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Roman Domination in Semi-Middle Graphs: Insights and Applications in Computer Network Design

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Abstract: Consider a $G=(V,E)$ and the function $f:V\to\{0,1,2\}$. Unguarded with regard to is defined as a node u with $f(u)=0$ that is not next to a node with 1 or 2. The function $f(V_0,V_1,V_2)$ fulfilling the condition, in which each node u for which $f(u)=0$ is adjoint to at least one node v to which $f(v)=2$, is referred to be a Roman dominant function, known as RDF of a graph G. Roman domination number (RDN) of graph represented by $\gamma_R(G)$, is the bare lowest count of guards that must be employed in any RDF. In this paper, we introduce a new form for graph called *Semi-Middle* Graph for any given graph and we find the RDN for the *Semi - Middle* Graph of some specific graphs. In the field of networking, the concept of the Semi-Middle Graph can be applied to network topology optimization. Specifically, it can be used in the design of communication networks where each direct connection (edge) between two nodes (devices or routers) has an intermediary node that facilitates more efficient data routing or enhances fault tolerance. Usefulness of Roman Domination Number of a Semi-Middle Graph in Networking includes Network Coverage and Monitoring, Fault Tolerance and Redundancy, Optimal Placement of Relays and Routers, Load Balancing and Resource Allocation.

Index Terms: *Semi – Middle* graph, RDF, RDN, Optimization, Network Analysis

1. Introduction

Consider a $G=(V,E)$ graph and defining function $f:V\to\{0,1,2\}$. Undefended with regard to f is defined as a node u with $f(u)=0$ that is not next to the node with a positive weight. For a real value function $f:V\to R$, the weight of f is $w(f)=\sum_{v\in V} f(v)$, and $S\subseteq V$, $f(S)=\sum_{v\in S} f(v)$ so, $w(f)=f(v)$. WRDF defined by Henning et al as, a node $u\in V_0$ is unprotected, if it is nonadjacent node of positive weight . RDF on a graph G be a function $f:V\to\{0,1,2\}$ fulfilling the condition that each node u for which $f(u)=0$ is adjoint to at least one node of v in which $f(v)=2$. The RDN is the bare minimum of guards that must be utilised in any RDF of G and it is represented as $\gamma_R(G)$. A leaf which has its degree one [14]. A node that is not a leaf is called as an internal node. The longest downward path's length from a node to a leaf in a rooted tree is the node's height. A star graph is a particular kind of graph in which a node has degree $n-1$ and $n-1$ nodes have degree one. This appears to connect $n-1$ node to a single centre node. S_n refers to a star graph with n total node points. The Sun graph S_n is a graph which contains a cycle C_n with every node of the cycle is adjoint to a leaf node. A graph G with vertex set V and edge set E is defined as split graph if its node-set can be partitioned into disjoint sets which is not empty such as S and K, i.e., $V = K \cup S$, such that S is a lasting set and a complete set K. A complete bipartite graph, whose nodes can be partitioned into two subsets V_1 and V_2 such that no line has both the ends in the same subset. Here we introduce a graph called as *Semi-Middle Graph* denoted by $SM(G)$ is demarcated as follows, let G be a graph with node set $V(G)$ and edge set $E(G)$. Then any graph G' which contains G as a subgraph is called the Semi-Middle Graph of G if for every edge $e = uv$ in $E(G)$, there occurs a node w in G' such that u and v are adjacent to w. The method of developing any given structure or model as Semi-Middle can assist in meeting the demand efficiently by saving time (a newly introduced node). Additionally, in order to serve the servers in a cost-effective way, we generalised the Roman dominance number for a few particular graphs, including split graphs, cartesian products of graphs, and complete bipartite graphs. The study of dominant set issues is inspired by their advantages to game theory, file sharing in distributed systems, and facility placement (minimising the number of services, subject to every request being close enough to any facility). In graph theory, the idea of dominance is important. It is very useful in various fields of computer science, together with networking, the problem of facility localization, wireless sensor networking, social networking, etc., which was initiated by applications discussed in [2]. In networking, especially in scenarios like sensor networks or security monitoring systems, the Roman Domination Number of the Semi-Middle Graph helps ensure that each critical node or communication path is monitored by either direct control or through adjacent nodes. This guarantees coverage even if some nodes are only indirectly connected through intermediary nodes (e.g., relay points). Fault Tolerance and Redundancy: By studying the RDN of a Semi-Middle Graph, network designers can determine the minimum number of nodes or resources required to maintain network functionality even during node failures. This is critical for ensuring that backup or alternative paths (represented by intermediary nodes) can take over if primary nodes fail. Optimal Placement of Relays and Routers: The properties of the RDN provide insights into where to place relay nodes (the intermediary www nodes) to achieve efficient monitoring with minimal resources. The Semi-Middle Graph highlights key positions in the network where placing such relay nodes can contribute to full coverage with the least redundancy. Load Balancing and Resource Allocation: In data networks, ensuring that key paths are covered while minimizing the load on any single node can be derived by analysing the RDN of a Semi-Middle Graph. This helps distribute traffic efficiently, preventing overload and ensuring balanced data routing through intermediary nodes [10]. If G' is the Semi-Middle Graph of a network G, calculating its Roman Domination Number can inform the minimum number of critical nodes or relay points needed to ensure full network control. This analysis can be extended to network resilience planning, ensuring that each part of the network can still operate or be dominated, even if certain nodes are non-functional or compromised.

2. Notation

In this study, we focus on undirected, finite deprived of loops and multiple edges. We define $G(V,E)$ is a graph with a node set V of size n and E is an edge of size m. For a node v in V, we define the open neighbourhood of v as $N(v) = \{u \in V : uv \in E\}$, which consists of the nodes adjacent to v. The closed neighbourhood of v is denoted as $N[u] = \{v\} \cup N(v)$, which includes v and its open neighbourhood. A leaf is a node in the graph with a degree of one, meaning it has only one adjacent node. A node that is adjoint to at least one leaf is known as a support. Specifically, a weak support is a node adjoint to just one leaf, while a strong support is a node adjoint to at least two leaf nodes. We can also consider rooted trees, where one node stands out from the others and is referred to as the distinctive node or the root of the tree. In the Cartesian product of two random graphs H and K, denoted as $H \square K$, the resulting graph has nodes of the form (u,v) for $u \in H$ and $v \in K$. If $u_1 = u_2$ and v_1 is adjoint to v_2 in K, or if $v_1 = v_2$ and u_1 is adjoint to u_2 in H, then the two nodes (u_1, v_1) and (u_2, v_2) are neighbouring in the $H \square K$. By considering the Cartesian product of two paths, and P_n, represented as $G_{m.n}$, we obtain the $m \times n$ grid graph [1,13]. This grid graph represents a rectangular grid where the nodes correspond to the intersections, and edges connect adjacent intersections.

Theorem 1:

Let G be any Split Graph that has bipartition (X,Y) where X is independent and Y is complete $\gamma_R(SM(G))$ is either $2Y$, $2(Y-1)$ or $2Y-3$.

Proof:

Let G be a Split Graph γ_R with X independent, Y complete with $|X| = x$ and $|Y| = y$

Consider the Roman dominating function for G as (V_0, V_1, V_2) .Then any node of v in V_2 can prevail at least $2(Y-1)$ nodes, whereas the nodes in V_1 can dominate at most one node of G , hence $|V_0| \neq \varnothing$

Let $V = \{v_1, v_2, v_3 ..., v_Y\}$ be the node set of Y

Let $f(V_0, V_1, V_2)$ be the γ_R function.

Then clearly $|V_2| = y-2, |V_1| = 1$ and $|V_0| = V - |V_2| + 1$

Now, we discuss the following three cases

Case 1:

$|V_2| = Y$

$\deg(y) > 2(Y-1) \forall y \in Y$

Then $|V_2| = \{v_1, v_2, v_3 ..., v_Y\}, V_1 = \varnothing$ and $|V_0| = |V| - |V_2|$

$$\begin{aligned} \gamma_R(SM(G)) &= 2|V_2| + |V_1| \\ &= 2Y \end{aligned} \tag{1}$$

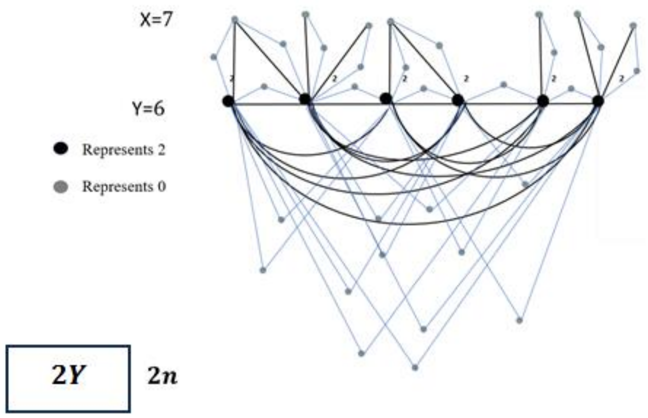


Fig. 1. case 1

Case 2:

$|V_2| = Y$

$\deg(y) > 2(Y-1)$ for $v_1, v_2, v_3 ..., v_{Y-1}$

Let, $\deg(V_Y) > 2(Y-1)$

since, k_Y is complete $|V_2| = \{v_1, v_2, v_3 ..., v_{Y-1}\}, V_1 = \varnothing$ and $|V_0| = |V| - |V_2|$ then,

$$\begin{aligned} \gamma_R(SM(G)) &= 2|V_2| + |V_1| \\ &= 2(Y-1) \end{aligned} \tag{2}$$

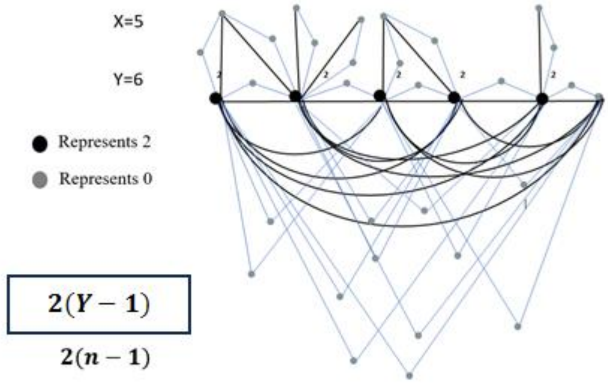


Fig. 2. Case 2

Case 3:

$|V_2| = Y - 2$

$\deg(y) > 2(Y-1)$ for $v_1, v_2, v_3 ..., v_{Y-1}$

thus clearly, $\deg(V_{Y-1}) = 2(Y-1)$ and $\deg(V_Y) = 2(Y-1)$

since, k_Y is complete $|V_2| = \{v_1, v_2, v_3 ..., v_{Y-1}\}, V_1 = \{v_{Y-1}\}$ and $V_0 = V - (V_1 \mathrm{U} V_2)$

then,

$$\begin{aligned} \gamma_R(SM(G)) &= 2|V_2| + |V_1| \\ &= 2|V_{Y-2}| + 1 = 2(Y-2) + 1 = 2Y - 3 \end{aligned} \tag{3}$$

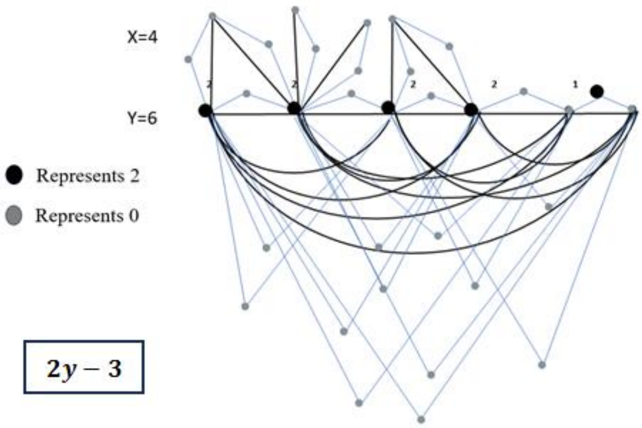


Fig. 3. Case 3

This insight assists in network monitoring and optimization, where the calculated bounds $2Y$, $2(Y-1)$ or $2Y-3$ help determine the minimum number of monitoring or control nodes needed. In sensor networks, this translates to deploying a minimal number of sensors for effective area coverage while maintaining connectivity and resilience. The results contribute to fault-tolerant network designs, ensuring backup nodes are strategically placed to maintain operational stability in case of failures [3].

Moreover, these findings are critical in ad hoc and wireless networks, where strategic placement of intermediary or relay nodes can ensure uninterrupted data flow with minimal communication overhead, thus extending the network's lifespan and improving energy efficiency [12]. The theorem also supports load balancing, helping distribute monitoring or routing responsibilities to prevent node congestion and enhance overall network performance.

In distributed systems and cloud environments, the bounds provided by $\gamma_R(SM(G))$ guide the optimal deployment of monitoring agents or backup units, improving reliability and uptime without excess resource use [8]. This application is essential for data centres, where maintaining network oversight with minimal hardware investment reduces costs. For security protocols, the theorem aids in designing networks that remain continuously monitored, enhancing detection of breaches and maintaining network integrity. It also assists in scaling networks, allowing new components to be added while preserving coverage efficiency.

Understanding these properties informs topology control strategies, where network planners can design efficient layouts that minimize energy consumption while maintaining full control of the network. Finally, these insights aid in resilient communication planning, supporting critical infrastructure networks where maintaining connectivity under varying conditions is crucial for safety and reliability.

Theorem 2:

For the Cartesian product of graph $G_{m,n}$,

$$\gamma_R(SM(G_{m,n})) = \begin{cases} mn, \text{ for any } m,n \text{ is even} \\ mn+1, \text{ for } m,n \text{ is odd} \end{cases} \tag{4}$$

Proof:

We frame RDF $f = (V_0, V_1, V_2)$ of the graph P_m, P_n

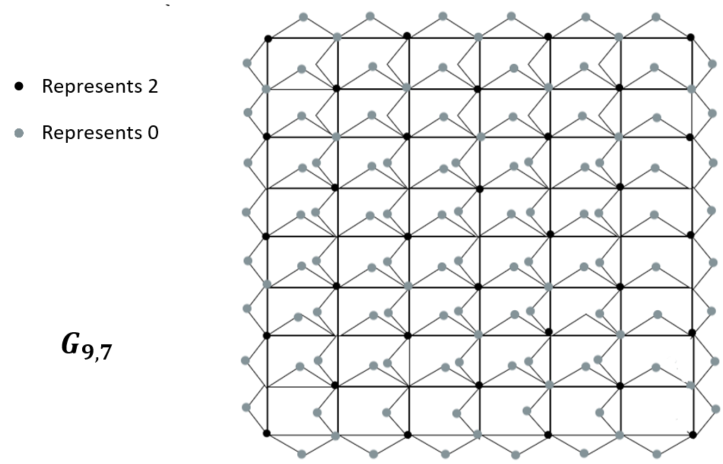


Fig. 4. Example - m, n is odd

We firstly place two guards at each node of set v_2 of the black nodes of the above figure. It is clear that only few nodes of $SM(G_{m,n})$ receives two guards and the remaining nodes of $SM(G_{m,n})$ receives no guards. Hence, as two guards placed at a region can protect at least four regions whereas, one guard placed at a region can protect only that region.

Hence, $V_1 = \varnothing$.

Case 1:

(i) m,n is even

Hence, $V_2 = \{v_{ij}\} \cup \{v_{kl}\}$ where $i = 1,3,5,...,m-1, l = 1,3,5, n-1, k = 2,4,6,...,m$ and $j = 2,4,...,n$

$V_1 = 0$ and $|V_0| = |V| - |V_2|$

Clearly,

$$\begin{aligned} |V_2| &= \left(\frac{m}{2}\right)\left(\frac{n}{2}\right) + \left(\frac{m}{2}\right)\left(\frac{n}{2}\right) \\ &= \frac{mn}{4} + \frac{mn}{4} = \frac{mn}{2} \end{aligned} \tag{5}$$

$$\begin{aligned}\gamma_R(SM(G_{m,n})) &= 2|V_2| + |V_1| \\ &= 2\left[\frac{mn}{2}\right] + 0 \\ &= mn\end{aligned} \tag{6}$$

(ii) m is odd, n is even

Hence, $V_2 = \{v_{ij}\} \cup \{v_{kl}\}$ where $i = 1,3,5,\ldots,m, l = 1,3,5, m-1, k = 2,4,6,\ldots,n$ and $j = 2,4,\ldots,n-1$

$V_1 = \varnothing$ and $|V_2| = \frac{mn}{2}$

Clearly,

$$\begin{aligned}|V_2| &= \left(\frac{m}{2}\right)\left(\frac{n}{2}\right) + \left(\frac{m}{2}\right)\left(\frac{n}{2}\right) \\ &= \frac{mn}{4} + \frac{mn}{4} = \frac{mn}{2}\end{aligned} \tag{7}$$

$$\begin{aligned}\gamma_R(SM(G_{m,n})) &= 2|V_2| + |V_1| \\ &= 2\left[\frac{mn}{2}\right] \\ &= mn\end{aligned} \tag{8}$$

Case 2:

m is odd, n is odd

Hence, $V_2 = \{v_{ij}\} \cup \{v_{kl}\}$ where $i = 1,3,5,\ldots,m-1, l = 1,3,5, n-1, k = 2,4,6,\ldots,n-1$ and $j = 2,4,\ldots,n$

Clearly,

$$\begin{aligned}|V_2| &= \left\lceil\frac{m}{2}\right\rceil\left\lceil\frac{n}{2}\right\rceil + \left\lfloor\frac{m}{2}\right\rfloor\left\lceil\frac{n-1}{2}\right\rceil \\ &= \left[\frac{m+1}{2}\right]\left[\frac{n+1}{2}\right] + \left[\frac{m-1}{2}\right]\left[\frac{n-1}{2}\right] \\ &= \frac{mn}{2} + \frac{1}{2} \\ &= \frac{mn+1}{2}\end{aligned} \tag{9}$$

$$\begin{aligned}\gamma_R(SM(G_{m,n})) &= 2|V_2| + |V_1| \\ &= 2\left[\frac{mn+1}{2}\right] \\ &= mn+1\end{aligned} \tag{10}$$

This result aids in designing optimal network coverage by determining the minimum number of monitoring nodes needed in grid-based network topologies. The Cartesian product structure reflects common real-world layouts such as mesh networks and sensor grids, making this theorem valuable for resource allocation in such environments.

In wireless sensor networks (WSNs), the theorem helps identify the precise number of sensors needed for full area monitoring, ensuring that no node is left undominated. When m or n is even, achieving coverage with exactly mn nodes simplify deployment and minimizes cost. For odd-dimensional grids, the extra node suggested by $mn+1$ highlights the need for additional coverage to maintain connectivity, aiding in fault tolerance and resilience planning [5,6].

This insight is also crucial for network reliability in ad hoc networks, where maintaining a dominating set helps ensure communication pathways remain robust against failures. In distributed computing systems, this theorem supports efficient task monitoring and node supervision, reducing the overhead needed to maintain full network oversight. By understanding $\gamma_R(SM(G_{m,n}))$, network engineers can develop strategies for load balancing and energy-efficient routing, ensuring optimal use of resources.

For data centre layouts that rely on grid structures, applying this result helps minimize the number of monitoring nodes while ensuring all server clusters remain within a controlled network. Additionally, in emergency communication networks, where grid-like setups are used for coverage, this theorem aids in determining the minimum relay points necessary to maintain operational effectiveness. Lastly, the insights support scalability in network design, allowing for expansions that uphold coverage with minimal changes to existing infrastructure.

Theorem 3:

For any complete Bipartite graph,

$$K_{p,q}, p \le q, \gamma_R(SM(K_{p,q})) = 2p \tag{11}$$

Proof:

Let (X,Y) be the bipartite of $K_{p,q}$ where $|X| = p$ and $|Y| = q$ with $X = \{x_1, x_2, x_3 ..., x_p\}$ and $Y = \{y_1, y_2, y_3 ..., y_q\}$.Let $SM(K_{p,q})$ be the *Semi-Middle Graph* of $K_{p,q}$. Since $K_{p,q}$ is a complete bipartite graph, q edges are incident with x_i in $K_{p,q}$. Since, G is a *Semi-Middle Graph* for every edge $e = x_i, y_i$ there exists q nodes say $i = 1, 2, 3, ..., p, w_1, w_2, ..., w_q$, which are adjacent to x_i and y_i

Hence, $\deg(x_i) = 2q$ for every $x_i \in X$ in G.

Now, we define a minimal RDF $f = (V_0, V_1, V_2)$ as follows:

Let's claim, $w \in W$ is not in V_2.Then the two legions placed at w can protect only three regions whereas, two legions placed at x can protect $2q+1 > 3$ regions. Hence, w does not belong to W for any w.

Next let us claim that y not belongs to Y is not in V_2

Then the two legions placed at y can protect only $p+1$ regions. Whereas two legions placed at x can protect $q+1$ regions. Since $p \le q, p+1 \le q+1$ which proves the contradiction to the minimal Roman dominating set.

Hence V_2 is contained in X

Now, we claim that X is contained in V_2

Let, $x \in X$

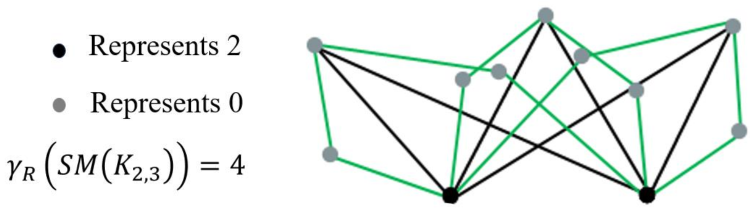


Fig. 5. Example – Semi – Middle graph of Complete Bipartite graph

Case (i): $V_1 = \varnothing$

Suppose not let, $x \in V_1$

Then $2q$ nodes are adjacent to x cannot protect $2q+1$ nodes, then one legion is to be placed at each neighbour of x or two legions must be placed at each node of Y which will contradict the Roman dominating function.

Case (ii): $x \in V_0$

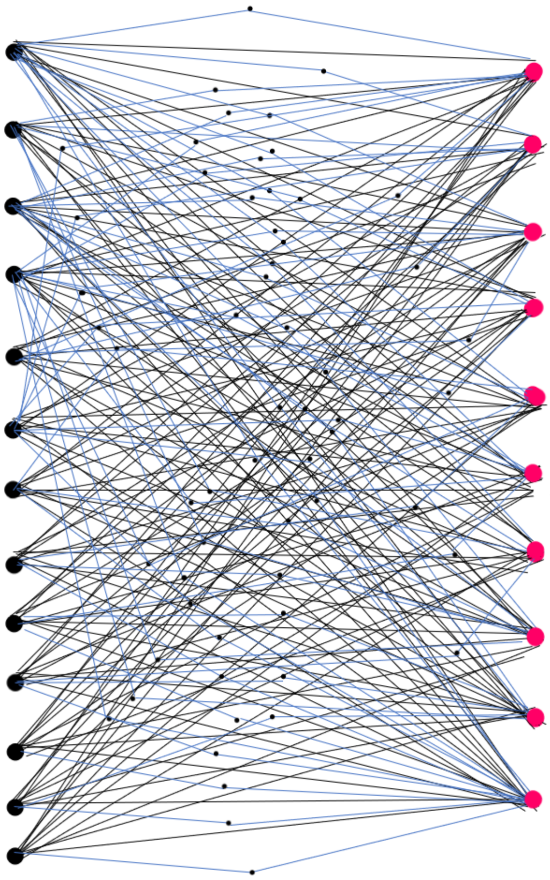
Then at each $N(x)$ in Y, two legions must be placed to protect $N[x]$ which is again a contradiction since, $p \le q$,

Hence, this implies $X = V_2$

Then,

$$\begin{aligned}\gamma_R(SM(G_{m,n})) &= 2|V_2| + |V_1| \\ &= 2p\end{aligned} \tag{12}$$

The above theorem concerning the Roman domination number $\gamma_R(SM(K_{p,q})) = 2P$ for the Semi-Middle Graph of a complete bipartite graph $k_{p,q}$ has several practical applications, particularly in fields that involve bipartite graph structures.



Represents the edges connecting the newly introduced vertices

Represents the edges existing in the original complete bipartite graph

Fig. 6. $SM(K_{10,13})$

This result helps optimize network resource allocation by indicating that the minimum number of monitoring or control nodes required in a bipartite network layout is $2p$. This insight is especially valuable in cloud computing architectures where data flows between distributed servers (partition p) and client systems (partition q) need efficient monitoring and management.

In IoT networks, which often adopt bipartite-like structures with devices communicating to centralized hubs, the result ensures that the placement of monitoring nodes is optimized to cover all interactions effectively. For server-client infrastructures in enterprise networks, knowing that $2p$ monitoring nodes suffice allows for a reduction in oversight complexity while maintaining robust system performance.

The theorem also benefits content delivery networks (CDNs), where strategic node placement can minimize latency by ensuring key content delivery points are monitored with $2p$ dominating nodes. In network security, this helps configure the minimum number of security checkpoints needed to oversee communication between two sets of devices, enhancing the detection of suspicious activities. [4,9]

In parallel processing and distributed systems, where data exchange occurs between worker nodes (partition p) and data storage units (partition q), this result aids in efficient node supervision without overloading the system [7]. It is also crucial for network topology design, simplifying the construction of balanced and monitored layouts for peer-to-peer networks or task allocation grids.

Finally, the theorem supports energy-efficient routing in power-constrained environments, such as wireless mesh networks, by guiding the minimum deployment of nodes for comprehensive coverage, ensuring energy is conserved while maintaining full connectivity [11].The simplicity of $2p$ as the Roman Domination Number allows for easy adaptation in rapidly changing network configurations, promoting adaptable and scalable network management strategies.

3. Conclusion

The Roman Domination Number of Semi-Middle Graphs offers a powerful tool for analysing and enhancing network efficiency and reliability. These theoretical results translate into practical benefits across sensor networks, distributed computing, IoT, and cloud infrastructures by informing decisions on node placement, energy conservation, fault tolerance, and security. Understanding $\gamma_R(SM(G))$ enables network designers to create robust, cost-effective, and scalable solutions, ensuring comprehensive coverage and optimal performance with minimal resource expenditure. This synthesis between graph theory and network applications underscores the value of mathematical models in solving complex real-world challenges in modern networking.

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