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SIP Model and Bifurcation Analysis for Spread of Misinformation in Online Social Networks

Nitesh Narayan
Department of Mathematics, R. D. and D. J. College, Munger University, Bihar, India
E-mail: narayan.nitesh26@gmail.com
ORCID iD: https://orcid.org/0000-0002-7536-3426

Kaushik Haldar*
Department of Mathematics, Marwari College Ranchi, Ranchi, India
E-mail: haldarkaushik@gmail.com
ORCID iD: https://orcid.org/0000-0001-8814-8429
*Corresponding Author

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Abstract: The spreading of misinformation in Online Social Networks (OSNs) is quite similar to the spreading of infection in biological diseases. As the biological virus spreads and makes one infected as well as those who come into contact with the infected person, the nature and behavior of the spread of misinformation in an online social network is similar. So in order to understand the functioning of misinformation and to control over epidemic outbreak of misinformation in OSNs, epidemic models can be quite handy. The introduction of Bifurcation theory in the epidemic model explains the qualitative behavior of the system with changes in parameters. In this paper, we have introduced the Susceptible, Infectious, and Protected (SIP) model with Bifurcation for the propagation of misinformation in OSNs. Here Bifurcation is due to the limited number of users who are ready to adopt the Security and Privacy Policies of Social Network Sites. We define the threshold number for the system of equations and explain the stability of an Infection Free Equilibrium (IFE), representing the absence of misinformation. We have discussed about the endemic equilibrium point and bifurcation conditions along with its nature and stability at those points. Also, we have shown global stability at the endemic Equilibrium of the system. Finally, numerical simulation has been used to show the existence of bifurcation in the system with a change in the value of parameters.

Index Terms: SIP Model, Bifurcation, Stability, Numerical Simulation, Misinformation, Social Network

1. Introduction

An Online Social Network is a web-based platform which is widely used by the people who share similar personal or career interest, activity, background or real-life connection to build social relations with each other. The online social network provides a wide range of services and thus it varies in format and features. The OSNs like Facebook, Instagram, and Twitter carry millions of data, information, photos, blogging, chats which may be both professional and personal. It is worth saying that social network sites have become an integral part of life as we spent hours of our routine life on these sites. As the millions of users of OSNs share a tremendous number of data and information in each and every sphere, social network sites have become data and information chest of different doing. It is oblivious that OSN, being a data and information chest is a soft target for hackers and attackers. The easy accessibility of the social network sites and unawareness of users regarding their online data makes social network sites most vulnerable. So it becomes a challenge for the Social Network provider to provide a safeguard to their users against such malicious threats. Many Social Network sites provide Security to their users and update from time to time but less awareness of technology among users and lack of concrete safeguard still make online social network's data and information easy hand for data attackers.

1.1. Related work

The transmission of malicious content in Social networks is similar to the nature of transmission of viruses in Biology. So the various existing epidemic model has a greater role to understand attacks on OSNs. In recent years, the traditional SIR model [1-2] in which the population is divided into Susceptible, Infectious and Recover compartments according to their epidemics stages is widely used in the non-biological fields. Cannarella and Spechler [3] use an epidemic model to introduce a new infectious recovery SIR (irSIR), model. Further Zhu, Nie and Li [3] proposed the irSIRS model in their work. In these models, adoption and abandonment of OSNs are considered as infection and recovery. Zhang and Wang [4] introduced an improved SIR model for OSNs with crowding or protection effects. Further Bifurcation theory is introduced in the field of epidemic models which help in the study of qualitative changes of the system with the change in the parameter. Wang and Raun [5] studied the simple SIR model with a constant removal rate under limited resources. The given model shows different types of bifurcation under constant removal rate. In our work, we introduced the Bifurcation theory for an epidemic model on OSNs. We proposed a Susceptible, Infectious, and Protected (SIP) model for online Social networks to understand the transmission of malicious objects with the role of Security and Privacy services provided by Social Network sites. The users of OSNs are less aware of these services due to which they may undergo various attacks. Only the limited numbers of users are aware of the safety services. Such users are referred to as a protected class since they are not available for any types of attacks. The number of users of a protected class is constant. Thus need to study about Bifurcation of the system with changes in parameters. The rest of this paper is organized as follows: Section 2 contains Model formulation; Section 3 explains the Infection free equilibrium. In Section 4 we find out the endemic equilibrium points and bifurcation, Section 5 describes the Nature and stability of these points. Section 6 performed numerical simulation. In Section 7 and 8 discussed about the real data set and experiment performed on this. Finally, section 9 contains the conclusion of the paper.

2. Model Formulation

Here a SIP (Susceptible- Infectious- Protected) model for Social Networking is proposed. We divided the social network users into Susceptible (S), Infectious (I), and Protected (P) classes. Any of the users who are associated with the network are susceptible. The users who make the new post, sharing or reacting on any existing post become malicious come into infectious class. Some of the users who follow the security measure provided by the online social sites are guarded against malicious attacks, come into a protected class. The users of this class are less available for attacks. Further, if the users belong to a protected class do not follow the updated privacy and security measure from time to time, they may also become available for attacks and come into susceptible classes. Again we have assumed that only the limited number of users belongs to the protected class because most of the social network users are less aware of the security services provided by online social sites. The compartmental model based on the above considerations is depicted in the figure1.

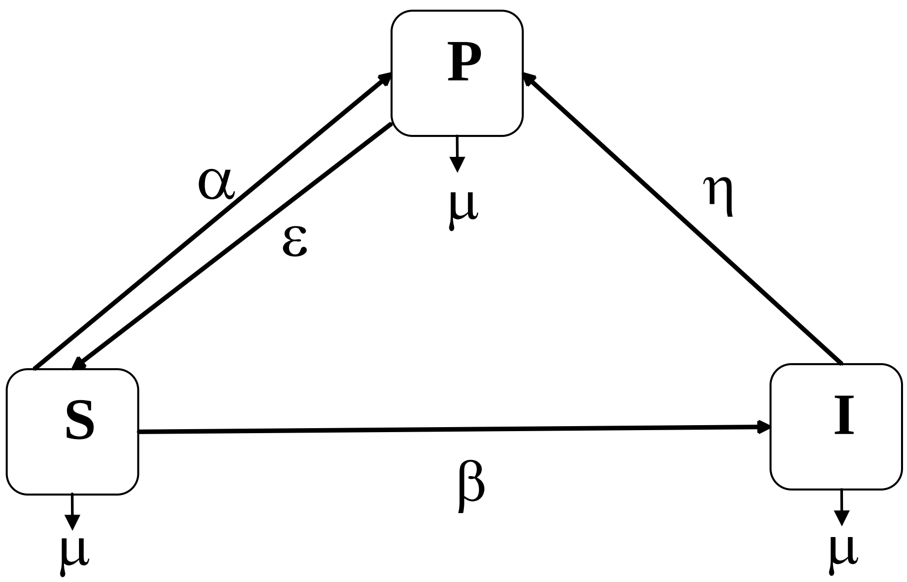

Fig. 1. Graphical Representation of the Model

The system of differential equations for the above model is given below

$$\frac{dS}{dt} = \lambda - \left(\beta I + \alpha + \mu\right) S + \varepsilon P \tag{1}$$

$$\frac{dI}{dt} = \beta S I - \eta - \left(\mu + \gamma\right) I \tag{2}$$

$$\frac{dP}{dt} = \alpha S + \eta - (\mu + \varepsilon)P \tag{3}$$

where $S + I + P = 1$ and λ is the rate of a new entry in population, β stands for the rate at which susceptible users become infected, γ shows the number of users removing from the network due to malicious behavior, η is the rate at which the infected users become protected. Here we considered η is constant. Again consider ε is the rate of Protected becomes susceptible, α stands for Susceptible becomes protected after following privacy policies and μ is a number of population leaving the network naturally.

3. Infection Free Equilibrium

Infection free equilibrium is defined as the stage at which social networks have no any rumors. Let Infection free equilibrium point for system of equations (1-3) is (S^0, I^0, P^0). Now solving the system (1-3) at Infection free equilibrium we get

$$S^{0} = \frac{\lambda(\mu + \varepsilon)}{\mu(\mu + \alpha + \varepsilon)} \tag{4}$$

$$P^{0} = \frac{\lambda\alpha}{\mu(\mu + \alpha + \varepsilon)} \tag{5}$$

$$I^{0} = 0 \tag{6}$$

Set Basic Reproduction Number for system (1-3),

$$R_o = \frac{\beta}{(\mu + \gamma)} \tag{7}$$

Also introduce a threshold number,

$$R_P = \frac{\beta\{\lambda(\mu + \varepsilon) + \eta\varepsilon\}}{\mu(\mu + \gamma)(\mu + \alpha + \varepsilon)} = R_o \frac{\lambda(\mu + \varepsilon) + \eta\varepsilon}{\mu(\mu + \alpha + \varepsilon)} \tag{8}$$

Theorem 3.1: System (1-3) is stable at Infection Free Equilibrium when $R_o < 1$ and unstable when $R_o > 1$.
Proof: Jacobian of the system at Infection free equilibrium is given as

$$J = \begin{bmatrix} -(\mu + \alpha) & -\beta & \varepsilon \\ 0 & \beta - (\mu + \gamma) & 0 \\ \alpha & \eta & -(\mu + \varepsilon) \end{bmatrix} \tag{9}$$

Here the stability of the system depends upon nature of roots of Jacobian (9).
For Eigen values consider $|J - \lambda I| = 0$, where λ is eigen value of (9).

On solving $|J - \lambda I| = 0$, we get

$$\{\beta - (\mu + \gamma) - \lambda\}\left[\lambda^2 + \{(\mu + \alpha) + (\mu + \varepsilon)\}\lambda + (\mu + \alpha)(\mu + \varepsilon) + \varepsilon\right] = 0 \tag{10}$$

Here, $\lambda^2 + \{(\mu + \alpha) + (\mu + \varepsilon)\}\lambda + (\mu + \alpha)(\mu + \varepsilon) + \varepsilon = 0$

Since above quadratic equation of λ has $tr < 0$ and $\det > 0$, so both roots of this equation will be negative. Again from (10), $\{\beta - (\mu + \gamma) - \lambda\} = 0$ gives $\lambda = (\mu + \gamma)(R_o - 1)$. Thus when $R_o < 1$ then all eigen values of (9) are negative. Hence the system (1-3) will be local asymptotically stable when $R_o < 1$ and unstable when $R_o > 1$.

4. Endemic Equilibrium and Bifurcation

Here Endemic Equilibrium is the stage at which system persists infection for long time. Let the system (1-3) has endemic equilibrium $\left(S^*, I^*, P^*\right)$. Then on solving (1-3) at this point, we get

$$S^* = \frac{\lambda(\mu+\varepsilon)+\eta\varepsilon}{\beta I^*(\mu+\varepsilon)+\mu(\mu+\alpha+\varepsilon)} \tag{11}$$

$$P^* = \frac{\alpha S^* + \eta}{(\mu+\varepsilon)} \tag{12}$$

The value of I^* is be given by the equation,

$$A\left(I^*\right)^2 + BI^* + C = 0 \tag{13}$$

where,

$$A = \frac{\beta(\mu+\varepsilon)}{\mu(\mu+\alpha+\varepsilon)} \tag{14}$$

$$B = \left(R_P - H - 1\right) \tag{15}$$

$$C = \frac{\eta}{\mu+\gamma} \tag{16}$$

Here, R_P is given by (8) and

$$H = \frac{\beta\eta(\mu+\varepsilon)}{\mu(\mu+\gamma)(\mu+\alpha+\varepsilon)}$$

Now value of I^* can be calculated from the equation $A\left(I^*\right)^2 + BI^* + C = 0$. The nature and value of solution depends upon the discriminant D of the given quadratic equation.

Here discriminant, $D = B^2 - 4AC$

Case 1: When $D < 0$ or $B^2 - 4AC < 0$ or $\left(\sqrt{R_P} - 1\right)^2 < H$

Hence equation (13) has no solution.

Case 2: When $D = 0$ or $B^2 - 4AC = 0$ or $\left(\sqrt{R_P} - 1\right)^2 = H$

In this case there is only one solution given by

$$I^* = \frac{B}{2A} \tag{17}$$

Case 3: When $D > 0$ or $B^2 - 4AC > 0$ or $\left(\sqrt{R_P} - 1\right)^2 > H$

Here there are two solutions which are given by

$$I_1^* = \frac{B + \sqrt{B^2 - 4AC}}{2A} \text{ and } I_2^* = \frac{B - \sqrt{B^2 - 4AC}}{2A} \tag{18}$$

Now following Theorem concludes the above results

Theorem 4.1:

(a) If $R_P \le 1$ then the system (1)-(3) has no positive solution.

(b) If $R_P > 1$ then system (1)-(3) has

(i) No solution if $B^2 - 4AC < 0$ or $\left(\sqrt{R_P} - 1\right)^2 < H.$

(ii) Only one solution if $B^2 - 4AC = 0$ or $\left(\sqrt{R_P} - 1\right)^2 = H.$

(iii) Two solutions if $B^2 - 4AC > 0$ or $\left(\sqrt{R_P} - 1\right)^2 > H.$

5. Nature and Stability at Endemic equilibrium points

In this section we discuss the nature of solution and stability of the system (1)-(3) at different endemic equilibrium points defined in theorem 4.1. For this purpose we reduce the system (1)-(3)

Reduced form of system (1)-(3) is given as

$$\begin{aligned} \frac{dI}{dt} &= \beta\left(1 - I - P\right)I - \eta - \left(\mu + \gamma\right)I \\ \frac{dP}{dt} &= \alpha\left(1 - I - P\right)I + \eta - \left(\mu + \varepsilon\right)P \end{aligned} \tag{5.1}$$

Here, $S + I + P = 1.$

Now Jacobian of reduced system (5.1) is given by

$$J = \begin{bmatrix} \beta - \beta P - 2\beta I - \left(\mu + \gamma\right) & -\beta I \\ -\alpha & -\alpha - \left(\mu + \varepsilon\right) \end{bmatrix}$$

$$= \left\{\beta P + 2\beta I + \left(\mu + \gamma\right) - \beta\right\}\left\{\alpha + \left(\mu + \varepsilon\right)\right\} - \alpha\beta I$$

$$= (\mu + \gamma)(\mu + \varepsilon) + (\mu + \varepsilon)(2\beta I + (\mu + \gamma) - \beta) + \alpha(\mu + \gamma) + \alpha\beta I + \alpha\beta P - \alpha\beta$$

$$= (\mu + \gamma)(\mu + \varepsilon) + (\mu + \varepsilon)(\beta I - \beta S) + \alpha(\mu + \gamma) + \alpha\beta(1 - S) - \alpha\beta$$

$$= (\mu + \varepsilon)\{(\mu + \gamma) + \beta(I - S)\} + \alpha\{(\mu + \gamma) - \beta S\}$$

$$= (\mu + \gamma)(\mu + \varepsilon)\left\{1 - \frac{\beta S - \beta I}{(\mu + \gamma)}\right\} + \alpha\{(\mu + \gamma)\left\{1 - \frac{\beta S}{(\mu + \gamma)}\right\}$$

So,

$$J = (\mu + \gamma)(\mu + \varepsilon + \alpha)\left\{1 - \frac{\beta S}{(\mu + \gamma)}\right\} + \beta I(\mu + \varepsilon) \tag{5.2}$$

As discussed above that system has three endemic equilibrium points I_1^*, I_2^* and I^* Thus we have to calculate the values of (5.2) on these points for the nature and stability of the system.

Case 1: When endemic equilibrium is I_1^*, then from (4.5)

$$I_1^* = \frac{B + \sqrt{B^2 - 4AC}}{2A}$$

Putting the values of A, B, C from (4.3) then

$$I_1^* = \frac{\left(R_P - H - 1\right) + \sqrt{\left(R_P - H - 1\right)^2 - 4H}}{2\beta(\mu + \varepsilon)} . \mu(\mu + \varepsilon + \alpha) \tag{5.3}$$

Again from (4.1),

$$S^* = \frac{\lambda(\mu+\varepsilon)+\eta\varepsilon}{\beta I^*(\mu+\varepsilon)+\mu(\mu+\varepsilon+\alpha)}$$

Putting $I^* = I_1^*$ in above and calculate the value of S^* we get

$$S^* = \frac{2R_P}{\left(R_P - H - 1\right) + \sqrt{\left(R_P - H - 1\right)^2 - 4H}} \cdot \frac{(\mu+\gamma)}{\beta} \tag{5.4}$$

Now putting values of S^* and I_1^* from (5.3) and (5.4) in (5.2) we get

$$|J| = -\mu(\mu+\alpha+\varepsilon)\left(\frac{4R_P - 2D + D^2}{2D}\right) - (\mu+\alpha+\varepsilon)\left(\frac{2R_P - D}{D}\right)$$

where, $D = \left(R_P - H - 1\right) + \sqrt{\left(R_P - H - 1\right)^2 - 4H}$

Here $|J| < 0$, so the product of eigen values is negative. Thus both the eigen values are different in sign. Hence I_1^* is saddle point and system is unstable at I_1^*.

(b) Similarly we can easily find that determinant of Jacobian 'J' at points I_2^* and S^* is positive and both Eigen values are negative. So point is a centre or focus or node and system is local asymptotically stable at this point.

(c) Now consider endemic equilibrium I^* from (4.4), so

$$I^* = \frac{B}{2A} = \frac{R_P - H - 1}{2\beta(\mu+\varepsilon)} . \mu(\mu+\varepsilon+\alpha)$$

From (4.1)

$$S^* = \frac{\lambda(\mu+\varepsilon)+\eta\varepsilon}{\beta I^*(\mu+\varepsilon)+\mu(\mu+\varepsilon+\alpha)}$$

$$P^* = \frac{\alpha S^* + \eta}{(\mu+\varepsilon)}$$

Putting these values in (5.2), the Jacobian becomes,

$$|J| = -H\mu(\mu+\alpha+\varepsilon) - (\mu+\alpha+\varepsilon)\gamma\left(\frac{R_P + H - 1}{2}\right)$$

Thus both the Eigen values of Jacobian 'J' have different sign. Hence the point is saddle point and system is unstable at this point.

6. Global stability at Endemic Equilibrium (S*, I1*, P*)

Theorem 5.1: The system (2.1) is global asymptotically stable at endemic equilibrium (S^*, I_2^*, P^*).

Proof: For global stability of the system (1)-(3) at Endemic equilibrium point (S^*, I_2^*, P^*). Consider Lapunov's function $L : \{(S, I, P) \in \Gamma : S, I, P > 0\} \to R$ by

$$L = (S - S^*) + (I - I^*) + (P - P^*) - (S^* + I^* + P^*)\ln\frac{S + I + P}{S^* + I^* + P^*}$$
$$+ \frac{(2\mu+\gamma)(S^* + I^* + P^*)}{\beta(I^* + P^*)}\left(I - I^* - I^*\ln\frac{I}{I^*}\right) + \frac{(2\mu+\gamma)}{\eta}\left(1 + \frac{S^*}{I^* + P^*}\right)\frac{\left(P - P^*\right)^2}{S + I + P}$$

$$L' = \frac{(S-S^*)+(I-I^*)+(P-P^*)}{S+I+P}\frac{d(S+I+P)}{dt} + \frac{(2\mu+\gamma)(S^*+I^*+P^*)}{\beta(I^*+P^*)}\frac{dI}{dt}$$
$$+\frac{(2\mu+\gamma)}{\eta}\left(1+\frac{S^*}{I^*+P^*}\right)\frac{dP}{dt} - \frac{(2\mu+\gamma)}{\eta}\left(1+\frac{S^*}{I^*+P^*}\right)\frac{\left(P-P^*\right)^2}{\left(S+I+P\right)^2}\frac{d(S+I+P)}{dt}$$

Putting the values from (1) to (3) we get,

$$L' = \frac{(S-S^*)+(I-I^*)+(P-P^*)}{S+I+P}\frac{d(S+I+P)}{dt} + \frac{(2\mu+\gamma)(S^*+I^*+P^*)}{\beta(I^*+P^*)}(\beta SI - \eta - (\mu+\gamma)I)$$
$$+\frac{(2\mu+\gamma)}{\eta}\left(1+\frac{S^*}{I^*+P^*}\right)(\alpha S+\eta-(\mu+\varepsilon)P) - \frac{(2\mu+\gamma)}{\eta}\left(1+\frac{S^*}{I^*+P^*}\right)\frac{\left(P-P^*\right)^2}{\left(S+I+P\right)^2}\frac{d(S+I+P)}{dt}$$

On simplification we get,

$$L' = -\frac{\mu\{(S-S^*)+(P-P^*)\}^2}{S+I+P} - \left(\mu+\gamma+\frac{(2\mu+\alpha)}{I^*+P^*}\right)\left(\frac{\left(I-I^*\right)^2}{S+I+P}\right)$$
$$-\frac{(2\mu+\alpha)}{2\eta}\left(1+\frac{S^*}{I^*+P^*}\right)\frac{\left(P-P^*\right)^2}{\left(S+I+P\right)^2}\frac{d(S+I+P)}{dt} + 2(\mu+\varepsilon)(S+I+P)$$

Putting the value of $\frac{d(S+I+P)}{dt}$ from (1) - (3)we get $L'<0$. Also $L'=0$ at $S=S^*, I=I_2^*$ and $P=P^*$.Therefore the largest compact invariant set in $\{(S,I,P)\in\Gamma : L'=0\}$ is singleton. So by LaSalle's invariance principal [6] the system is globally asymptotically stable. Hence the theorem is proved.

7. Numerical Simulations

In this section, we present the time-series plots of S(t), I(t), and P(t) in Figures 2, 3, and 4, respectively. Additionally, the phase-plane diagrams for P-I, S-I, and S-P are shown in Figures 6, 7, and 8. We assume a constant flow of infected users transitioning to the protected class due to limited awareness of social network privacy policies and infrequent service updates. This behavior induces bifurcation in the system, which is clearly observed in Figures 2–4 and 5–7 across all scenarios, further validating the analytical results discussed in the previous section. The parameters used for these simulations are: λ=0.9, β=0.9, α=0.2, η=0.1, γ=0.1, ϵ=0.2, and μ=0.1.

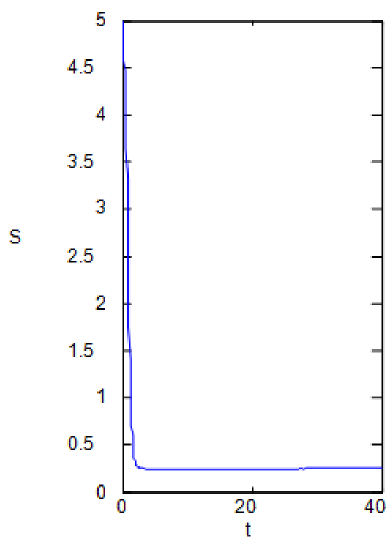

Fig. 2. Time-series plots of S(t)

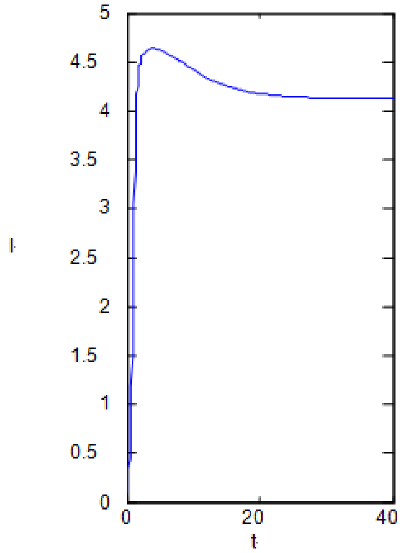

Fig. 3. Time-series plots I(t)

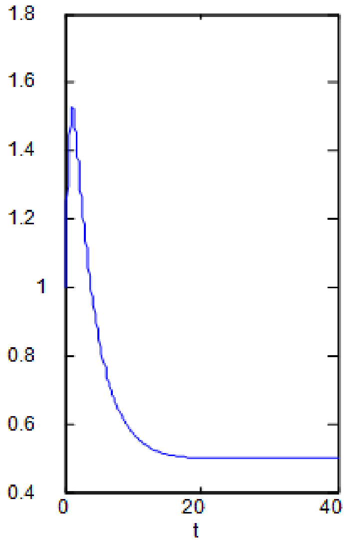

Fig. 4. Time-series plots of P(t)

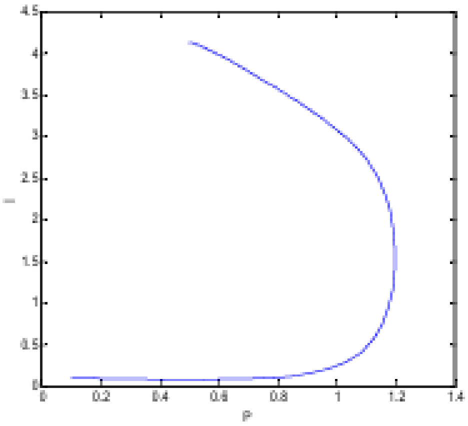

Fig. 5. Phase-plane diagrams for P-I

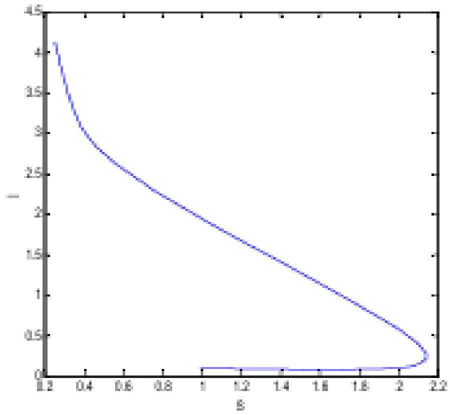

Fig. 6. Phase-plane diagrams for S-I

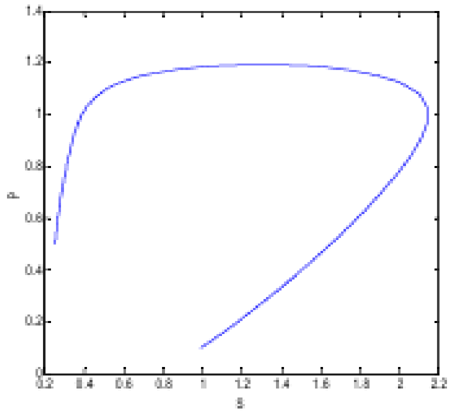

Fig. 7. Phase-plane diagrams S-P

We next perform sensitivity analysis to find the important parameters of the model and how they impact the working of the model.

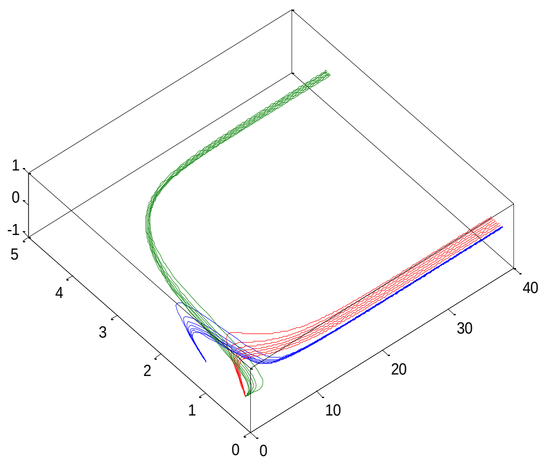

Fig. 8. Variations for a given range of η

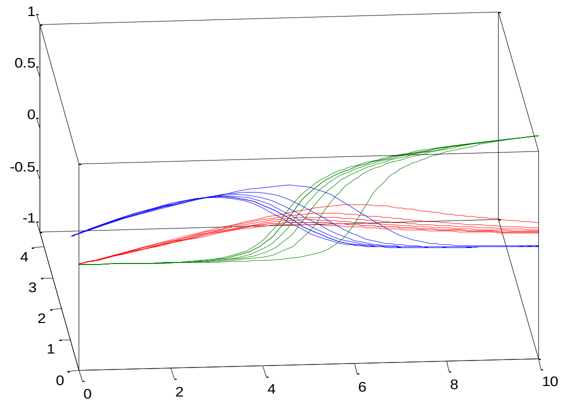

Fig. 9. Variations for a given range of β

In the first fig. 8 $\boldsymbol{\eta}$ varies from certain range. We have got that after the certain range of $\boldsymbol{\eta}$, the system becomes unstable, which actually satisfied the real world condition in OSNs. Here, $\boldsymbol{\eta}$ has taken the rate from which infected users become protected because if the rate will decrease from a given range then number of users which become protected will decrease and in that case, the system will become unstable because most of the users will be available for attacks. Again, if value of $\boldsymbol{\eta}$ exceeds from a given range then the users become more aware than required and they would not show their usual activity on the network due to their concern about privacy. This is not good for social network society because we will not get the true information regarding any concern.

Again, we see in fig. 9 that if we vary the value of β from specific range then system becomes unstable. Here β stands for rate at which susceptible users become infected. If value of β is very low from given range then minimum number of users becomes infected which is our purpose but at the same time in real world situation it is not possible that rate of infection can minimize at that extend. Similarly, if we maximize the beta from given value in the system, number of infected users becomes very large and will damage the network abruptly and making the environment ideal becomes impossible.

Thus we observe from above situations that the value of both the parameters should be in the given range to make it rumor free situation in the network. Both the conditions in which value of the parameters either decrease or exceed from the given range make the network unstable for spreading of true information. Hence our consideration about the given parameter is valid.

8. Data Collection Methodology

In this section, we describe the real-time data collected from Twitter for our experiment. Using the Twitter API and Martin Hawskey's spreadsheet tool, TAGS v6.1 [7], we gathered data for the hashtags #BJPBengal and #TMCBengal from January 2, 2021, to January 17, 2021. This dataset includes approximately 4,400 tweets, retweets, replies, and mentions for #BJPBengal and 4,000 for #TMCBengal**.**

To estimate the total number of Twitter users supporting each party, we used publicly available data [8], accounting for the annual increase in new Twitter users while excluding individuals under the age of 18 [9-10]. Based on this analysis, the total number of Twitter users for BJP Bengal is approximately 107,059**,** while TMC Bengal has 473,396 users.

Additionally, we found that 14% of Twitter users utilize all available privacy features on the platform [11]. Furthermore, 15% of users express regret over their published posts due to ineffective update policies [12]. Among those who regret their posts, 38% subsequently modify their privacy settings by enabling the "Protect My Tweets" option [12].

9. Experimental Findings and Analysis

We analyzed the spread of misinformation by plotting the I(t) (infected users over time) and P(t) (protected users over time) graphs for both #BJPBengal and #TMCBengal, as shown in Figures 10, 11, 12, and 13.

The results from Figures 9 and 11 indicate that the number of infected users in both parties initially increases rapidly, followed by a gradual decline over time, eventually stabilizing at a constant value. This stabilization suggests the emergence of a rumor-free environment within each party as misinformation spread diminishes.

Similarly, Figures 10 and 12 show that the number of protected users for both hashtags decreases over time before reaching a constant value. This implies that some users actively adopt privacy measures, making them less susceptible to misinformation and reducing their participation in its spread.

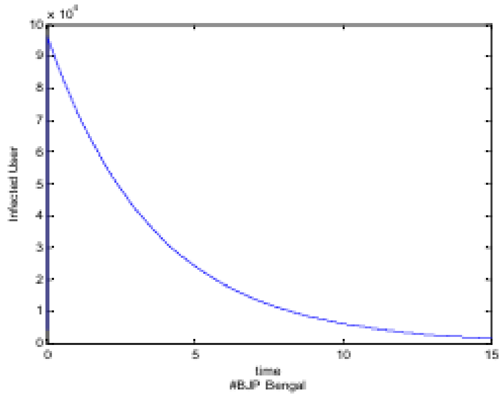

Fig. 10. Emergence of Rumor Free Environment

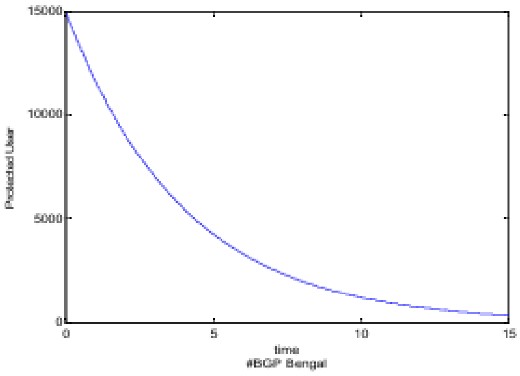

Fig. 11. Role of Adopting Privacy Measures

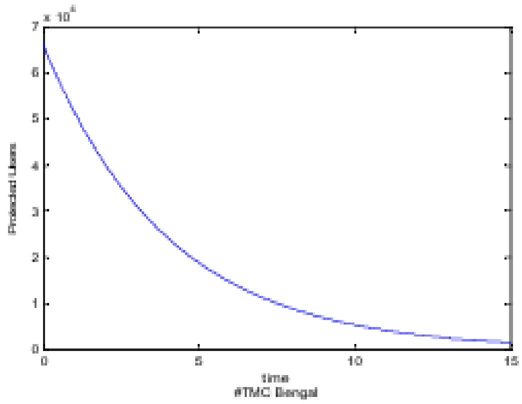

Fig. 12. Emergence of Rumor Free Environmen

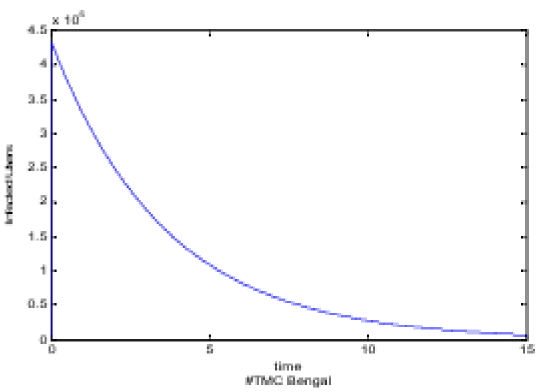

Fig. 13. Role of Adopting Privacy Measures

10. Conclusion

In this paper, we analyze an SIP bifurcation model for Online Social Networks, where bifurcation arises due to the limited awareness of Security and Privacy services among users. Our analysis shows that the infection-free equilibrium is stable when $R_0 < 1$ and unstable when $R_0 > 1$. However, the stability of the endemic equilibrium depends not only on the basic reproduction number R_0 but also on a critical threshold value R_p. We observe that the system exhibits unstable endemic equilibria at $I = I^*$ and I_1^*, while $I = I_2^*$ is a stable equilibrium**.** Numerical simulations confirm the occurrence of bifurcation in the system. Finally, we validate our theoretical findings through experiments using real-time Twitter data and discuss the results in detail.

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Authors’ Profiles

Nitesh Narayan is presently working as an assistant professor in the department of Mathematics, R.D. & D.J., College, Munger, Bihar, India. He has completed his Ph.D. from National Institute of Technology, Patna. He has master’s degrees in Mathematics and Technology(Scientific Computing). His area of interest for research includes Mathematical Modeling on various types of attacks, their defense and information transmission in the field of mobile network, social network and other related fields.

Kaushik Haldar is a seasoned expert in the field of mathematical modeling. He has a strong academic and research background in mathematics and computing. He has been actively working to develop mathematical theory of different phenomenon that arise in modern society because of the diffusion of human and technological spaces. He attributes his research success to the experience he acquired in the theoretical and practical applications of mathematical modeling and scientific computing during the time that he spent at the Birla Institute of Technology. At BIT, he pursued his M.Tech. in Scientific Computing and then Ph.D. in Applied Mathematics. Holding advanced degrees in these fields, he has spent years exploring the intersection of theoretical models and real-world applications, making complex data-driven problems more accessible and solvable. He has been the recipient of various awards including Young Scientist Award and the Omar Khayyam Distinguished Mathematician Award, among others. Beyond research, Dr Kaushik Haldar is a passionate educator, dedicated to making data analysis, mathematics and computing approachable for professionals and students alike.

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