

Some Properties of Anti Imprecise Subgroup and its Application

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Received: 14 August, 2024; Revised: 28 December, 2024; Accepted: 24 March, 2025; Published: 08 June, 2025

Abstract: Various authors from around the world have extended the fuzzy concept to study the uncertainty condition and define its degree of certainty in various real-life experiments. At the same time, many authors have discussed the shortcomings of the definition of fuzzy sets that currently exist. However, no author has properly highlighted the problem of not following the two main classical set theories logically. To address this issue, an imprecise set definition is introduced as an extended definition of fuzzy sets, where the new concept applies two parameters, namely the functions of membership and reference, instead of one, and is helpful in defining the uncertainty problem in a more convenient manner than the existing one. In our previous work, we have studied imprecise subgroup using this new concept addressed by Baruah. In this paper, using the concept of complement of imprecise subgroup, we have introduced anti imprecise subgroup and some properties of anti imprecise subgroup with examples. Imprecise subgroup is an extended version of fuzzy group theory developed using the definition of imprecise set defined by Baruah. In addition, we expected an application developed from an anti imprecise subgroup that can be used to resolve various networking problems.

Index Terms: Imprecise Set, Imprecise Subgroup, Anti Imprecise Set, Anti Imprecise Subgroup

1. Introduction

The notion of classical sets is generalized in the theory of FSs. It was in 1965 that Zadeh [1] first proposed this notion. Zadeh [1] began the study of FS and since then this particular branch of mathematics has been brilliantly focused for its diverse applications. Numerous authors have extended this theory to a broad spectrum of mathematics and combined it with different uncertainty approaches. Rosenfeld [2] initiated the study of algebraic structures on FSs by defining FGs. This FG theory is one of the most impressive uses of Zadeh's FS theory. The idea of an FS_G was first proposed by Rosenfeld [2] in 1971 using the idea of Zadeh's FS_b of a set. Rosenfeld's research [2] served as an inspiration for fuzzy abstract algebra's creation. Das [23], Mukherjee and Bhattacharya [19,20,21,22] have all conducted additional research on this topic. Biswas [3] introduced the concept of anti FS_G which is nothing but the complement of Rosenfeld's [2] FS_G theory, constructed using Zadeh's complement definition of FS and established some fundamental properties of this phenomenon. Onasanya [4] established many important anti fuzzy properties of FGs. However, the claim that FS theory is an extension of traditional set theory is refuted by Zadeh's [1] formulation of FS complement, which does not adhere to the two universal laws of the traditional set theory. Regarding this, Baruah [6] came to the conclusion that the fuzzy complement definition had a flaw because the two functions—fuzzy MF and fuzzy RF—were regarded as one and the same in the previous definition of FS. This ultimately led to the conclusion that Zadeh's concept of the fuzzy complement set does not adhere to the two key principles of traditional set theory: intersection of a FS and its complement is null set; and union of a FS and its complement is a universal set. In 1999, H. K Baruah [5,6,7,8] redefined the definition of the complement of a FS to overcome such shortcomings in the existing definition of a FS. We were then able to obtain a new definition of the fuzzy complement of an FS after Baruah [5,6,7,8] provided a new definition of FSs in terms of MF and RF. Additionally, this expanded definition of FS can get around the problem with

Zadeh's [1] FS and provide us with the intersection of an FS and its complement as null set; and the union of a FS and its complement as a universal set. This expanded definition of an FS with a new complement form is called an IS. Numerous authors have written about this IS, which satisfies many of the traditional set theoretical features. Later, the theory is used in numerous fuzzy number extension investigations. This new definition has been addressed by many other authors, which can be viewed in [9,10,11,12,13,14,15,16,17]. In 2024, Narzary and Borgoyary [18], used the definition of an IS to study the algebraic structure of FG and FS_G in terms of MF and RF to construct an extended version called IG, which is basically an extension of a FG theory. As the extended concept of IS is more convenient than the existing FS so, the properties and results satisfied by IG is more convenient and satisfactory than existing FG theory. In the same year, Narzary [18] used the concept of IGs to develop many other fundamental characterization of the IS_G .

The main objective of this paper is to introduce anti IG and therefore anti IS_G which have been established to study some fundamental properties of traditional group theory with respect to anti IS_G . This study of anti IG is important as it is based on Baruah's [7] extension of existing FS definition which can overcome the shortcomings of Zadeh's FS definition. Here, the concept of anti IG is basically the complement of IG which is defined in terms of MF and RF based on the complement definition of IS by Baruah [7]. This paper is a continuation of the idea of IG and IS_G s defined in [18] using the concept of IS by Baruah [7]. We also used this anti IS_G to construct a network structure, which shows that every anti IS_G is a closed network structure. And, this structure can be used to solve some networking problems. The properties of anti imprecise groups enable the creation of a wide range of fascinating network diagrams that could have practical uses in the domains of invention and research.

For our entire study 'G' is considered as the precise group and 'X' as the universal set.

2. Preliminaries

2.1. Definition ([2])

A $FS_b u_f$ is said of to be a FS_G of G if

- (i) $u_f(t_1 t_2) \geq u_f(t_1) \wedge u_f(t_2); \forall t_1, t_2 \in G$
- (ii) $u_f(t_1^{-1}) \geq u_f(t_1); \forall t_1 \in G$
- (iii) $u_f(e) \geq u_f(t_1); \forall t_1 \in G$

2.2. Definition ([7])

If $u_m^1(t_1)$ is a fuzzy MF and $u_r^2(t_1)$ is a fuzzy RF such that $0 \leq u_r^2(t_1) \leq u_m^1(t_1) \leq 1$, then the IS is defined as $A_i = \{t_1, u_m^1(t_1), u_r^2(t_1); t_1 \in X\}$ where X is the universal set and $u_A^1(t_1) = u_m^1(t_1) - u_r^2(t_1)$ gives the actual membership value for all $t_1 \in X$. For convenient of writing above imprecise set is denoted by $(u_m^1, u_r^2)(t_1); t_1 \in X$.

2.3. Definition ([24])

If $A_i = \{t_1, u_m^1(t_1), u_r^2(t_1); t_1 \in X\}$ and $B_i = \{t_1, u_m^3(t_1), u_r^4(t_1); t_1 \in X\}$ are two IS over the universe X then $A_i \subseteq B_i$ iff $u_m^1(t_1) \subseteq u_m^3(t_1)$ and $u_r^4(t_1) \subseteq u_r^2(t_1)$.

2.4. Definition ([7])

If (u_m^1, u_r^2) is an IS_b of X , then the complement of (u_m^1, u_r^2) denoted by $(u_m^1, u_r^2)^c$, is the IS of X given by $(u_m^1, u_r^2)^c(t_1) = \{t_1, 1, u_m^1(t_1); t_1 \in X\}$ where $1 - u_m^1$ is the non MV of t_1 in X .

2.5. Definition ([7])

If A_i^c and B_i^c are the complements of the two ISs $A_i = \{t_1, u_m^1(t_1), u_r^2(t_1); t_1 \in X\}$ and $B_i = \{t_1, u_m^3(t_1), u_r^4(t_1); t_1 \in X\}$ respectively then their union and intersection are respectively,
 $A_i^c \cup B_i^c = \{t_1, 1, u_m^1(t_1)\} \cup \{t_1, 1, u_m^3(t_1)\} = \{t_1, \max(1,1), \min(u_m^1(t_1), u_m^3(t_1)); t_1 \in X\}$
 $A_i^c \cap B_i^c = \{t_1, 1, u_m^1(t_1)\} \cap \{t_1, 1, u_m^3(t_1)\} = \{t_1, \min(1,1), \max(u_m^1(t_1), u_m^3(t_1)); t_1 \in X\}$

2.6. Definition [12]

Let an $IS_b (u_m^1, u_r^2)$ of G together with a binary composition '*' be called an IG $[(u_m^1, u_r^2), *]$, if

- (i) $(u_m^1, u_r^2)(t_1 t_2) \geq (u_m^1, u_r^2)(t_1) \wedge (u_m^1, u_r^2)(t_2); t_1, t_2 \in G$, where ' $\wedge$ ' is the minimum operator
- (ii) $(u_m^1, u_r^2)(t_1^{-1}) \geq (u_m^1, u_r^2)(t_1); \forall t_1 \in G$
- (iii) $(u_m^1, u_r^2)(e) \geq (u_m^1, u_r^2)(t_1); \forall t_1 \in G$

2.7. Definition

A Cayley table is a technique for describing an algebraic structure (usually a finite group) by putting all the elements in a square array to show the binary operation of the elements in a group. In our study, Cayley table is used to present binary operations of the elements of a precise group.

3. Anti IS_G

The concept of complement definition of IS by Baruah [7] has been applied in the Definition 2.6 of IS_G by Narzary and Borgoyary [18] to establish the concept of anti IG and therefore of anti IS_G to study some fundamental properties with respect to anti IS_G . So, the concept of anti IS_G is basically the complement of IS_G which is defined in terms of MF and RF based on the complement definition of IS by Baruah [7].

3.1. Definition

Let G be a group. A $FS_b(u_m^1, u_r^2)^c$ of G is called anti IS_G of G if for $t_1, t_2 \in G$

(i) $(u_m^1, u_r^2)^c(t_1, t_2) \leq \max\{(u_m^1, u_r^2)^c(t_1), (u_m^1, u_r^2)^c(t_2)\}; \forall t_1, t_2 \in G$
 where $\max\{(u_m^1, u_r^2)^c(t_1), (u_m^1, u_r^2)^c(t_2)\} = (u_m^1, u_r^2)^c \vee (u_m^1, u_r^2)^c(t_2) = (u_m^1(t_1) \vee u_m^1(t_2), u_r^2(t_1) \wedge u_r^2(t_2))$ and ' $\vee$ ' is the maximum operator.

(ii) $(u_m^1, u_r^2)^c(t_1^{-1}) \leq (u_m^1, u_r^2)^c(t_1); \forall t_1 \in G$

4. Some Properties of Anti IS_G

4.1. Property

If A_i and B_i are anti IS_G then the complement of the union of A_i and B_i is not an anti IS_G i.e; $(A_i \cup B_i)^c$ is not an anti IS_G .

Proof:

To prove this statement Example 4.1.1 is cited.

4.1.1 Example

Let $G = \{1, -1, i, -i\}$ be a group under multiplication.

Table 1. Cayley table for $G = \{1, -1, i, -i\}$

*	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

where 1 is the identity element, $i^2 = -1, (-i)^2 = -1, (i)(-i) = 1$

Let

$$(u_m^1, u_r^2)(t_1) = \begin{cases} (1,0); & t_1 = 1, -1 \\ (0.92, 0.08); & t_1 = i, -i \end{cases} \quad (1)$$

and

$$(u_m^3, u_r^4)(t_1) = \begin{cases} (1,0); & t_1 = 1, -1 \\ (0.9, 0.1); & t_1 = i, -i \end{cases} \quad (2)$$

Equation (1) and (2) are clearly IS_G s by Definition 2.6

$$A_i = (u_m^1, u_r^2)^c(t_1) = \begin{cases} (1,1); & t_1 = 1, -1 \\ (1, 0.92); & t_1 = i, -i \end{cases} \quad (3)$$

$$B_i = (u_m^3, u_r^4)^c(t_1) = \begin{cases} (1,1); & \text{if } t_1 = 1, -1 \\ (1, 0.9); & \text{if } t_1 = i, -i \end{cases} \quad (4)$$

Again, by Definition 2.4 and 3.1, equation (3) and equation (4) are respective anti IS_G s of A_i and B_i . Now, the union of equation (3) and equation (4) is given by

$$A_i \cup B_i = \begin{cases} (1,1); & \text{if } t_1 = 1, -1 \\ (1, 0.9); & \text{if } t_1 = i, -i \end{cases} \quad (5)$$

which is nothing but equal to B_i and so equation (5) is an anti IS_G .

Then the complement of equation (5) is given by

$$A_i \cup B_i^c = \begin{cases} (1,0); & \text{if } t_1 = 1, -1 \\ (0.9,0.1); & \text{if } t_1 = i, -i \end{cases} \quad (6)$$

which is nothing but equation (2), so equation (6) is an IS_G not an anti IS_G .

4.2. Property

If A_i^c and B_i^c are anti IS_G such that $(A_i^c \cup B_i^c) \subseteq C^c$ then C^c is an anti IS_G .

Proof:

To prove this statement Example 4.2.1 is cited.

4.2.1 Example

From previous Example 4.1.1 let,

$$A_i^c = \begin{cases} (1,1); & t_1 = 1, -1 \\ (1,0.92); & t_1 = i, -i \end{cases} \quad (7)$$

$$B_i^c = \begin{cases} (1,1); & t_1 = 1, -1 \\ (1,0.9); & t_1 = i, -i \end{cases} \quad (8)$$

Let us consider another IS_b , C on $G = \{1, -1, i, -i\}$ by

$$C = (u_m^5, u_r^6)(t_1) = \begin{cases} (0.9,0); & t_1 = 1 \\ (0.4,0.4); & t_1 = -1 \\ (0.3,0.3); & t_1 = i, -i \end{cases} \quad (9)$$

then by Definition 2.4, anti imprecise subsets of equation (9) is,

$$C^c = (u_m^5, u_r^6)^c(t_1) = \begin{cases} (1,0.9); & t_1 = 1 \\ (1,0.4); & t_1 = -1 \\ (1,0.3); & t_1 = i, -i \end{cases} \quad (10)$$

such that

$$A_i^c \cup B_i^c = \begin{cases} (1,1); & t_1 = 1, -1 \\ (1,0.9); & t_1 = i, -i \end{cases} \subseteq C^c \quad (11)$$

Thus, by Definition 2.3, $A_i^c \cup B_i^c \subseteq C^c$.

And, equation (11) and by Definition 2.4 and 3.1, implies that C^c must be an anti IS_G .

4.3. Property

If A_i^c and B_i^c are anti IS_G s such that $(A_i^c \cup B_i^c)^c \subseteq C$ then C must be an IS_G .

Proof:

To prove this statement Example 4.3.1 is cited.

4.3.1 Example

From previous Example 4.1.1, let

$$A_i^c = \begin{cases} (1,1); & t_1 = 1, -1 \\ (1,0.92); & t_1 = i, -i \end{cases} \quad (12)$$

$$B_i^c = \begin{cases} (1,1); & t_1 = 1, -1 \\ (1,0.9); & t_1 = i, -i \end{cases} \quad (13)$$

Then equation (12) and (13) are two anti IS_G s on $G = \{1, -1, i, -i\}$ by Definition 2.4 and 3.1

And, let

$$C = \begin{cases} (1,0); & t_1 = 1, -1 \\ (0.93,0.05); & t_1 = i, -i \end{cases} \quad (14)$$

be an IS_b on $G = \{1, -1, i, -i\}$

Then, the complement of the union of equation (12) and (13) by Definition 2.4, 2.5 and equation (2) is

$$(A_i^c \cup B_i^c)^c = \begin{cases} (1,0); & t_1 = 1, -1 \\ (0.9,0.1); & t_1 = i, -i \end{cases} \quad (15)$$

which is clearly an IS_G by Definition 2.6 such that $(A_i^c \cup B_i^c)^c \subseteq C$ by Definition 2.3 then C must be an IS_G by Definition 2.6.

4.4. Proposition

Union of two anti IS_G s is again an anti IS_G .

Proof:

Let $(u_m^1, u_r^2)^c$ and $(u_m^3, u_r^4)^c$ be an anti IS_G s of G Then

$$\begin{aligned} ((u_m^1, u_r^2)^c \cup (u_m^3, u_r^4)^c)(t_1 t_2) &= \max[(u_m^1, u_r^2)^c(t_1 t_2), (u_m^3, u_r^4)^c(t_1 t_2)] \\ &\leq \max\{\max\{(u_m^1, u_r^2)^c(t_1), (u_m^1, u_r^2)^c(t_2)\}, \max\{(u_m^3, u_r^4)^c(t_1), (u_m^3, u_r^4)^c(t_2)\}\} \\ &= \max\{\max\{(u_m^1, u_r^2)^c(t_1), (u_m^3, u_r^4)^c(t_1)\}, \max\{(u_m^1, u_r^2)^c(t_2), (u_m^3, u_r^4)^c(t_2)\}\} \\ &= \max\{((u_m^1, u_r^2)^c \cup (u_m^3, u_r^4)^c)(t_1), ((u_m^1, u_r^2)^c \cup (u_m^3, u_r^4)^c)(t_2)\} \end{aligned}$$

$$\text{Thus, } ((u_m^1, u_r^2)^c \cup (u_m^3, u_r^4)^c)(t_1 t_2) \leq \max\{((u_m^1, u_r^2)^c \cup (u_m^3, u_r^4)^c)(t_1), ((u_m^1, u_r^2)^c \cup (u_m^3, u_r^4)^c)(t_2)\}$$

$$\begin{aligned} \text{Also, } ((u_m^1, u_r^2)^c \cup (u_m^3, u_r^4)^c)(t_1^{-1}) &= \max\{(u_m^1, u_r^2)^c(t_1^{-1}), (u_m^3, u_r^4)^c(t_1^{-1})\} \\ &= \max\{(u_m^1, u_r^2)^c(t_1), (u_m^3, u_r^4)^c(t_1)\} \text{ by Definition 3.1} \\ &= ((u_m^1, u_r^2)^c \cup (u_m^3, u_r^4)^c)(t_1) \end{aligned}$$

Hence, $((u_m^1, u_r^2)^c \cup (u_m^3, u_r^4)^c)$ is an anti IS_G of G .

4.5. Corollary

Union of a family of anti IS_G s of a group is again an anti IS_G of the group.

4.6. Proposition

Intersection of any two anti IS_G s is not necessarily an anti IS_G .

Proof:

Let us consider two anti IS_G s

$$(u_m^1, u_r^2)^c = \begin{cases} (1,0.9); & t_1 = e, c \\ (1,0.5); & t_1 = a, b \end{cases} \quad (16)$$

and

$$(u_m^3, u_r^4)^c = \begin{cases} (1,0.8); & t_1 = e, a \\ (1,0.7); & t_1 = b, c \end{cases} \quad (17)$$

over the multiplicative group $V_4 = \{e, a, b, c\}$ and Cayley table for V_4 is given in Table 2; where e is the identity element, $a^2 = b^2 = c^2 = e, ab = c, ac = b, bc = a$

Table 2. Cayley table for V_4

*	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Then, the intersection of equation (16) and (17) by Definition 2.5 is

$$(u_m^1, u_r^2)^c \cap (u_m^3, u_r^4)^c = \begin{cases} (1,0.9); & t_1 = e, c \\ (1,0.8); & t_1 = a \\ (1,0.7); & t_1 = b \end{cases} \quad (18)$$

But,

$$\begin{aligned} [(u_m^1, u_r^2)^c \cap (u_m^3, u_r^4)^c](ca) &= (1,0.7) \\ &\not\leq (1,0.8) \\ &= \max\{[(u_m^1, u_r^2)^c \cap (u_m^3, u_r^4)^c](c), [(u_m^1, u_r^2)^c \cap (u_m^3, u_r^4)^c](a)\} \end{aligned}$$

So, $(u_m^1, u_r^2)^c \cap (u_m^3, u_r^4)^c$ is not an anti IS_G by Definition 3.1.

5. Application of an Anti IG in Networking Problem

There are several applications of group theory in various areas of mathematics. In geometry, groups are used to express concepts like symmetry and specific kinds of transformations. For instance, in symmetry of molecules, one symmetric figure A is transformed to another symmetric figure B. But in between this transformation from A to B there are a number of imprecise transformed figures of A to B. Here, the concept of an imprecise subgroup is used to present the journey of these imprecise transformed figures during the interval in between two precise symmetric figures A and B. And, the overall set of precise symmetric figures and imprecise symmetric transformed figures represents the concept of an imprecise subgroup with respect to the operation of composition. And, the overall set of complement of such imprecise transformed figures are used to explain the concept of anti IS_G with respect to the operation of composition. This anti IS_G then represents a closed network figure which can be applicable in various networking problem. Here, a highly symmetric molecule CH_4 is studied by the use of anti IS_G . The elements in $\mathfrak{T}_4 = CH_4$ are the transformed figures of CH_4 due to different compositions, given by $\mathfrak{T}_4 = \{r^0, r^{90}, r^{180}, r^{270}, h1, v1, \mathfrak{D}, \mathfrak{D}'\}$ where r^0 is rotation of 0° , r^{90} is rotation of 90° , r^{180} is rotation of 180° , r^{270} is rotation of 270° , $h1$ is flip about a horizontal axis, $v1$ is flip about a vertical axis, $\mathfrak{D}$ is flip about the main diagonal and $\mathfrak{D}'$ is flip about the other diagonal. The Cayley table of $\mathfrak{T}_4$ is given in Table 3.

Table 3. Cayley Table of $\mathfrak{T}_4$

*	r^0	r^{90}	r^{180}	r^{270}	$h1$	$v1$	$\mathfrak{D}$	$\mathfrak{D}'$
r^0	r^0	r^{90}	r^{180}	r^{270}	$h1$	$v1$	$\mathfrak{D}$	$\mathfrak{D}'$
r^{90}	r^{90}	r^{180}	r^{270}	r^0	$\mathfrak{D}$	$\mathfrak{D}'$	$v1$	$h1$
r^{180}	r^{180}	r^{270}	r^0	r^{90}	$v1$	$h1$	$\mathfrak{D}'$	$\mathfrak{D}$
r^{270}	r^{270}	r^0	r^{90}	r^{180}	$\mathfrak{D}'$	$\mathfrak{D}$	$h1$	$v1$
$h1$	$h1$	$\mathfrak{D}'$	$v1$	$\mathfrak{D}$	r^0	r^{180}	r^{270}	r^{90}
$v1$	$v1$	$\mathfrak{D}$	$h1$	$\mathfrak{D}'$	r^{180}	r^0	r^{90}	r^{270}
$\mathfrak{D}$	$\mathfrak{D}$	$h1$	$\mathfrak{D}'$	$v1$	r^{90}	r^{270}	r^0	r^{180}
$\mathfrak{D}'$	$\mathfrak{D}'$	$v1$	$\mathfrak{D}$	$h1$	r^{270}	r^{90}	r^{180}	r^0

Then the grades of MF and RF are assigned to each of the elements of $\mathfrak{T}_4$ given in equation (19)

$$(u_m^1, u_r^2)(t_1) = \begin{cases} (0.9, 0.1); & \text{if } t_1 = r^0, r^{90}, r^{180}, r^{270} \\ (0.75, 0.2); & \text{if } t_1 = h1, v1, \mathfrak{D}, \mathfrak{D}' \end{cases} \quad (19)$$

which is clearly an IS_G .

Now,

$$(u_m^1, u_r^2)^c(t_1) = \begin{cases} (1, 0.9); & \text{if } t_1 = r^0, r^{90}, r^{180}, r^{270} \\ (1, 0.75); & \text{if } t_1 = h1, v1, \mathfrak{D}, \mathfrak{D}' \end{cases} \quad (20)$$

is an anti IS_G of equation (19).

Now, we can construct a network figure by using the elements of the precise group as nodes of the network figure and the imprecise value of the same elements as the imprecise path between two elements indicated by the connecting lines.

These connecting lines are given by

$$\begin{aligned} (u_m^1, u_r^2)(r^{90}h1) &= (u_m^1, u_r^2)(\mathfrak{D}) = (0.75, 0.2); \\ (u_m^1, u_r^2)(r^{90}v1) &= (u_m^1, u_r^2)(\mathfrak{D}') = (0.75, 0.2); \\ (u_m^1, u_r^2)(r^{90}\mathfrak{D}) &= (u_m^1, u_r^2)(v1) = (0.75, 0.2); \\ (u_m^1, u_r^2)(r^{90}\mathfrak{D}') &= (u_m^1, u_r^2)(h1) = (0.75, 0.2); \\ (u_m^1, u_r^2)(r^{180}h1) &= (u_m^1, u_r^2)(v1) = (0.75, 0.2); \\ (u_m^1, u_r^2)(r^{180}v1) &= (u_m^1, u_r^2)(h1) = (0.75, 0.2); \\ (u_m^1, u_r^2)(r^{180}\mathfrak{D}) &= (u_m^1, u_r^2)(\mathfrak{D}') = (0.75, 0.2); \\ (u_m^1, u_r^2)(r^{180}\mathfrak{D}') &= (u_m^1, u_r^2)(\mathfrak{D}) = (0.75, 0.2); \\ (u_m^1, u_r^2)(r^{270}h1) &= (u_m^1, u_r^2)(\mathfrak{D}') = (0.75, 0.2); \\ (u_m^1, u_r^2)(r^{270}v1) &= (u_m^1, u_r^2)(\mathfrak{D}) = (0.75, 0.2); \\ (u_m^1, u_r^2)(r^{270}\mathfrak{D}) &= (u_m^1, u_r^2)(h1) = (0.75, 0.2); \\ (u_m^1, u_r^2)(r^{270}\mathfrak{D}') &= (u_m^1, u_r^2)(v1) = (0.75, 0.2); \end{aligned}$$

$$\begin{aligned}
 (u_m^1, u_r^2)(r^{90}r^{90}) &= (u_m^1, u_r^2)(r^{180}) = (0.9, 0.1); \\
 (u_m^1, u_r^2)(r^{90}r^{180}) &= (u_m^1, u_r^2)(r^{270}) = (0.9, 0.1); \\
 (u_m^1, u_r^2)(r^{90}r^{270}) &= (u_m^1, u_r^2)(r^0) = (0.9, 0.1); \\
 (u_m^1, u_r^2)(r^{180}r^{270}) &= (u_m^1, u_r^2)(r^{90}) = (0.9, 0.1); \\
 (u_m^1, u_r^2)(r^{270}r^{270}) &= (u_m^1, u_r^2)(r^{180}) = (0.9, 0.1); \\
 (u_m^1, u_r^2)(h1h1) &= (u_m^1, u_r^2)(r^0) = (0.9, 0.1); \\
 (u_m^1, u_r^2)(h1v1) &= (u_m^1, u_r^2)(r^{180}) = (0.9, 0.1); \\
 (u_m^1, u_r^2)(h1\mathfrak{D}) &= (u_m^1, u_r^2)(r^{270}) = (0.9, 0.1); \\
 (u_m^1, u_r^2)(h1\mathfrak{D}') &= (u_m^1, u_r^2)(r^{90}) = (0.9, 0.1); \\
 (u_m^1, u_r^2)(v1v1) &= (u_m^1, u_r^2)(r^0) = (0.9, 0.1); \\
 (u_m^1, u_r^2)(v1\mathfrak{D}) &= (u_m^1, u_r^2)(r^{90}) = (0.9, 0.1); \\
 (u_m^1, u_r^2)(v1\mathfrak{D}') &= (u_m^1, u_r^2)(r^{270}) = (0.9, 0.1); \\
 (u_m^1, u_r^2)(\mathfrak{D}\mathfrak{D}) &= (u_m^1, u_r^2)(r^0) = (0.9, 0.1); \\
 (u_m^1, u_r^2)(\mathfrak{D}\mathfrak{D}') &= (u_m^1, u_r^2)(r^{180}) = (0.9, 0.1); \\
 (u_m^1, u_r^2)(\mathfrak{D}'\mathfrak{D}') &= (u_m^1, u_r^2)(r^0) = (0.9, 0.1)
 \end{aligned}$$

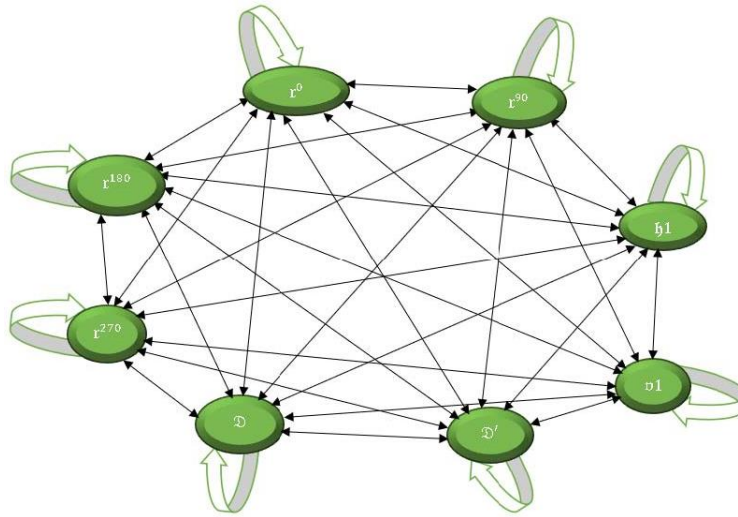


Fig. 1. Network figure of equation (20).

Since the imprecise values are commutative so each connecting lines again get connected towards the opposite node. Here, the elements of $\mathfrak{T}_4$ represents the nodes for the model of Fig. 1 and the connecting line between two elements are the imprecise path which represents a journey of converting from one element figure to the another due to composition and resulting a closed network Fig. 1. And, the anti imprecise value of each element represents the non completion journey of one element to the another. Since each of the elements of $\mathfrak{T}_4$ under composition of symmetries with identity element r^0 gives out the self element so it results in creating a self loop around each element. And, the composition of symmetries between two different elements gives an imprecise path of either $(0.9, 0.1)$ or $(0.75, 0.2)$ resulting in anti imprecise path of either $(1, 0.9)$ or $(1, 0.75)$ elements. So each anti imprecise value represents a not completing journey from one node to the other. And, all these connecting lines together with the loops form an anti IG which in return can be represented in the form of a closed network figure. The number of nodes and connecting lines may differ depending upon the anti IG we are considering. Thus we can generate several network structures depending on the number of elements in the group.

6. Conclusion

The main objective of our work is to lead algebraic properties in a more conventional way in the world of imprecision that could hold traditional set theory. Therefore, we have introduced IS_G with fundamental properties in our previous work which is based on the definition of IS. In this article, we have introduced anti IS_G based on the complement definition of IS and established some interesting properties with some examples. This anti IS_G together with the IS_G can create the closed network diagram that is discussed in the article's final part. The features of anti IS_G s allow for the discovery of a variety of intriguing network diagrams, which may find useful applications in the fields of research and innovations. There are, however, a great deal of theories and properties that need to be looked into for the study of anti IG.

Acknowledgment

The first author acknowledges the financial support received from the University Grant Commission, New Delhi under the Scheme of the National Fellowship for Higher Education (NFHE) vide award letter-number 202021-NFST-ASS-01210, Dated 20th September 2021 to carry out this research work.

List of Abbreviations

FS	Fuzzy Set
FS _b	Fuzzy Subset
MF	Membership Function
RF	Reference Function
MV	Membership Value
FG	Fuzzy Group
FS _G	Fuzzy Subgroup
IS	Imprecise Set
IS _b	Imprecise Subset
IG	Imprecise Group
IS _G	Imprecise Subgroup

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How to cite this paper: Jaba Rani Narzary, Sahalad Borgoyary, "Some Properties of Anti Imprecise Subgroup and its Application", *International Journal of Mathematical Sciences and Computing(IJMSC)*, Vol.11, No.2, pp. 1-9, 2025. DOI: 10.5815/ijmsc.2025.02.01