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Edge-balanced Index Sets of the Nested Graph with Power-cycle $C_{5^m} \times P_{m_5}$ (I)

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***Abstract* —** Based on the power-cycle nested graph brought before, using the research methods and techniques of graph theory and combinatorial mathematics, through studying the new design idea about the basic graph,nested-cycle subgraph with gear and five-vertex sector subgraph-group, the edge-balanced index sets of the power-cycle nested graph $n=5$ are provided here, for $m \equiv 1(\bmod 3)$ and $m \geq 4$, and the proofs of the computational of formulas and the construction of the corresponding graphs also give out.

***Index Terms* —** Edge-friendly Labeling, Edge-balanced Index Set, Graph $C_{5^m} \times P_{m_5}$, The Nested-cycle Graph With Gear ,Five-vertices Sector Subgraph-group

I. INTRODUCTION

Graph theory is an important branch of discrete mathematics which regards figure as the research object. B.M.Stewart introduced a theory in 1966 which use the vertices of graph and the label function of edges making the vertices of graph correspond to the edges, through the mapping function and then studies the characteristics and inherent characteristics of all kinds of figure. Its theory can be applied to the information engineering, communication network, computer science, economics and management, medicine and so on. Over the years many domestic and foreign researchers devote themselves to this area, and have access to a series of results.

In 1995, M.C.Kong and others defined the edge-balanced graph and strong edge-balanced graph in reference [1], and put forward two conjectures: Conjecture 1: All trees except S_n , n is odd, are edge-balanced. Conjecture 2: All connected regular graphs except K_2 are edge-balanced. The reference [2] extends the concept of edge-balanced labeling to multigraphs and completely characterizes the edge-balanced multigraphs, and proves the two conjectures are true in the reference [1], at the same time proves the problem of decide a graph is edge-balanced does not belong to NP-hard. In 2008, Alexander and Harris studied the vertex -balanced index and friendly index sets of the figure in the literature [3-5].

Since 2009, author and her student Juan Lu solved the edge-balanced index sets of chain graph. In 2010, Ying Wang started the research about the edge-balanced index sets of the equal-cycle nested graph [7].From 2010, Hongjuan Tian[8] and others began the study of edge-balanced index sets of the nested graph with power-cycle. With the increase of n, the difficulty of graph design of odd n increases, ingenious design of classification is required for a nested transform edge index. In this paper, lay emphasis on the research about the graph when $n=5$ .Using the innovative methods and techniques , categories design, proof method of five-vertices sector sub graph-group and other methods of Graph theory and Combinatorial mathematics, which have Completely solved the existence of the edge-balanced index set, construction methods and formulas proof. We choose different n, the subgraph of construction is different in actual operation. The research of power-cycle nested graph $C_{5^m} \times P_{m_5}$ is divided into three parts.But in this paper we only research it when $m \equiv 1(\bmod 3)$ and $m \geq 4$.

The paper is organized as follows. In Section 2 ,we give the definitions that we shall consider. In section 3 includes the graphs with maximum edge-balanced index and using five-vertices sector subgraph-group to proof the lemmas. The main theorem is presented in Section 4. The final Section 5 contains the conclusion.

II. PRELIMINARY NOTES

In graph theory, a graph G is an ordered pair $(V(G), E(G))$ consists of vertex set $V(G)$ and edge set $E(G)$, together with an incidence function that associates with each edge of G an ordered pair of vertices of G .

Definition 1 In graph G , let: $f: E(G) \rightarrow Z$ be an edge labeling function, that is to say, $\forall e \in E(G)$ to define $f(e)=0$ or 1.The edge set labeled 0 or 1 is recorded as $E(0)$ and $E(1)$, using $e(0), e(1)$ to present the number of $E(0), E(1)$. According to the edge labeling f, we can define $f^+ : V(G) \to \{0,1\}$. If $e_x(0)$ is more than $e_x(1)$, $f^+(x)=0$, the vertex x is labeled 0; if $e_x(0)$ is less than $e_x(1)$, $f^+(x)=1$, the vertex x is labeled 1; otherwise, the vertex x is unlabeled. $e_x(0)$ or $e_x(1)$ presents the radix number of the edge collection that the edges linked x are labeled 0 or 1 .The radix number of the vertex set $V(0), V(1)$ is presented by $v(0)$ or $v(1)$.

Definition 2 Let: $f : E(G) \to Z$ is an edge labeling function of G. If $|e(0)-e(1)| \le 1$, f is considered as an edge-friendly labeling of the graph G.

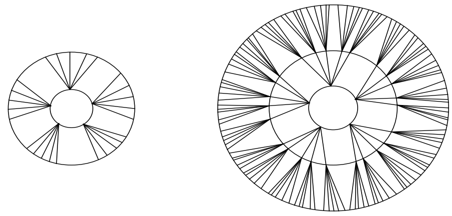
Definition 3 $EBI(G)=\{|v(0)-v(1)|$: the edge labeling f is edge-friendly $\}$ is thought as the edge-balanced index set of the graph G, if there is an edge-friendly labeling f in a graph G.

Definition 4 C_{5^m} shows the graph contains m cycles, and the number of the vertices is added by power exponent from the inner cycle to the outer.

Definition 5 P_{m_5} is a ray path that every road contains m points, and there are five branches at any points except the terminal point of each road.

Definition 6 The power-cycle nested graph $C_{5^m} \times P_{m_5}$ is the Cartesian product of C_{5^m} and P_{m_5}.

Example. Fig. 1 illustrates two graphs of $C_{5^m} \times P_{m_5}$



$C_{5^2} \times P_{2_5}$ $\qquad$ $C_{5^3} \times P_{3_5}$

Fig. 1: two graphs of $C_{5^m} \times P_{m_5}$

For convenience ,we will have the following label for the nested network graph: The vertices of the most inner circle in clockwise order are labeled as follows: $(1)_1, (1)_2, (1)_3, (1)_4, (1)_5$. Similarly, from inside to outside ,the vertices of the most outside circle in clockwise are labeled as follows: $(m)_1, (m)_2, \cdots, (m)_{5^m-1}, (m)_{5^m}$.

Among them, symbols $(j)_i$ says the i th vertex in the j th circle.The ray paths in the power-cycle nested graph, as follows:

$$(1)_{i_1} \to (2)_{i_2} \to (3)_{i_3} \to \cdots \to (m-2)_{i_{m-2}} \to (m-1)_{i_{m-1}} \to (m)_{i_m}$$

$$(i_k = 5(i_{k-1}-1) + j_{k-1}; 1 \le i_1 \le 5; 1 \le j_{k-1} \le 5; 2 \le k \le m)$$

That is:

$$(1)_1 \to (2)_1 \to (3)_1 \to \cdots \to (m-1)_1 \to (m)_1;$$

$$(1)_1 \to (2)_1 \to (3)_1 \to \cdots \to (m-1)_1 \to (m)_2;$$

$$\cdots$$

$$(1)_1 \to (2)_{5^1} \to (3)_{5^2} \to \cdots \to (m-1)_{5^{m-2}} \to (m)_{5(5^{m-2}-1)+4};$$

$$(1)_1 \to (2)_{5^1} \to (3)_{5^2} \to \cdots \to (m-1)_{5^{m-2}} \to (m)_{5^{m-1}};$$

$$\cdots$$

$$(1)_5 \to (2)_{5^2} \to (3)_{5^3} \to \cdots \to (m-1)_{5^{m-1}} \to (m)_{5(5^{m-1}-1)+4};$$

$$(1)_5 \to (2)_{5^2} \to (3)_{5^3} \to \cdots \to (m-1)_{5^{m-1}} \to (m)_{5^m}$$

The above says that the $C_{5^m} \times P_{m_5}$ has 5^m paths.

Definition 7 A 1-vertex x is considered to be saturated, if the n edges linked to x satisfy $e_x(1)=e_x(0)+1$,when n is an odd number; $e_x(1)=e_x(0)+2$,when n is an even number ;otherwise, the 1-vertex x is unsaturated.A 0-vertex x is considered to be saturated, if the n edges linked to x are all 0-edges; otherwise, the 0-vertex x is unsaturated.

Definition 8 Let $m=3t+1 (t \in N^+)$,the induced subgraph of the vertices, which the ray paths through the points with the vertices on the $3t+1 (t \in N^+)$ cycle as starting points and the vertices on the $3(t+1)+1 (t \in N^+)$ .cycle as the terminal points,is thought as the nested-cycle subgraph,denoted by $V_t \left(t = 1, 2, \cdots, \frac{m-4}{3} \right)$.

Definition 9 For the nested-cycle subgraph ,based on starting points of the ray paths, $V_t\left(t=1,2,\cdots,\frac{m-4}{3}\right)(m\geq 4)$ is subdivided into sector subgraph equally.If the ray path's starting points is on the ∂th cycle.Then the nested-cycle subgraph can be divided into 5^{∂} sector graphs,denoted separately as $S_1, S_2, \cdots, S_{5^{\partial}}$.

Definition 10 For the sector subgraphs in the given nested-cycle subgraph, denoted $S(i)=S_{5i+1}\cup S_{5i+2}\cup S_{5i+3}\cup S_{5i+4}\cup S_{5i+5}(0\leq i\leq 5^{\partial-1}-1)$ as five-vertices sector subgraph-group, and the label of the $S(i)$ is in the same, denoted by $S=S(i)$.

III. LEMMAS AND PROOF

Lemma 1 For the power-cycle nested graph $C_{5^m}\times P_{m_5}$, when $m=4$, $\max\left\{EBI(C_{5^4}\times P_{4_5})\right\}=594$

Proof: Firstly, construct the maximum edge-balanced index about $C_{5^4}\times P_{4_5}$ **:**

Because the power-circle nested graph is nested by the power-circle graph and the ray paths graph. We will label respectively the power circle graph and the ray paths graph.

Step 1: We mark all edges as 0-edge which are in the paths of <1>,2>,<3>and in the circles of <4>,<5>,<6>.

$$<1>(1)_a(2)_b(3)_c(4)_d\begin{pmatrix} a\in[1,5];b=3+w+5s,w\in[0,2],s\in[0,4]; \\ c=11+w+25s,w\in[0,14],s\in[0,4]; \\ d=51+w+125s,w\in[0,74],s\in[0,4] \end{pmatrix}$$

$$<2>(2)_b(3)_c(4)_d\begin{pmatrix} b=1+w+5s,w\in[0,1],s\in[0,4]; \\ c=5+w+25s,w\in\{0,5\},s\in[0,4]; \\ d=21+w+125s,w\in[0,4]\cup[24,29],s\in[0,4] \end{pmatrix}$$

$$<3>(3)_c(4)_d\begin{pmatrix} c=1+w+25s,w\in[0,3]\cup[5,8],s\in[0,4]; \\ d=1+5w+125s,w\in[0,3]\cup[5,8],s\in[0,4] \end{pmatrix}$$

<4> The 2th circle and the 3th circle.

$$\langle 5\rangle(4)_d(4)_{d+1}(\ d=2+2w+5s+125f,w\in[0,1];s\in[0,3]\cup[5,8];f\in[0,4])$$

$$<6>(4)_d(4)_{d+1}(d=47+2w,w\in[0,16])$$

Step 2: On the basis of step1,

$(3)_c(4)_d(c=11+w,w\in[0,5],d=51+w,w\in[0,29])$, $(2)_2(3)_{10}$, $(3)_{10}(4)_{47}$, $(3)_{10}(4)_{48}$, $(3)_{10}(4)_{49}$ and $(3)_{10}(4)_{50}$ have been marked 0-edges,then turn them from 0-edges to 1-edges.

Step 3: The other edges in the $C_{5^4}\times P_{4_5}$ are labeled as1.

In the graph, there are 1555 edges,of which 777 are the 0-edges.According to the definition of edge-friendly labeling, $|e(0)-e(1)|\leq 1$,we can get the edge-friendly labeling of the labeled graph by calculation. All the vertices

$(2)_i\left(i=3+w+5s,w\in[0,2],s\in[0,4]\right)$,
$(3)_i\left(i=5+25w,w\in[0,4]\right)$, $(3)_i\left(i=17+w,w\in[0,8]\right)$,
$(3)_i\left(i=35+w+25s,w\in[0,15],s\in[0,3]\right)$ are labeled as 0.There are 93 0-vertices,so $|v(1)-v(0)|=|687-93|=594$.

We can prove $EBI(C_{5^4}\times P_{4_5})=594$ is the maximum of edge-balanced index in the graph $C_{5^4}\times P_{4_5}$ **.**

In this labeled graph, the vertices on the first cycle have degree7.The vertices on the 4th cycle have degree 3,and the others all have degree 8. The edges linking the 0-vertex are labeled 0; the 1-vertex are all saturated, except that the vertex $(2)_2$, which links two 0-edges.

In order to change the label of any 0-vertex into 1-vertex or unlabeled, we need to interchange 4 0-edges and 1-edges at least, but any of the 1-vertex's label changes, we only need to interchange 2 0-edges and 1-edge at least. It is obvious that the interchanging must bring $v(0)$ increases or remains unchanged, and meanwhile $v(1)$ decreases definitely. Thus the value of $|v(1)-v(0)|$ reduces. That's to say:

$$\max\left\{EBI(C_{5^4}\times P_{4_5})\right\}=|v(1)-v(0)|=594$$

Lemma 2 In the $C_{5^m}\times P_{m_5}$, for the five-vertices sector subgraph group S , $\max\{EBI(S)\}=589$.

Proof:Because these five-vertices sector subgraph-group have the same characteristics of edge and vertex. We only give the label of $S=S(0)=S_1\cup S_2\cup S_3\cup S_4\cup S_5$, the other five-vertices $S(i)(1\leq i\leq 5^{\partial-1}-1)$ sector subgraph-group have the same label as $S(0)$.

Firstly, construct the maximum edge-balanced index about S :

Step 1: We mark all edges for 0-edge which are in the paths of <1>,<2>,<3>and in the circle of <4>,<5>,<6>,<7>.

$$<1>(\partial)_a(\partial+1)_b(\partial+2)_c(\partial+3)_d\begin{pmatrix} a\in[1,5];b=1+w+5s,w\in[0,1],s\in[0,4]; \\ c=1+w+25s,w\in[0,9],s\in[0,4]; \\ d=1+w+125s,w\in[0,49],s\in[0,4], \end{pmatrix}$$

$$<2> (\partial+1)_b (\partial+2)_c (\partial+3)_d \left(\begin{array}{l} b = 3 + w + 5s, w \in [0,2], s \in [0,4]; \\ c = 11 + w + 25s, w \in [0,1] \cup [5,7] \cup [10,11], s \in [0,4]; \\ d = 51 + w, w \in [0,9] \cup [25,39] \cup [50,59] \end{array} \right)$$

$$<3> (\partial+2)_c (\partial+3)_d \left(\begin{array}{l} c = 13 + w + 25s, w \in [0,2] \cup [6,7] \cup [10,12], s \in [0,4]; \\ d = 61 + w + 125s, w \in \{0,5,10,30,35,50,55,60\}, s \in [0,4] \end{array} \right)$$

$$<4> (\partial+2)th$$

$$<5> (\partial+1)_b (\partial+1)_{b+1} \left(\begin{array}{l} b = 1 + w + 5s, \\ w \in [0,1] \cup \{4\}, s \in [0,4] \end{array} \right)$$

$$<6> (\partial+3)_d (\partial+3)_{d+1} \left(\begin{array}{l} d = 62 + 2w + 5s + 125f, w \in [0,1]; \\ s \in [0,2] \cup [6,7] \cup [10,12], f \in [0,4] \end{array} \right)$$

$$<7> (\partial+3)_d (\partial+3)_{d+1} (d = 1 + 2w, w \in [0,4])$$

Step 2: On the basis of step1,

$$(\partial+2)_c (\partial+3)_d \left(\begin{array}{l} c = 1 + w, w \in [0,1], \\ d = 1 + w, w \in [0,9] \end{array} \right)$$

have been marked 0-edges.We turn them to 1-edges.

Step 3: The other edges in the S are labeled as 1.

In the S graph, there are 1500 edges,of which 775 are the 0-edges.According to the definition of edge-friendly labeling, $|e(0)-e(1)| \le 1$,we can get the edge-friendly labeling of the labeled graph by calculation. In this labeled graph, all the vertices

$$(\partial+1)_b (b = 1 + w + 5s, w \in [0,1], s \in [0,4])$$
$$(\partial+2)_c (c = 1 + w + 25s, w \in [2,11] \cup [15,17] \cup [20,21], s \in [0,4])$$
$$(\partial+2)_c (c = 26 + w + 25s, w \in [0,1], s \in [0,3])$$

are 0-vertices, $v(0)$=93 .In order to satisfy the index calculation of nested-cycle,the number of vertex of S doses't include $(\partial)_a (a \in [1,5])$.Thus $|v(1)-v(0)| = |682-93| = 589$.

We can prove $EBI(S) = 589$ is the maximum of edge-balanced index in the graph S.

In this labeled graph, the vertices in S are all saturated. In order to turn the label of any 0-vertex to1-vertex or unlabeled, we need to interchange 4 0-edges and 1-edges at least, but any of the 1-vertex's label changes, only need to interchange 3 0-edges and 1-edges at least.Obviously, if change 0-vertex's label ,then $v(0)$ increases or remains unchanged, and meanwhile $v(1)$ decreases, so the value of $|v(1)-v(0)|$ reduces. That's to say: $\max\{EBI(S)\} = 589$.

Lemma 3 For the power-cycle nested graph $C_{5^m} \times P_{m_5}$, when $m \equiv 1 \pmod 3$ and $m \ge 4$,

$$\max\{EBI(C_{5^m} \times P_{m_5})\} = \frac{19 \times 5^{m-1} + 1}{4}$$

Proof: For $m = 4$, the formula can be proved by Lemma 1.

When m>4 , firstly,construct the graphics with the maximum edge-balanced index of $C_{5^m} \times P_{m_5}$.

Obviously, $C_{5^m} \times P_{m_5} = C_{5^4} \times P_{4_5} \bigcup \left(\bigcup_{t=1}^{\frac{m-4}{3}} V_t \right)$,the maximum edge-balanced index labeling method of $C_{5^4} \times P_{4_5}$ is the same as we structure in lemma 1. For the nested-cycle subgraph $\bigcup_{i=1}^{\frac{m-4}{3}} V_i$ consists $\frac{5^{m-1}-125}{124}$ S ,the S in the $V_t \left(t = 1, 2, \cdots, \frac{m-4}{3} \right)$,whose maximum edge-balanced index labeling method is the same as we construct in lemma 2.During the process of label , we know the $C_{5^m} \times P_{m_5}$ satisfies the friendly labeling,and the characteristic of vertices remain unchanged,so this graphics is the maximum edge-balanced index.

We shall prove $\max\{EBI(C_{5^m} \times P_{m_5})\} = \frac{19 \times 5^{m-1} + 1}{4}$.

According to lemma 2,we know $\max\{EBI(S)\} = 589$, if the starting points at the $\partial th = 3t + 1 (t \in N^+)$ cycle,then the graph can be divided into 5^{∂} sector subgraphs and $5^{\partial-1}$ five-vertices sector subgraph-group.That's to say $V_t\left(t=1,2,\cdots,\frac{m-4}{3}\right)$ has 5^{3t} five-vertices sector subgraph-group.By computing, we can get the maximum edge-balanced index of $C_{5^m} \times P_{m_5}$.

$$\max\{EBI(C_{5^m} \times P_{m_5})\} = \max\left\{EBI\left(C_{5^4} \times P_{4_5}\right)\right\} + \sum_{t=1}^{\frac{m-4}{3}} \max\left\{EBI\left(V_t\right)\right\}$$
$$= 594 + 589 \times 5^3 + 589 \times 5^6 + \cdots + 589 \times 5^{m-4}$$
$$= \frac{19 \times 5^{m-1} + 1}{4}$$

The proof is completed.

Lemma 4 For S in nested-cycle subgraph $V_{\frac{m-4}{3}}(m \ge 7)$, $\{588,587,\cdots,1,0\} \subset EBI(S)$.

Proof: In the lemma 2,we get $\max\{EBI(S)\} = 589$.

We know all the five-vertices sector graph-group have the same label.Below transform is based on the biggest index labeled graph constructed from lemma 2, and the change of every step is based on the previous step .The $(2k-1)_r(2k)_s \leftrightarrow (2k)_s(2k)_{s+1}$ means that we exchange $(2k-1)_r(2k)_s$ and $(2k)_s(2k)_{s+1}$ which are 0-edge and 1-edge respectively.

First, build an odd index set:

Step 1: In turn exchange

$$(\partial+2)_c(\partial+3)_d \leftrightarrow (\partial+3)_d(\partial+3)_{d+1} \begin{pmatrix} c \in [3,10], \\ d = 11 + w + 5s, \\ w \in [0,2], s \in [0,7] \end{pmatrix},$$ we can obtain 24 labeled graphs of odd edge-balanced indexes which are $\{587,585,\cdots,543,541\}$.

Step 2: In turn exchange

$$(\partial+3)_c(\partial+4)_d \leftrightarrow (\partial+3)_d(\partial+3)_{d+1} \begin{pmatrix} c = 26 + w + 25s, \\ w \in [0,9], s \in [0,3]; \\ d = 126 + w + 5s + 125f, \\ w \in [0,2], s \in [0,9], f \in [0,3] \end{pmatrix},$$

we can obtain 120labeled graphs of odd edge-balanced indexes which are $\{539,537,\cdots,299,301\}$.

Step 3: In turn exchange

$$(\partial+3)_c(\partial+4)_d \leftrightarrow (\partial+3)_d(\partial+3)_{d+1} \begin{pmatrix} c = 11 + w + 25s, \\ w \in [0,1] \cup [5,7] \cup [10,11], s \in [0,4]; \\ d = 51 + w + 5s + 125f, \\ w \in [0,2], \\ s \in [0,1] \cup [5,7] \cup [10,11], f \in [0,4] \end{pmatrix},$$

we can obtain 105 labeled graphs of odd edge-balanced indexes which are $\{299,333,\cdots,93,91\}$.

Step 4: In turn exchange

$$(\partial+2)_c(\partial+3)_d \leftrightarrow (\partial+3)_d(\partial+3)_{d+1} \begin{pmatrix} c = 13 + w + 5s + 25f, \\ w \in [0,2], \ s \in \{0,2\}, f \in [0,4]; \\ d = 61 + 5w + 125s, \\ w \in [0,2] \cup [10,11], s \in [0,4] \end{pmatrix},$$

we can obtain 25 labeled graphs of odd edge-balanced indexes which are $\{89,87,\cdots,41\}$

Step5: In turn exchange

$$(\partial+2)_c(\partial+2)_{c+1} \leftrightarrow (\partial+2)_{c+1}(\partial+3)_d \begin{pmatrix} c = 13 + w + 5s + 25f, \\ w \in [0,1], \ s \in \{0,2\}, f \in [0,4]; \\ d = 70 + 5w + 125s, \\ w \in \{0,1,10,11\}, s \in [0,4] \end{pmatrix},$$

we can obtain 20 labeled graphs of odd edge-balanced indexes which are $\{39,37,\cdots 3,1\}$

Even index set structures as follows:

Step1: Exchange $(\partial+3)_1(\partial+3)_2 \leftrightarrow (\partial+2)_1(\partial+3)_2$, we can obtain one labeled graphs of even edge-balanced indexes which is $\{588\}$.

Step2:Repeat from the first to fifth step as we construct the odd index sets .Thus we can obtain these labeled graphs of even edge-balanced indexes which are respectively $\{586,584,\cdots,2,0\}$.

The proof is completed.

Lemma 5 For every of S in the nested-cycle subgraph,

$$V_t(1 \le t \le \frac{m-7}{3})(m \ge 10), \ \{588,587,\cdots,1,0\} \subset EBI(S).$$

Proof: Below transform is based on the biggest index labeled graph constructed form lemma3, and every step all proceeds is based on the previous step to transform, which in turn constructs labeling graphs corresponding with the index.One of S in the,is denoted as S_{V_t}.

We know the $(\partial)_a\left(a \in [1,5]\right)$ of $S_{V_{t+1}}$ regards as the poinds on the $(\partial+3)th$ circle of S_{V_t} when we calculate .One S_{V_t} of V_t corresponds to 125 $S_{V_{t+1}}$, we select 117 $S_{V_{t+1}}$ from 125 $S_{V_{t+1}}$,we change them as the following order ,in turn exchange $(\partial+1)_{b+1}(\partial+1)_b \leftrightarrow (\partial+1)_b(\partial)_a \begin{pmatrix} a\in[1,5] \\ b=5+5w, w\in[0,4] \end{pmatrix}$,we can change 585 1-vertex on the third circle of S_{V_t} into unlabeled, so that the index can decrease by 585, respectively 585 labeled graphs are obtained. By the lemma 2, $\max\{EBI(S)\}=589$,so $\{588,587,\cdots,5\}\subset EBI(S_{V_t})$, then we select one S_{V_t} again, in turn exchange

$$(\partial+1)_{b+1}(\partial+1)_b \leftrightarrow (\partial+1)_b(\partial)_a \begin{pmatrix} a\in[1,4] \\ b=5+5w, w\in[0,3] \end{pmatrix},$$

similarly, we can change 4 1-vertex on the third circle of S_{V_t} into unlabeled,so that the index can decrease by 4,respectively 4 labeled graphs are obtained, so $\{4,3\ 2\ 1\}\subset EBI(S_{V_t})$.That is $\{588,587,\cdots,0\}\subset EBI(S_{V_t})(1\le t\le \frac{m-7}{3})$.

The proof is completed.

Lemma 6 In nested-cycle subgraph $C_{5^m}\times P_{m_5}$,when $m=4$, $\{593,591,\ldots,1,0\}\subset EBI(C_{5^4}\times P_{4_5})$.

Proof:Below transform is based on the biggest index labeled graph constructed form lemma 1, and every step is based on the previous step to transform, which in turn constructs labeling graphs corresponding with the index.

First, build an even index set:

Step 1: In turn exchange

$$(3)_c(4)_d \leftrightarrow (4)_d(4)_{d+1} \begin{pmatrix} c=1+w+5s+25f, \\ w\in[0,3], s\in[0,1], f\in[0,4]; \\ d=1+5w+25s+125f, \\ w\in[0,3], s\in[0,1], f\in[0,4] \end{pmatrix},$$

we can obtain 40 labeled graphs of even edge-balanced indexes which are $\{592,590,\cdots,516,514\}$.

Step 2: In turn exchange

$$(3)_c(4)_d \leftrightarrow (4)_d(4)_{d+1} \begin{pmatrix} c=36+w+5s, w\in[0,14], \\ s\in[0,3]; \\ d=176+w+5s+125f, \\ w\in[0,2], s\in[0,14], f\in[0,3] \end{pmatrix},$$

we can obtain 180 labeled graphs of even edge-balanced indexes which are $\{512,508,\cdots,156,154\}$.

Step 3: In turn exchange

$$(3)_c(4)_d \leftrightarrow (4)_d(4)_{d+1} \begin{pmatrix} c=30+5w+25s, \\ w\in[0,1],\ s\in[0,3]; \\ d=146+w+25s+125f, \\ w\in[0,2], s\in[0,1], f\in[0,3] \end{pmatrix},$$

we can obtain 24 labeled graphs of even edge-balanced indexes which are $\{152,150,\cdots,108\ 106\}$.

Step 4: In turn exchange

$$(3)_c(4)_d \leftrightarrow (4)_d(4)_{d+1} \begin{pmatrix} c=17+w, w\in[0,8]; \\ d=81+w+5s, w\in[0,2], s\in[0,8] \end{pmatrix},$$

we can obtain 27 labeled graphs of even edge-balanced indexes which are $\{104,102,\cdots,54,52\}$.

Step5: In turn exchange

$$(3)_c(3)_{c+1} \leftrightarrow (3)_c(4)_d \begin{pmatrix} c=26+w+5s+25f, \\ w\in[0,2],\ s\in[0,1], f\in[0,3]; \\ d=126+5w+125s, \\ w\in[0,2]\cup[5,7], s\in[0,3] \end{pmatrix}$$

we can obtain 24 labeled graphs of even edge-balanced indexes which are $\{50,48,\cdots,6,4\}$.

Step6: In turn exchange $(1)_1(2)_1 \leftrightarrow (2)_1(2)_2, (1)_2(2)_6 \leftrightarrow (2)_6(2)_7$,we can obtain 2 labeled graphs of even edge-balanced indexes which are $\{2,0\}$.

Odd index set structures as follows:

Step1: Exchange $(1)_1(2)_3 \leftrightarrow (1)_1(1)_2$,we can obtain one edge-balanced indexes which is $\{593\}$.

Step2: Repeat from the first to fifth step as we construct the odd index sets .Thus we can obtain these labeled graphs of even edge-balanced indexes which are respectively $\{586,584,\cdots,2,0\}$.

Step3: Exchange $(1)_1(2)_1 \leftrightarrow (2)_1(2)_2$,we can obtain one edge-balanced indexes which is $\{1\}$.

The proof is completed.

Lemma 7 In the nested graph with power-cycle $C_{5^m}\times P_{m_5}$, when $m\equiv 1 (\bmod 3)$ and $m\ge 4$,

$$\{\frac{19\times 5^{m-1}-3}{4},\frac{19\times 5^{m-1}-7}{4},\cdots,1,0\}\subset EBI(C_{5^m}\times P_{m_5})$$

Proof: ①When $m=4$, the formula can be proved by lemma 6.

Now, we prove the formula was established when $m>4$. Below transform is based on the biggest index labeled graph constructed form lemma 3.

②When $m=7, C_{5^7}\times P_{7_5} = C_{5^4}\times P_{4_5}\cup V_1$,

Step 1: According to changing method of S in lemma 4.By lemma 4,every S_{v_1} 's index in V_1 can reduces by 589，when we exchange，we can obtain 589 labeled graphs conversely, so $\{589, 588, \cdots, 0\} \subset EBI(S)$.

V_1 has 125 S_{v_1} ,which have the same changing method as S in lemma 4,so that the index can decrease by 589×125 ,when we exchange， we can obtain 73625 edge-balanced conversely.so $\{74218, 74217, \cdots, 594\} \subset EBI(C_{5^7} \times P_{7_5})$

Step 2: To select 118 S_{v_1} from V_1 , in turn exchange $(5)_{b+1}(5)_b \leftrightarrow (5)_b(4)_a \begin{pmatrix} a\in[1,5] \\ b=5+5w, w\in[0,4] \end{pmatrix}$, we can obtain 590 labeled graphs' edge-balanced indexes which are $\{593, 592, \cdots, 4\}$,so $\{593, 592, \cdots, 4\} \subset EBI(C_{5^7} \times P_{7_5})$

Step3: Then we select one S_{v_1} from V_1 , in turn exchange $(5)_{b+1}(5)_b \leftrightarrow (5)_b(4)_a \begin{pmatrix} a\in[1,4] \\ b=5+5w, w\in[0,3] \end{pmatrix}$, we can obtain 4 edge-balanced indexes which are $\{3,2,1,0\}$,so $\{3,2,1,0\} \subset EBI(C_{5^7} \times P_{7_5})$.

③When $m\ge10, C_{5^m} \times P_{m_5} = C_{5^4} \times P_{4_5} \bigcup \left(\bigcup_{t=1}^{\frac{m-4}{3}} V_t \right)$,

Step 1: According to changing method of S in lemma 4.By lemma 4,every $S_{V_{\frac{m-4}{3}}}$'s index in $V_{\frac{m-4}{3}}$ can reduces by 589，when we exchange，we can obtain 589 labeled graphs conversely, $V_{\frac{m-4}{3}}$ has 5^{m-4} $S_{V_{\frac{m-4}{3}}}$,which have the same change method as S in lemma 4,so that the index can decrease by $589\times5^{m-4}$,when we exchange，we can obtain $589\times5^{m-4}$ labeled graphs. So $\{\frac{19\times5^{m-1}-3}{4}, \frac{19\times5^{m-1}-7}{4}, \cdots, \frac{19\times5^{m-1}-2356\times5^{m-4}+1}{4}\} \subset EBI(C_{5^m} \times P_{m_5})$

Step 2: According to changing method of S in lemma 5.All the S of $V_t (1\le t\le \frac{m-7}{3})$ have the same changing method as S in lemma 5,so that the index can decrease by $\frac{19\times5^{m-4}-2375}{4}$ .That is $\{\frac{19\times5^{m-4}-3}{4}, \frac{19\times5^{m-4}-7}{4}, \cdots, 595, 594\} \subset EBI(C_{5^m} \times P_{m_5})$.

Step 3: Repeat step 2 and step 3, when $m=7$, we can obtain 594 labeled edge-balanced graphs whose indexes are $\{593, 592, \ldots, 0\}$,So

$$\{\frac{19\times5^{m-1}-3}{4}, \frac{19\times5^{m-1}-7}{4}, \cdots, 1, 0\} \subset EBI(C_{5^m} \times P_{m_5}) (m\ge10).$$

The proof is completed.

IV. THEOREM

According to lemma 3 and lemma7, we can get the following theorem.

Theorem 1: In the nested graph with power-cycle $C_{5^m} \times P_{m_5}$ when $m\equiv1(\bmod 3)$ and $m\ge4$,

$$EBI(C_{5^m} \times P_{m_5}) = \{\frac{19\times5^m+1}{4}, \frac{19\times5^{m-1}-3}{4}, \cdots, 1, 0\}$$

V. CONCLUSION

In this paper ,in order to study the edge-balanced index sets of the nested graph with power-cycle $C_{5^m} \times P_{m_5}$,we introduced the five-vertices sector subgraph-group, showing the proof of the computational formula, at the same time giving the construction of the corresponding graphs. The resulting work using the same method will be investigated in a future paper.

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