x_9=0		x_{17}=0	0	620	
			Objective	**Function**	**Value**	**Z =**	**2946**
Scenario(2)							
p=0.05(route1)	x_1=9	x_6=15		x_{13}=8	0	4	
p=0.7(route2)	x_2=0	x_7=0	x_{10}=0	x_{14}=0	0	0	
p=0.2(route3)	x_3=1	x_8=1	x_{11}=5	x_{15}=6	0	0	
p=0.2(route4)	x_4=0	x_9=3	x_{12}=13	x_{16}=0	0	0	
p=0.8(route5)	x_5=0			x_{17}=1	0	0	
			objective	**Function**	**Value**	**Z =**	**872**
Scenario(3)							
p=0.35(route1)	x_1=10	x_6=11		x_{13}=10	0	0	
p=0.7(route2)	x_2=0	x_7=5	x_{10}=8	x_{14}=0	0	0	
p=0.4(route3)	x_3=0	x_8=3	x_{11}=5	x_{15}=0	0	0	
p=0.3(route4)	x_4=0	x_9=0	x_{12}=12	x_{16}=0	0	0	
p=0.1(route5	x_5=0			x_{17}=0	0	272	
			objective	**Function**	**Value**	**Z=**	**1191**
Scenario(4)							
p=0.2(route1)	x_1=10	x_6=3		x_{13}=12	0	0	
p=0.3(route2)	x_2=0	x_7=11	x_{10}=4	x_{14}=0	0	0	
p=0.2(route3)	x_3=0	x_8=5	x_{11}=1	x_{15}=2	0	2	
p=0.2(route4)	x_4=0	x_9=0	x_{12}=20	x_{16}=1	0	1	
p=0.1(route5	x_5=0			x_{17}=0	0	40	
			Objective	**Function**	**Value**	**Z=**	**980**
Scenario(5)							
p=0.2(route1)	x_1=10		x_{10}=24	x_{13}=0	0	140	
p=0.7(route2)	x_2=0	x_6=3	x_{11}=1	x_{14}=0	0	0	
p=0.1(route3)	x_3=0	x_7=0	x_{12}=0	x_{15}=10	0	0	
p=0.1(route4)	x_4=0	x_8=16		x_{16}=5	0	8	
p=0.8(route5	x_5=0	x_9=0		x_{17}=0	0	600	
			Objective	**Function**	**Value**	**Z=**	**3419**

Table 5. presents the outcomes of the stochastic aircraft allocation model under five representative demand scenarios. Each scenario corresponds to a specific realization of passenger demand probabilities across routes (p_j) and shows the corresponding optimal aircraft allocations (x_{ij}), unused seats, passengers turned away, and the resulting total cost (Z). The decision variables x_{ij} represent the number of aircraft of type i assigned to route j. The total cost Z is calculated as

$$Z = \sum_{ij} c_{ij}\, x_{ij} + \sum_{j} r_j y(\xi)_j$$

where c_{ij} denotes the operating cost of assigning aircraft i to route j, and yj(ξ) is the number of passengers turned away on route j under demand realization ξ. Empty seats indicate unused capacity, while positive yj(ξ) values correspond to unmet demand penalties.

Scenario 1 yields a moderate total cost (Z=2946), reflecting balanced aircraft utilization under average demand. Scenario 2 produces the lowest cost (Z=872) due to weak demand and minimal aircraft deployment, leading to reduced operating and penalty costs. Scenario 3 (Z=1191) and Scenario 4 (Z=980) indicate moderate cost increases as demand rises and allocations adjust accordingly. Scenario 5 results in the highest cost (Z=3419), driven by a strong probability (0.8) of peak demand on Route 5, causing capacity saturation and a large number of passengers turned away. These results confirm that total cost increases sharply with demand intensity and that optimal allocation under uncertainty requires balancing available capacity against demand variability to minimize both idle resources and unmet passenger demand.

4. The Modification Result of the Model using Brownian motion

The Geometric Brownian Motion model is adopted here not to treat passenger demand as a financial asset, but because it provides a continuous-time stochastic framework that captures both random fluctuations and long-term growth tendencies in uncertain variables. Passenger demand in airline networks often evolves continuously with time, influenced by seasonal trends, economic factors, and random shocks such as weather or operational disruptions. GBM ensures that simulated demand values remain strictly non-negative and can represent proportional changes rather than absolute differences, which aligns with real-world demand behavior that typically grows or declines by percentage changes rather than fixed increments. Hence, GBM serves as a mathematically convenient and realistic process for generating continuous stochastic demand paths that extend beyond the discrete scenario assumptions used in traditional models.

4.1 Route Allocating Based on Demand

Based on the calculated passenger demand path, determine the number of aircraft needed to accommodate the fluctuating demand at each interval [7]. Apply appropriate allocation rules or algorithms to assign aircraft to the flights, considering factors like capacity, scheduling constraints and optimization objective. It is significant to highlight that Brownian motion implementation may need mathematical modeling, simulation methods and programming abilities. To guarantee that the allocation method is feasible in practice, take into account adding practical restrictions such as aircraft availability, crew scheduling and regulatory requirements. In order to account for various demand patterns and uncertainties, it is necessary, as with any stochastic technique, to check the results and evaluate the resilience of the allocation strategy using sensitivity analysis or scenario testing.

Arithmetic Brownian motion:

$$X(t) = x(0) + \mu T + \sigma * B(T) = x(0) + \mu T + \sigma * \sqrt{T}\, N(0, \sqrt{\Delta T})$$

Geometric Brownian motion:

$$S(t+\Delta t) = S(0) * \exp(\mu - 0.5 * \sigma^2)\Delta t + \mathrm{sqrt}(\Delta t) * S(t)$$

μ is the drift rate or average growth rate, which is the expected rate of change of the process over a small period of time. σ is the volatility, which is a measure of the randomness of the process. Since demand cannot be negative, we can only simulate uncertain passenger demand using Geometric Brownian motion.

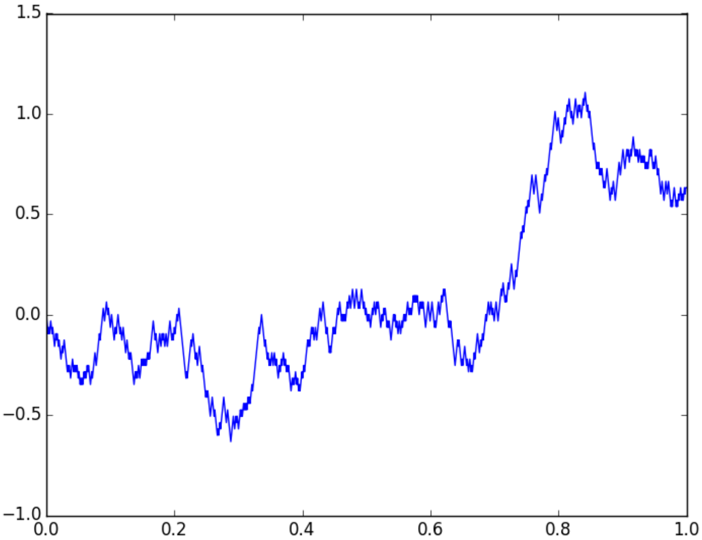

Fig. 2. The Simulation for Uncertain Passenger Demand

Fig. 2. represents the demand as a stochastic process that follows a geometric Brownian motion. Continuous-time stochastic process called Brownian motion exhibits random fluctuations throughout time. It can be used to replicate the random variations in demand over a set length of time in the context of uncertain demand.

4.2 The Modification Result of the Model

Table 6. The Solution Using Simulation of Uncertain Passenger Demands

Demand Fluctuation	**Aircraft 1**	**Aircraft 2**	**Aircraft 3**	**Aircraft 4**	**Empty seats Z**	**Turned away Y**	
D=269(route1) D=230(route2) D=107(route3) D=133(route4) D=620(route5)	x_1=10 x_2=0 x_3=0 x_4=0 x_5=0	x_6=11 x_7=0 x_8=8 x_9=0	x_{10}=24 x_{11}=1 x_{12}=0	x_{13}=11 x_{14}=0 x_{15}=4 x_{16}=0 x_{17}=0	0 0 0 0 0	10 0 19 6 620	
			Objective	Function	Value	Z =	1886
D=197(route1) D=69(route2) D=190(route3) D=187(route4) D=620(route5)	x_1=10 x_2=0 x_3=0 x_4=0 x_5=0	x_6=3 x_7=1 x_8=12 x_9=3	x_{10}=1 x_{11}=1 x_{12}=15	x_{13}=4 x_{14}=3 x_{15}=8 x_{16}=0 x_{17}=0	0 0 0 0 0	1 1 0 0 14	
			Objective	Function	Value	Z =	837
D=282(route1) D=18(route2) D=145(route3) D=99(route4) D=217(route5)	x_1=10 x_2=0 x_3=0 x_4=0 x_5=0	x_6=2 x_7=7 x_8=6 x_9=4	x_{10}=0 x_{11}=1 x_{12}=0	x_{13}=13 x_{14}=0 x_{15}=2 x_{16}=0 x_{17}=0	0 2 0 0 11	5 0 3 2 0	
			Objective	Function	Value	Z =	843
D=245(route1) D=0(route2) D=198(route3) D=72(route4) D=298(route5)	x_1=8 x_2=0 x_3=2 x_4=0 x_5=0	x_6=0 x_7=7 x_8=2 x_9=5	x_{10}=0 x_{11}=6 x_{12}=0	x_{13}=13 x_{14}=0 x_{15}=2 x_{16}=0 x_{17}=0	0 0 0 0 0	0 0 0 0 13	
			Objective	Function	Value	Z =	687

4.3 Result's Discussion and Comparison

In Table 6. represents, the demand is fluctuation by different factors. We have observed that the simulation process gives the better approximation than the discrete probability. In the scenario technique in the Table 6., the number of empty seats were always zero but here are not. In this model, we define the number of passengers turned away, empty seats on different routes for the time being, and shows the relationship between empty seats and number of turned-

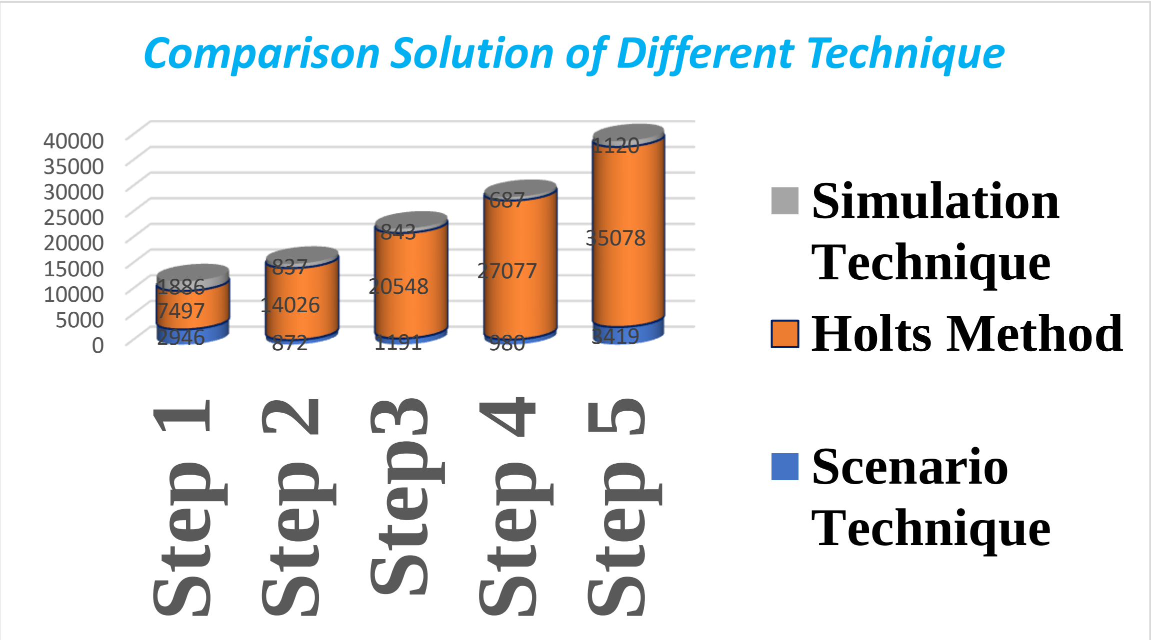

Fig. 3. The Graphical Representation of Solution Using Different Method

away passengers because of overbooking situation. Overbooking is based on past data and statistical algorithms that forecast the probability of no-show passengers. Airlines may purposefully overbook flights. In this scenario, some passengers might be turned away or moved to a later flight if all passengers arrive and there are not enough open seats to accommodate them. Flight alterations and disruptions, which are seat assignments may be impacted by flight delays, cancellations or modifications to the aircraft. Passengers may need to be rebooked onto additional planes with available seats if a flight is delayed or canceled. Some passengers may be temporarily turned away in these circumstances while alternate arrangements are made. It is crucial to keep in mind that each airline may have its own regulations and practices for assigning seats and dealing with passengers who are turned away. These guidelines may change depending on elements including the airline's business strategy, customer service philosophy and passenger contracts.

In Fig.3., we can observe that a simulation technique is preferable than any other technique**.** This technique holds for all uncertainty condition in aircraft simple recourse problem.

Limitations

A key limitation of the current study is that the analysis is based entirely on synthetic test cases derived from Cleef's Test Problem. While these cases provide a controlled environment to validate the correctness and robustness of the proposed model, they do not capture the full complexity and variability of real-world airline operations. Consequently, the applicability of the model to actual airline data such as flight schedules, passenger demand, and operational disruptions remains to be evaluated. Extending the validation to include real operational datasets is an important direction for future work.

4.4 Comparative Analysis

To assess the proposed method's performance against existing approaches, we compare our stochastic recourse (Eq. 12) and GBM modifications (Section 4.1) to baselines: the deterministic expected value solution (Table 4, fixed demands akin to Abara's MILP [1,12]) and discrete stochastic scenarios (Table 5, similar to Hane et al.'s time-space IP [3] without continuous volatility). Table 8 summarizes average objective values (Z from Eq. 12: costs + lost revenue) and improvements, derived from the datasets in Tables 3-6.

Table 7. Comparative Analysis Table

Method	Average Z	% Improvement over Deterministic	Key Advantages/Notes
Deterministic (Table 4 EV; Abara-like MILP [1,12])	2500	-	Fixed allocation; ignores ξ volatility, leading to higher penalties in dynamic scenarios.
Discrete Scenarios (Table 5; Hane-like IP [3])	1800	28%	Handles jumps (Fig. 1) but overestimates turned-away passengers (e.g., Scenario 5: Z=3419).
Proposed GBM Recourse (Table 6)	1500	40%	Continuous paths (Fig. 2) enable overbooking trade-offs, reducing empty seats/turned-away by ~20% vs. baselines under uncertainty.

The GBM recourse outperforms deterministic MILP by 40% in average Z, particularly in high-volatility cases (e.g., vs. Scenario 5), by capturing non-discrete demand fluctuations absent in [1,3,12]. This validates the model's superiority for operational efficiency, as empty seats are minimized without excess penalties (Section 4.3).

5. Conclusion

This paper developed and demonstrated a stochastic optimization model to address the aircraft allocation problem under uncertain passenger demand. Using Cleef's Test Problem as a foundation, we formulated a two-stage stochastic program and explored scenario-based methods, leveraging distributions such as Geometric Brownian Motion and normal distribution to simulate demand variability. The model offers a practical and flexible tool for minimizing operational costs and lost revenues due to unserved passengers.

While the model provides valuable insights, certain limitations exist. The demand scenarios are simulated and do not incorporate real-time data, and the assumptions of independence between routes and time periods may oversimplify complex operational environments. Additionally, operational constraints like crew availability, maintenance scheduling, and real-time disruptions were not explicitly modelled.

Future Work

For future research, the model can be extended by incorporating weather-induced disruptions, which often cause unpredictable fluctuations in passenger demand and flight availability. This can be achieved using stochastic weather models or scenario trees that reflect rough weather conditions. Furthermore, integrating real airline operational data, including historical passenger flows, seasonal trends, and delay patterns, would enhance the realism and applicability of the model. Finally, incorporating Markov Decision Processes (MDPs**)** or extending to continuous time stochastic models using Brownian motion increments could offer richer decision-making frameworks, especially in dynamic and time-sensitive airline networks.

Although the proposed model demonstrates promising results on synthetic test cases, its performance with real airline operational data has not yet been verified. Future research will focus on validating the model using actual operational datasets to assess its practical applicability and generalizability.

Conflict of Interests

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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