I. J. Mathematical Sciences and Computing, **2025, 1, 55-61**
Published Online on April 8, 2025 by MECS Press (http://www.mecs-press.org/)
DOI: 10.5815/ijmsc.2025.01.04

Pseudo-Complemented Semigroups

G. Sujatha*
Department of Mathematics, Jawaharlal Nehru Technological University Kakinada, JNTUK, A.P., India
E-mail: sujathaedyth@gmail.com
ORCID ID: https://orcid.org/0009-0001-4762-5652
*Corresponding Author

Ch. Santhi Sundar Raj
Department of Engineering Mathematics, Andhra University, Visakhapatnam-530003, A.P., India
E-mail: santhisundarraj@yahoo.com
ORCID ID: https://orcid.org/0000-0002-7800-9609

U. M. Swamy
Department of Mathematics, Andhra University, Visakhapatnam-530003, A.P., India
E-mail: umswamy@yahoo.com

Received: 10 November, 2024; Revised: 20 December, 2024; Accepted: 11 January, 2025; Published: 08 April, 2025

Abstract: We shall introduce the notion of pseudo – complemented semigroup, which is a natural generalization of the notion of pseudo- complemented semi-lattices, and give certain properties of such semi-groups. We also introduced the notion of Baer- Stone semigroup, which is Pseudo complimented semigroup satisfy certain additional properties.

Index Terms: Semigroup, Pseudo-Complemented Semigroup, Baer-Stone Semigroup

1. Introduction

A theory of pseudo-complements in lattices, and particularly in distributive lattices was first developed by V. Glivenko [14], M. H. Stone [8] and Garrett Birkhoff [3]. Later, O. Frink [9] extended the concept of pseudo-complements to semilattices as a generalization of the theory of lattices. A semilattice $(L, \wedge)$ with least element 0 is said to be pseudo- complemented if, for any $a \in L$, there exists an element a^* satisfying the condition that $a \wedge x = 0$ if and only if $x \leq a^*$. Further, it is proved that $L^* = \{a^* : a \in L\}$ becomes a Boolean algebra. Lee [7] proved that any pseudo-complemented semilattice is equationally definable. A semigroup with zero is an algebra of type (2, 0) satisfying $a(bc) = (ab)c$ and $a0 = 0 = 0a$ for all $a, b, c \in S$. Let $E(S)$ denote the central idempotent e in a semigroup S, i.e., $e^2 (= e.e) = e$ and $ex = xe$ for all $x \in S$. Then $E(S)$ is a subsemigroup of S. In this paper, we shall introduce the notion of a pseudo- complemented semigroup $(S,.)$ with 0 and give certain important properties of these. Mainly, we prove that the set of all pseudo-complements of elements of S is a Boolean algebra.

The concept of Baer-Stone semigroup was first introduced by U.M.Swamy [13]. Here, we prove that a Baer-Stone semigroup is a common abstraction of the multiplicative semi group of Baer- ring [13] and the meet semi-lattice structure of a Stone lattice [1]. In this paper we mainly pay attention to pseudo-complemented semigroup.

2. Pseudo-Complemented Semigroup

In this section, we give the definition of a pseudo-complemented semigroup and prove some basic properties of a pseudo-complemented semigroup.

Definition 2.1. Let S be a commutative semigroup with zero. For any $x \in S$ and $X \subseteq S$, define the sets, as given below:

$$xS = \{xa : a \in S\}$$
$$X^* = \{a \in S : ax = 0 \; for \; all \; x \in S\}.$$

X^* is called the annihilator of X in S. If $X = \{x\}$, then simply we write $(x)^*$ for $\{x\}^*$.

Definition 2.2. A commutative semigroup S with zero is said to be a pseudo-complemented semigroup if, for each $x \in S$, there exists an idempotent e in S such that $(x)^* = eS$.

Observe that for any idempotents e and f in S, $eS = fS$ implies $e = f$ (for, $e \in eS = fS \Rightarrow e = fa \Rightarrow fe = f^2a = fa = e$ and similarly $ef = f$). This shows that in a pseudo-complemented semigroup S there can be atmost one idempotent, which will be denoted by x^* such that x^* $=x^*S$. In this case, the map $x\,' \to x^*$ is called a pseudo-complementation on S. Note that these can be at most one pseudo- complementation on any semigroup.

The following are some examples of pseudo-complemented semigroup.

Example 2.3. If $(R, +, .)$ is an integral domain, then

$$(\mathrm{x})^* = \begin{cases} \{0\} & \text{if } \mathrm{x} \neq 0 \\ \mathrm{R} & \text{if } \mathrm{x} = 0 \end{cases}$$

and hence $(R, .)$ is a pseudo-complemented semigroup.

Example 2.4. The set $\mathbb{N}$ of non-negative integers together with the usual multiplication is a pseudo-complemented semi- group, in which

$$\mathrm{x}^* = \begin{cases} 0 & \text{if } \mathrm{x} \neq 0 \\ 1 & \text{if } \mathrm{x} = 0 \end{cases}$$

Example 2.5. Let S be the set of positive divisors of 50 i.e., $S = \{1, 2, 5, 10, 25, 50\}$. If we define for any $x, y \in S, xy = gcd\,\{a, b\}$.

Then $(S, .)$ is a pseudo-complemented semigroup with 1 as the zero element, in which

$$1^* = 50, 2^* = 25, 10^* = 0 = 50^* and\; 5^* = 2 = 25^*.$$

In the following lemma we give some properties of a pseudo-complemented semigroup.

Lemma 2.6. Let S be a pseudo-complemented semigroup. Then for any $x, y \in S$, we have the following.

(1) 0^* is the unity in S.
(2) $x^*x^* = x^*$
(3) $xx^* = 0$
(4) $xy = 0 \Leftrightarrow x^*y = y$
(5) $x^{**}x = x = xx^{**}$
(6) $x^{***} = x^*$
(7) $(xy)^{**} = x^{**}y^{**}$

Proof.

(1) It follows by the fact that $S = (0)^* = 0 * S$.
(2) and (3) are direct implications of the definition of pseudo-complemented semigroup.
(4) $xy = 0 \Leftrightarrow y = x^*s$ for some $s \in S \Leftrightarrow x * y = y$.
(5) It follows by (3) and (4).
(6) We have $xy = 0 \Leftrightarrow x^{**}y = 0$. It follows that $x^{***} = x^*$
(7) Let $x, y \in S$. We have

$$\begin{aligned} xy(xy)^* = 0 &\Rightarrow x^*y(xy)^* = y(xy)^* \text{ (by(4))} \\ &\Rightarrow x^{**}y(xy)^* = 0 \\ &\Rightarrow yx^{**}(xy)^* = 0 \\ &\Rightarrow y^*x^{**}(xy)^* = x^{**}(xy)^* \text{ (by(4))} \\ &\Rightarrow y^{**}x^{**}(xy)^* = 0 \\ &\Rightarrow (xy)^*x^{**}y^{**} = x^{**}y^{**} \\ \text{and } x^*xy = 0 &\Rightarrow (xy)^*x^* = x^* \\ &\Rightarrow (xy)^{**}x^* = 0 \\ &\Rightarrow x^{**}(xy)^{**} = (xy)^{**} \\ \text{and similarly } & y^{**}(xy)^{**} = (xy)^{**} \\ &\Rightarrow x^{**}y^{**}(xy)^{**} = x^{**}(xy)^{**} = (xy)^{**}. \end{aligned}$$

From the above two arguments, $(xy)^{**} = x^{**}y^{**}$.

For any idempotent e in a commutative semigroup S, the set $eS = \{ex : x \in S\}$ is a subsemigroup of S, since $(ex)(ey) = exy$.

Theorem 2.7. Let S be a pseudo-complemented semigroup and e an idempotent in S, then eS is a pseudo-complemented semigroup.

Proof. Let $x \to x^*$ be a pseudo-complementation on S. We have eS is a commutative semigroup with zero. For any ex and $ey \in eS$ with x and $y \in S$, we have

$$(ex)(ey) = 0 \Leftrightarrow xey = 0 \text{ (since } e^2 = e)$$
$$\Leftrightarrow x^*ey = ey$$
$$\Leftrightarrow ex^*ey = ey$$

and hence $(ex)^* = ex^*$ in eS. Therefore $ex \to ex^*$ is a pseudo-complementation on eS. Thus eS is pseudo-complemented semigroup.

Definition 2.8. Let S be a pseudo-complemented semigroup. For any subset X of S, define

$$X_* = \{x^*: x \in X\}.$$

For any pseudo-complemented semigroup S, we have

$$S_* = E(S)_* = \{e \in E(S): e^{**} = e\} = \{x \in S: x^{**} = x\}$$
$$= \{x \in S: x = y^* \; for \; some \; y \in S\}$$

Definition 2.9. An element x in a pseudo-complemented semigroup S is called closed if $x^{**} = x$; The set of all closed elements in S is called the centre of S and is denoted by $B(S)$.

We observe that $(S) = S_* = E(S)_*$. Now, we prove that $B(S)$ is a Boolean algebra of all closed elements of S.

Theorem 2.10. Let S be a pseudo-complemented semigroup. For any $x, y \in B(S)$, define

$$x \wedge y = xy \text{ and } x \vee y = (x^*y^*)^*.$$

Then $(B(S),\wedge,\vee,*)$ is a Boolean algebra.

Proof. Clearly $(B(S),\wedge)$ is a semilattice and we have a partial order $\leq$ on $B(S)$ defined by $a \leq b \Leftrightarrow a \wedge b = a$ for any $a, b \in B(S)$ and $a \wedge b = glb\,\{a, b\}$. Let $a, b \in B(S)$. Since $a^*b^*a = 0 = a^*b^*b$, $(a^*b^*)^*a = a \; and \; (a^*b^*)^*b = b$, which implies that $a \leq (a^*b^*)^* = a \vee b$ and $b \leq a \vee b$ and hence $a \vee b$ is an upper bound of $\{a, b\}$ in $B(S)$. Now, for any $c \in B(S)$,

$$a \leq c \text{ and } b \leq \text{c} \Rightarrow ac = a \text{ and } bc = b$$
$$\Rightarrow c^*a = 0 = c^*b$$
$$\Rightarrow a^*c^* = c^* = b^*c^*$$
$$\Rightarrow a^*b^*c^* = c^*$$
$$\Rightarrow (a^*b^*)^*c^* = 0$$
$$\Rightarrow (a^*b^*)^*c^{**} = (a^*b^*)^*$$
$$\Rightarrow (a \vee b)c = a \vee b \text{ (since } c^{**} = c)$$
$$\Rightarrow a \vee b \leq c$$

Therefore $a \vee b$ is the lub of $\{a, b\}$ in $B(S)$. Now

$$a \wedge (a \vee b) = a(a^*b^*)^* = a \text{ (since } (a^*b^*)a = 0)$$

Also, $a^*(ab) = 0$ and hence $(ab)^*a^* = a^*$ which implies that $a \vee (a \wedge b) = (a^*(ab)^*)^* = a^{**} = a$. Thus $(B(S),\wedge,\vee)$ is a lattice with smallest element 0 and greatest element $1(= 0^*)$. Further, for any $a \in B(S)$, $a \wedge a^* = aa^* = 0$ and $a \vee a^* = (a^*a^{**})^* = 0^* = 1$. Thus a^* is a complement of a in $B(S)$. Finally, let $a, b, c \in B(S)$. Put $x = a \wedge (b \vee c)$ and $y = (a \wedge b) \vee (a \wedge c)$. In any lattice, it is true that $y \leq x$. We have $a \wedge b \leq y \Rightarrow y^*ab = 0 \Rightarrow y^*ab^* = y^*a$ and $a \wedge c \leq y \Rightarrow y^*ac = 0 \Rightarrow y^*ac^* = y^*a$.

Therefore $y^*ab^*c^* = y^*a$ which implies that $y^*a(b^*c^*)* = 0$ and hence $y^{**}x = x$. Since $y^{**} = y$, We have $yx = x$ and hence $x \leq y$,thus $x = y$ and therefore $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$.

Therefore $(B(S),\wedge,\vee)$ is a distributive complemented lattice and hence a Boolean Algebra.

The following is a consequence of above theorem.

Corollary 2.11. Let $B(S)$ be the centre of a pseudo-complemented semigroup. For any$x$ and y in $B(S)$ define

$$x.y = xy \; and \; x + y = ((xy^*)^*(x^*y)^*)^*.$$

Then $(B(S), +, .)$ is a Boolean ring; that is a ring with unity in which every element is idempotent.

3. Baer-Stone Semigroups

The concept of Baer-Stone semigroup was first introduced by U. M. Swamy [13] in a short note published in the journal 'Semigroup Forum' and mentioned a characterization theorem without proof. In this section, we characterize Baer-Stone semigroups, as a common abstraction of Baer-rings and Stone lattices. First, we start with the following theorem.

Theorem 3.1. Let S_1 and S_2 be pseudo-complemented semi groups. Then the product $S_1 \times S_2$ is a pseudo-complemented semigroup under the coordinate-wise operations.

Proof. SinceS_1 S₁ and S_2 are commutative semigroups with zero, so is $S_1 \times S_2$ and $(0,0)$ is the zero in $S_1 \times S_2$.

For any (x_1, x_2) and (y_1, y_2) in $S_1 \times S_2$,

$$(x_1, x_2)\ (y_1, y_2) = (0,0) \Leftrightarrow x_1 y_1 = 0 \text{ and } x_2 y_2 = 0$$
$$\Leftrightarrow x_1 * y_1 = y_1 \; and \; x_2 * y_2 = y_2$$
$$\Leftrightarrow ({x_1}^*, {x_2}^*)\ (y_1, y_2) = (y_1, y_2)$$

and hence $({x_1}^*, {x_2}^*)$ is the pseudo-complement of (x_1, x_2) in $S_1 \times S_2$.

Note that, the converse of above theorem is also true; that is, if $S_1 \times S_2$ is pseudo-complement semigroup then so are S_1 and S_2. If $(x_1, 0)^* = (a, b)$ then ${x_1}^* = a$ Also, if $(0, x_2)^* = (s, t)$, then ${x_2}^* = t$.

The above result can be extended to arbitrary products of pseudo-complemented semigroups.

Theorem 3.2. Let $(S, .)$ be a pseudo-complemented semigroup and $x \in S$. Define

$$d_x : S \rightarrow x^*S \times x^{**}S \; by \; d_x(y) = (x^*y, x^{**}y).$$

Then d_x is a homomorphism of semigroups and preserves pseudo-complements.

Proof. By theorem 2.7, x^*S and $x^{**}S$ are pseudo-complemented semi groups such that

$$(x^*y)^* = x^*y^* \text{ in } x^*S \text{ and } (x^{**}y)^* = x^{**}y^* \text{ in } x^{**}S.$$

Since x^* is an idempotent and S is commutative, we have

$$d_x(yz) = (x^*yz, x^{**}yz) = (x^*yx^*z, x^{**}yx^{**}z)$$
$$= (x^*y, x^{**}y)(x^*z, x^{**}z)$$
$$= d_x(y)d_x(z)$$
$$\text{and} \quad d_x(y^*) = (x^*y^*, x^{**}y^*)$$
$$= ((x^*y)^*, (x^{**}y)^*)$$
$$= (x^*y,\ x^{**}y)^* = d_x(y)^*, \text{ the p.c of } d_x(y).$$

Thus d_x is a homomorphism and preserves pseudo-complements.

Definition 3.3. For any pseudo-complemented semigroup S, the map $d_x : S \rightarrow x^*S \times x^{**}S$ defined by $d_x(y) = (x^*y, x^{**}y)$ for all $y \in S$ is called the decomposition map corresponding to $x \in S$.

Definition 3.4. A pseudo-complemented semigroup S is called a Baer-Stone semigroup if for each $x \in S$, the decomposition map d_x is an isomorphism of S onto $x^*S \times x^{**}S$.

Theorem 3.5. Let $(S, .)$ be a Baer-Stone semigroup and e an idempotent in S. Then eS is Baer-Stone semigroup.

Proof. By theorem 2.7, eS is a pseudo-complemented semigroup in which the pseudo-complemented of any ex is precisely ex^*. Now, for any $ex \in eS$, consider the map

$$f : eS \rightarrow ex^*S \times ex^{**}S \text{ defined by}$$
$$f\ (ey) = (ex^*ey, ex^{**}ey) = (x^*ey, x^{**}ey).$$

Clearly f is a homomorphism and an injection, since the decomposition d_x is so. To prove f is surjective, let $(ex^*y, ex^{**}z) \in ex^*S \times ex^{**}S$. Then there exists $s \in S$ such that $d_x(s) = (x^*ey, x^{**}ez)$ and therefore $x^*s = x^*ey$ and $x^{**}s = x^{**}ez$. Which implies that $f\ (es) = (x^*ey, x^{**}ez)$. Thus f is an isomorphism. Therefore eS is a Baer-Stone semigroup.

Next, we establish a correspondence between the closed elements in a Baer-Stone semigroup S and decompositions of S into products of two semigroups with zero and unity.

Theorem 3.6. Let $(S, .)$ be a Baer-Stone semigroup and $x \in S$. Then $x \in B(S)$ if and only if there exist Baer-Stone semigroups S_1 and S_2 and an isomorphism $f : S \rightarrow S1 \times S_2$ such that $f\ (x) = (1,0)$; in this case $f\ (x^*) = (0,1)$.

Proof. Suppose that $x \in B(S)$. Then $x^{**} = x$. Note that both x and $x^{**}$are idempotents. Put $S_1 = xS$ and $S_2 = x^*S$. Then, by theorem 3.5, S_1 and S_2 are Baer-Stone semigroups in which x and x^* are unities in S_1 and S_2respectively. Define $f : S \to S_1 \times S_2$by $f(y) = (xy, x^*y)$ for all $y \in S$.

Then $f(x) = (x, 0)$ and $f(x^*) = (0, x^*)$. Since S is Baer-Stone semigroup, it can be easily verified that f is an isomorphism. Conversely, suppose that S_1 and S_2 are Baer-Stone semigroups and $f : S \to S_1 \times S_2$is an isomorphism such that $f(x) = (1, 0)$. Then $f(x^*) = (0, 1)$ and $\mathrm{f}(x^{**}) = (1, 0)$. Since f is an injection, $x^{**} = x$. Thus $x \in B(S)$.

Definition 3.7. [6]. A commutative ring $(R, +, .)$ is called a Baer-ring if, for each $a \in R$, there exists an idempotent e in

R such that the principal ideal generated by e is the annihilator of a in R; that is, $(a)^* = eR$.

Note that for any element a in a commutative ring R, there can be at most one idempotent e in R such that $(a)^* = eR$; for , $eR = fR$ for any idempotents e and f implies $e = fe$ and $f = ef$ and hence $e = f$. Also, note that every Baer ring R has unity, since $(0)^*(= R) = eR$ for an idempotent e, which becomes the unity in R. We prove that the multiplicative semigroup of a Baer ring is a Baer-Stone semigroup.

Theorem 3.8. If $(R, +, .)$ is a Baer-ring, then $(R, .)$ is a Baer-Stone semigroup.

Proof. Let $(R, +, .)$ be a Baer-ring. Then, for any $a \in R$, there exists unique idempotent e in R such that $(a)^* = eR$. Define $a^* = e$. Then, for any $x \in R$, $ax = 0 \Leftrightarrow x \in a^*R \Leftrightarrow a^*x = x$.

Therefore, the mapping $a \to a^*$ is the pseudo-complementation on R. Thus $(R, .)$ is a p.c.semigroup. Next, we observe that, for any $a \in R$, a is an idempotent if and only if $a + a^* = 1$; if a is an idempotent, then $1 - a$ is also an idempotent and, for any $x \in R$, $xa = 0 \Leftrightarrow (1 - a)x = x$ and hence $(a)^* = (1 - a)R$, so that $a^* = 1 - a$.

To prove the decomposition map d_x is an isomorphism, let $y, z \in R$ such that

$$x^*y = x^*z \text{ and } x^{**}y = x^{**}z.$$

Then

$$y = (x^* + x^{**})y = x^*y + x^{**}y$$
$$= x^*z + x^{**}z = (x^* + x^{**})z = z$$

Therefore $d_x : R \to x^*R \times x^{**}R$ is a monomorphism. Further,
let $(x^*y, x^{**}z) \in x^*R \times x^{**}R$ with $y, z \in R$.
Put $s = (x^*y + x^{**}z)$. Then since $x^*x^{**} = 0$, we get

$$x^*s = x^*(x^*y + x^{**}z) = x^*y\ ;$$
$$x^{**}s = x^{**}(x^*y + x^{**}z) = x^{**}z.$$

Therefore d_x is a surjection also. Hence $d_x : R \to x^*R \times x^{**}R$ is an isomorphism and thus $(R, .)$ is a Baer-Stone semigroup.

Next, we prove that meet-semi lattice of a Stone lattice is a Baer-Stone semigroup. Let us recall from [1, 2, 5] that, a lattice $L = (L, \wedge, \vee)$ with 0 is called a pseudo-complemented lattice if $(L, \wedge)$ is a pseudo-complemented semi lattice that is, there exists a mapping $x \to x^*$of L into itself such that, for any x and y in L, $x \wedge y = 0 \Leftrightarrow x^* \wedge y = y$; equivalently $(x)^* = \{x^* \wedge y : y \in L\}$, where $(x)^* = \{y \in L : x \wedge y = 0\}$, the annihilator of x. In this case x^* is the largest element in L such that $x \wedge x^* = 0$.

Definition 3.9. [1]. A bounded distributive lattice $(L, \wedge, \vee)$ is called a Stone lattice if it is pseudo-complemented and

$$x^* \vee x^{**} = 1 \; for\ all\ x \in L.$$

Now, we prove that the decomposition map on a Stone lattice is an isomorphism.

Lemma 3.10. Let $(L, \wedge, \vee)$ be a Stone lattice. Then, for any $x \in L$, the mapping $d_x : L \to x^*L \times x^{**}L$ defined by $d_x(y) = (x^* \wedge y, x^{**} \wedge y)$ is an isomorphism.

Proof. By the distributivity in L, it is clear that

$$d_x(y \wedge z) = d_x(y) \wedge d_x(z)$$
$$\text{and } d_x(y \vee z) = d_x(y) \vee d_x(z) \text{ for all } y, z \in L.$$

Note that x^* and x^{**} are the greatest elements in x^*L and $x^{**}L$ respectively.

Also, $d_x(0) = (0, 0)$ and $d_x(1) = (x^*, x^{**})$. Further, for any $y, z \in L$,

$$d_x(y) = d_x(z) \Rightarrow x^* \wedge y = x^* \wedge z \; and \; x^{**} \wedge y = x^{**} \wedge z$$
$$\Rightarrow y = (x^* \vee x^{**}) \wedge y = (x^* \wedge y) \vee (x^{**}y)$$
$$= (x^* \wedge z) \vee (x^{**} \wedge z)$$
$$= (x^* \vee x^{**}) \wedge z = 1 \wedge z = z.$$

Therefore f is a monomorphism. Next, for any $(x^* \wedge y, x^{**} \wedge z) \in x^*L \times x^{**}L$,
Put $s = (x^* \wedge y) \vee (x^{**} \wedge z)$. Then

$$x^* \wedge s = x^* \wedge y \quad (\text{since } x^* \wedge x^{**} \wedge z = 0)$$
$$\text{and } x^{**} \wedge s = x^{**} \wedge z \quad (\text{since } x^{**} \wedge x^* \wedge y = 0)$$

and therefore $d_x(s) = (x^* \wedge y, x^{**} \wedge z)$. Therefore d_x is an epimorphism. Thus d_x is an isomorphism of L onto $x^*L \times x^{**}L$.

Theorem 3.11. If $(L,\wedge,\vee)$ is a Stone lattice, then $(L,\wedge)$ is a Baer-Stone semigroup.

Proof. It follows the above lemma.

4. Conclusion

This is the initial work on Pseudo-complement semigroup and the authors are working on more details which will be adding in subsequent publications.

References

[1] C. C. Chen and G. Gratzer, Stone lattices: I: construction theorems, Canadian journal of Mathematics, 21(1969), 884-894.
[2] C. C. Chen and G. Gratzer, Stone lattices II. Structure Theorems, 21(1969), 895-903.
[3] G. Birkhoff, Lattice Theory, (revised edition), American Mathematical Society Colloquium Publications, Vol.25, New York, 1948.
[4] G. Gatzer, Lattice Theory: First concepts and distributive lattices, W.H. Freeman, (1971).
[5] J. E. Kist, Two characterizations of commutative Baer rings, Pacific journal of Mathematics, 50(1974), 125-134
[6] J. Varlet, On the Characterization of Stone lattices, Acta sci. Math., 27(1966), 81-84.
[7] K. B. Lee, Equational class of distributive pseudo-complemented lattices, Canadian J. Math., Vol.22(1970), 881- 891.
[8] M. H. Stone, Topological representation of distributive lattices and Brouwerian logic, casopis Pest.Math. Figs., 67(1938).
[9] O. Frink, Pseudo-complements in semi-lattices, Duke. Math J., Vol. 29(1961), 505-514.
[10] P. V. R. Murthy and V. V. R. Rao, characterization of certain classes of pseudo-complemented semi lattices, Algebra Universities, 4(1974), 289-300.
[11] T. P. Speed and M. W. Evans, A note on commutative Baer rings, J. Austral, Math. Soc., 13(1972), 1-6.
[12] T. S. Blyth, Ideals and filters of pseudo-complemented semilattices, Proc. of the Edinburgh Mathematical soc., 23(1980), 301-316.
[13] U. M. Swamy, Baer-Stone semigroups, Semigroup Forum, 19(1980), 385-386.
[14] V. Glivenko, Sur quelque points de la logique de Brouwer, Bulletin Academie des Sciences de Belgique, Vol.15(1929) pp. 183-188.
[15] W. H. Cornish, congruences on distributive pseudo-complemented lattices, Bull. Austral .Math. Soc., 8(1973), 161-179.

Authors' Profiles

Sujatha. G is an Assistant Professor at Dr.Lankapalli Bullayya College, Andhra Pradesh, India. She is a research scholar in the Department of Mathematics at Jawaharlal Nehru Technological University kakinada(JNTUK) , Kakinada , Andhra Pradesh, India.

Ch. Santhi Sundar Raj is working as a Professor in the Department of Engineering Mathematic at Andhra University College of Engineering (AUCE), Andhra University, Visakhapatnam, Andhra Pradesh, India.

Swamy. U. M. is working as a Professor in the Department of Mathematics at Andhra University, Visakhapatnam, and Andhra Pradesh, India.

How to cite this paper: G. Sujatha, Ch. Santhi Sundar Raj, U. M. Swamy, "Pseudo-Complemented Semigroups", International Journal of Mathematical Sciences and Computing(IJMSC), Vol.11, No.1, pp. 55-61, 2025. DOI: 10.5815/ijmsc.2025.01.04