

Matrix Approach to Rough Sets Based on Tolerance Relation

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Received: 07 November, 2024; Revised: 14 December, 2024; Accepted: 07 January, 2025; Published: 08 April, 2025

Abstract: There are many complex issues with incomplete data to make decisions in the field of computer science. These issues can be resolved with the aid of mathematical instruments. When dealing with incomplete data, rough set theory is a useful technique. In the classical rough set theory the information granules are equivalence classes. However, in real life scenario tolerance relations play a major role. By employing rough sets with Maximal Compatibility Blocks (MCBs) rather than equivalence classes, we were able to handle the challenges in this research with ease. A novel approach to define matrices on MCBs and operations on them is proposed. Additionally, applied the rough matrix approach to locate a consistent block related to any set in the universal set.

Index Terms: Rough Matrix (RM), Maximal Compatibility Block (MCB), Matrix approach of MCB Covering

1. Introduction

T Pawlak [1] examines inadequate information systems by introducing the theory of rough sets. The equivalence classes' definitions of information granules serve as the foundation for the hypothesis. With the universe serving as their covering and tolerance relations defining the information granules, the concept is later extended in a variety of ways. J. Mahanta and P.K. Das introduced Covering Based Rough Sets [2].

MCBs serve as information granules in a covering-based rough set technique defined and explored by the authors, N. Kishore Kumar, M.P.K. Kishore, and S.K. Vali and Ten additional forms of rough sets based on MCBs are equal to the types of rough sets stated in [3, 4]. These forms also study the structure of rough approximations in terms of rough inclusion ordering.

We present the matrix approach based on the papers [5, 6, 7]. Since linear systems can handle matrices with ease, we would like to introduce the matrix technique employing maximal compatibility blocks on rough sets. Information systems rely on a variety of databases.

2. Basic Definitions

This section serves as a helpful reminder of the fundamental definitions of covering a universe, rough set on covering, rough matrix notation, and key operations on rough matrices.

Definition 2.1: Consider a universe set U and a family of subsets of U , denoted by C . If $\cup C = U$ and none of the subsets in C are empty, then C is a covering of a universe set.

Definition 2.2: (Maximal Compatibility Block) Let U be a universe and R be a tolerance relation on U (i.e., R satisfies solely symmetric and reflexive conditions). Let $B \subseteq U$ if every element of B is related to every other element of B with respect to R and no element of $U - B$ is related to every element of B , then a subset $B \subseteq U$ is a Maximal Compatibility Block (MCB) with respect to R .

Definition 2.3: Let C be a covering $\{K_1, K_2 \dots K_n\}$ and R be a tolerance relation on a universe. For any $X \subseteq U$ $\bar{X} = \cup \{K \in C / K \cap X \neq \phi\}$ are known as the upper and lower approximation operators, $r \underline{X} = \cup \{K \in C / K \subseteq X\}$ respectively, and the pair $(\underline{X}, \bar{X})$ is referred to as the rough set.

Definition 2.4: Let matrix $M_C = (a_{ij})_{n \times m}$ known as a matrix of covering C , where $C = \{K_1, K_2 \dots K_n\}$ to be a covering of $U = \{x_1, x_2 \dots x_m\}$, where $a_{ij} = \begin{cases} 1 & \text{for } x_j \in K_i \\ 0 & \text{for } x_j \notin K_i \end{cases}$.

The following example provides a matrix representation of a covering based on the concept given above.

Example 2.5: Assuming

$C = \{\{1,2\}, \{1,2,3\}, \{3,4,5\}, \{2,3\}\}$, $R = \{(1,2)(2,1)(2,3)(3,2)(3,4)(4,3)(1,3)(3,1)(3,5)(5,3)(4,5)(5,4)\}$ is a tolerance relation for $U = \{1,2,3,4,5\}$. Then, a matrix approach to the covering C

$$M_C = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Where rows denote blocks and columns represent elements in the universal set U .

3. Rough Matrix on MCB Covering

This section presents the novel covering C_{MCB} with Maximal Compatibility Blocks (MCB), the matrix strategy for defining rough sets on C_{MCB} , the matrix approach for a set that is a subset of the universe set, and some rough matrix observations.

Definition 3.1: A covering labeled as C_{MCB} is formed by the set of all MCBs (specified in definition 2.2), and it is defined as follows:

$$C_{MCB} = \cup \{B / B \text{ is Maximal Compatibility Block w.r.t. } R\}.$$

Definition 3.2: (Matrix on MCB Covering) Let U be a universe R be a universe's tolerance relation. $C_{MCB} = \{B_1, B_2 \dots B_n\}$ be a covering of U with MCBs, and $U = \{x_1, x_2 \dots x_m\}$. As a matrix approach of C_{MCB} , define matrix $M_{CM} = (b_{ij})_{n \times m}$, where $b_{ij} = \begin{cases} 1 & \text{for } x_j \in B_i \\ 0 & \text{for } x_j \notin B_i \end{cases}$.

Example 3.3: Suppose $U = \{1,2,3,4,5\}$. Let $R = \{(1,2)(2,1)(2,3)(3,2)(3,4)(4,3)(1,3)(3,1)(3,5)(5,3)(4,5)(5,4)\}$ is a tolerance relation, and $C_{MCB} = \{B_1 = \{1, 2, 3\}, B_2 = \{3, 4, 5\}\}$. Then a matrix approach of the covering C_{MCB} is

$$M_{CM} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \text{ (From Example 2.5).}$$

Definition 3.2: (Matrix approach of a set $X \subseteq U$) Let R be a tolerance relation on $U = \{x_1, x_2 \dots x_m\}$, $C_{MCB} = \{B_1, B_2 \dots B_n\}$ be a covering of U with MCB's. Let $X \subseteq U$, define matrix on X as $X_{CM} = (x_{ij})_{n \times m}$, where

$$x_{ij} = \begin{cases} 1 & \text{if } x_i \in X \text{ and } x_i \in B_i \text{ where } 1 \leq i \leq m, 1 \leq j \leq n \\ 0 & \text{otherwise.} \end{cases}$$

Here Rows represent Blocks and the column represents the elements in the universal set U .

Example 3.5: (continued from example 3.3) Let $X = \{1, 3, 4\} \subseteq U$. Then, $X_{CM} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$ (By using the definition 3.4).

Definition 3.6: Let R be a tolerance relation on a universe U . $C_{MCB} = \{B_1, B_2 \dots B_n\}$ Where n is finite on U , be the covering formed by MCB's, for all $X \subseteq U$ define

$$X_\star = \cup \{B \in C_{MCB} / B \subseteq X\}$$

$$X^* = \cup \{B \in \mathcal{C}_{MCB} / B \cap X \neq \phi\}$$

as the lower approximation operator and the upper approximation operator, respectively then the pair (X_*, X^*) is called Rough Set.

The following two new operations are defined to obtain X_* and X^*

Definition 3.7: Let $A = (x_{ij})_{n \times m}$ and $B = (y_{ji})_{m \times n}$ be two matrices. We define a new operation ‘*’ as

$$A * B = (z_{ij})_{n \times n} = \begin{cases} 1 & \text{if } x_{ij} \in A \Rightarrow 1 = y_{ij} \in B, \forall 1 \leq i \leq m, 1 \leq j \leq n \\ 0 & \text{otherwise.} \end{cases}$$

Definition 3.8: Let $A = (x_{ij})_{n \times m}$ and $B = (y_{ji})_{m \times n}$ be two matrices. We define a new operation ‘ $\odot$ ’ as

$$A \odot B = (z_{ij})_{n \times n} = \bigvee_{j=1}^n (x_{ij} \wedge y_{ji}) \text{ for } 1 \leq j \leq n, 1 \leq i \leq m.$$

Example 3.9: (continued from example 3.3, 3.5) Let the matrix on covering and set, do the above operation as in definition of 3.7 in the following way from example 3.3 $M_{CM} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$ it is of order 2×5 and from

example 3.5 $X_{CM} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$ but it is of order 2×5 so that take it's transpose as $X_{CM}^T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ now it is of order 5×2 .

Now operate M_{CM} and X_{CM}^T with respect to the operation ‘*’ as

$$M_{CM} * X_{CM}^T = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$M_{CM} * X_{CM}^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\text{and } M_{CM} \odot X_{CM}^T = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$M_{CM} \odot X_{CM}^T = \begin{bmatrix} \bigvee(1 \wedge 1, 1 \wedge 0, 1 \wedge 1, 0 \wedge 0, 0 \wedge 0) & \bigvee(1 \wedge 0, 1 \wedge 0, 1 \wedge 1, 0 \wedge 1, 0 \wedge 0) \\ \bigvee(0 \wedge 1, 0 \wedge 0, 1 \wedge 1, 1 \wedge 0, 1 \wedge 0) & \bigvee(0 \wedge 0, 0 \wedge 0, 1 \wedge 1, 1 \wedge 1, 1 \wedge 0) \end{bmatrix}$$

$$M_{CM} \odot X_{CM}^T = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Definition 3.10: (Matrix approach of Rough set) Let R be a tolerance relation on a universe $U = \{x_1, x_2, \dots, x_m\}$ $\mathcal{C}_{MCB} = \{B_1, B_2, \dots, B_n \mid \text{Where } n \text{ is finite on } U\}$ be a covering of U with MCB's. For any $X \subseteq U$ the lower and upper approximations are defined in matrix form as $(X_*)_{n \times 1} = x_{ij}$ where $x_{ij} = \bigvee_{k=1}^n a_{ik}$ for $1 \leq i \leq n$, and $(X^*)_{n \times 1} = y_{ij}$ where $y_{ij} = \bigvee_{k=1}^n b_{ik}$ for $1 \leq i \leq n$ where $a_{ik} \in M_{CM} * X_{CM}^T$ and $b_{ik} \in M_{CM} \odot X_{CM}^T$.

Example 3.11: (continued from example 3.9)

$$\text{Since } M_{CM} * X_{CM}^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2 \times 2} \text{ and } M_{CM} \odot X_{CM}^T = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}_{2 \times 2}$$

Since $(X_*)_{n \times 1} = x_{ij}$ where $x_{ij} = \bigvee_{k=1}^n a_{ik}$ for $1 \leq i \leq n$ where $a_{ik} \in M_{CM} * X_{CM}^T$ thus $(X_*)_{n \times 1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $(X^*)_{n \times 1} = y_{ij}$ where $y_{ij} = \bigvee_{k=1}^n b_{ik}$ for $1 \leq i \leq n$ where $b_{ik} \in M_{CM} \odot X_{CM}^T$ thus $(X^*)_{n \times 1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, it implies $X_* = \emptyset$ and $X^* = \cup \{B_1, B_2\} = \{1, 2, 3, 4, 5\}$.

4. Identifying Consistent block in Social Networks using Rough Matrix in C_{MCB}

Definition 4.1: Let U be a universe set, R be a tolerance relation defined on U , the coverings $C_1, C_2, \dots, C_n$ on MCB's with respect to the tolerance relation R formed in different time slots and $X \subseteq U$. The blocks in the covering C_i are consistent than the other all blocks which are in remaining coverings, if the Lower approximation of X is minimum and its Upper approximation is Maximum on the covering C_i than all lower and upper approximations of X on remaining Coverings.

Example 4.2: The above lower and upper approximations can be found manually using the set operation (by definition 3.6). However, if we wish to discover them with a machine in large amounts of data, using set notation may be difficult. In this case, the matrix approach makes it relatively straightforward to identify lower and upper approximations in such data. This method works well for large data sets seen in social networks. Consider a social media network with seven users. The lower and upper estimates of various MCB coverings that take place in various time slots are shown in the example that follows.

Assume $U = \{1, 2, 3, 4, 5, 6, 7\}$ and various MCB covering kinds. $C_1 = \{\{1, 2, 3\}, \{3, 4, 5\}, \{5, 6, 7\}\}$, $C_2 = \{\{1, 2, 3\}, \{4, 5, 6\}, \{5, 7\}\}$, $C_3 = \{\{1, 2, 3\}, \{4, 5\}, \{5, 6, 7\}\}$, $C_4 = \{\{1, 2\}, \{3, 4, 5\}, \{5, 6, 7\}\}$ are taken at different times t_1, t_2, t_3, t_4 . The problem is to find the minimum (Lower Approximations) relationships between all of the elements in set X with respect to all four of the aforementioned coverings and the maximum (Upper approximation) relationships between X and the remaining elements (apart from the set X) are necessary if you want to identify the most consistent block across all coverings. Consider $X = \{3, 4, 5, 6\}$.

Let us find

$$M_{C_1M} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

and if

$$X = \{3, 4, 5, 6\}$$

then

$$X_{C_1M} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Implies

$$X_{C_1M}^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus

$$M_{C_1M} * X_{C_1M}^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and

$$M_{C_1M} \odot X_{C_1M}^T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Therefore from the definition of 3.10 $X_{C_{1*}} = \begin{bmatrix} 0V0V0 \\ 0V1V0 \\ 0V0V0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ which means there is only one middle block $\{3,4,5\}$ in C_1 contained in X therefore $X_{C_{1*}} = \{3,4,5\}$,

Also, from the definition of 3.10 $X_{C_1}^* = \begin{bmatrix} 1V1V0 \\ 1V1V1 \\ 0V1V1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ which means all blocks in C_1 intersecting with X .

Therefore $X_{C_1}^* = \{1,2,3,4,5,6,7\} = U$.

Similarly we can obtain

$$X_{C_{2*}} = \{4,5,6\}, X_{C_2}^* = \{1,2,3,4,5,6,7\} = U,$$

$$X_{C_{3*}} = \{4,5\}, X_{C_3}^* = \{1,2,3,4,5,6,7\} = U$$

and

$$X_{C_{4*}} = \{3,4,5\}, X_{C_4}^* = \{3,4,5,6,7\} = U.$$

since

$$X_{C_{3*}} \subseteq X_{C_{1*}}, X_{C_{3*}} \subseteq X_{C_{2*}}$$

and

$$X_{C_{3*}} \subseteq X_{C_{4*}} \text{ thus } X_{C_{3*}} = \{4,5\}$$

is the minimum block than remaining and

$$X_4^* \subseteq X_{C_1}^* = X_{C_2}^* = X_{C_3}^*.$$

Thus, the lower and upper approximations of X on the covering C_3 are minimal and maximum with respect To other coverings respectively, when compared to the rest of blocks; hence the block C_3 is more consistent in all of the remaining MCB Coverings.

5. Conclusion

In this paper new matrices defined with respect to blocks and observed how they can be useful in finding consistent blocks with respect to a given set. This approach can be used to model and solve problems related to social media networks.

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How to cite this paper: N. Kishore Kumar, M. P. K. Kishore, S. K. Vali, "Matrix Approach to Rough Sets Based on Tolerance Relation", International Journal of Mathematical Sciences and Computing(IJMSC), Vol.11, No.1, pp. 31-36, 2025. DOI: 10.5815/ijmsc.2025.01.02