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Stabilitty of Anti-periodic Solutions for Certain Shunting Inhibitory Cellular Neural Networks

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***Abstract*—In this paper, the existence and exponential stability of anti-periodic solutions for shunting inhibitory cellular neural networks (SICNNs) with continuously distributed delays are considered by constructing suitable Lyapunov fuctions and applying some critial analysis techniques. Our results remove restrictive conditions of the global Lipschitz and bounded conditions of activation functions and new sufficient conditions ensuring the existence and exponential stability of anti-periodic solutions for SICNNs are obtained. Moreover, an example is given to illustrate the feasibility of the conditions in our results.**

***Index Terms*—Global exponential stability, Shunting inhibitory cellular neural networks, Anti-periodic solution, Continuously distributed delays, Lyapunov fuctions.**

I. INTRODUCTION

Recently, cellular neural networks (CNNs) have shown great potential as information-processing systems, and many researchers have paid much attention to the research on the theory and application of the CNNs. Some sufficient conditions are given to ensure the existence and stability of the equilibrium point for the CNNs. The shunting inhibitory cellular neural networks (SICNNs) are a new class of CNNs, which were introduced by Bouzerdoum and Pinter in [1-3], and have been extensively applied in psychophysics, speech, perception, robotics, adaptive pattern recognition, vision, and image processing. Hence, they have been the object of intensive analysis by numerous researchers in recent years. In particular , there have been extensive results on the problem of the existence and stability of periodic and almost periodic solutions of SICNNs in the literature (see, e.g., [4-13] and the references therein). In contrast, however, very few results are available on a generic, in-depth, existence and exponential stability of anti-periodic solutions for SICNNs (1.1). Moreover, it is well known that the existence of anti-periodic solutions play a key role in characterizing the behavior of nonlinear differential equations (see [14-17]). Since SICNNs can be analog voltage transmission, and voltage transmission process is often an anti-periodic process. Thus, it is worth while to continue to investigate the existence and stability of anti-periodic solutions of SICNNs.

Consider shunting inhibitory cellular neural networks (SICNNs) with continuously distributed delays described by

$$x'_{ij}(t) = -a_{ij}(t)x_{ij}(t) - \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl}(t) f(x_{kl}(t-\tau(t))) x_{ij}(t) + L_{ij}(t), \quad (1.1)$$

where $i = 1,2,\cdots,m$, $j = 1,2,\cdots,n$, C_{ij} denotes the cell at the (i, j) position of the lattice, the r-neighborhood $N_r(i,j)$ of C_{ij} is given by

$$N_r(i,j) = \{C_{ij}^{kl} : \max(|k-i|,|l-j|) \le r, 1 \le k \le m, 1 \le l \le n\}.$$

x_{ij} acts as the activity of the cell C_{ij}, $L_{ij}(t)$ is the external Input to C_{ij}, the variable $a_{ij}(t) > 0$ represent the passive decay rate of the cell activity, $C_{ij}(t) \ge 0$ is the connection or coupling strength of postsynaptic activity of the cell transmitted to the cell C_{ij}, and the activity function $f(\cdot)$ is a continuous function representing the output or firing rate of the cell C_{kl}, and $\tau(t) \ge 0$ corresponds to the transmission delay.

However, we note that, in most of the above-mentioned literature, the activity function f(·) is assumed to be global Lipschitz continuous and bounded, that is, there exist constants μ_f and M_f such that for all $x, y \in R$

(T_0) $\quad |f(x) - f(y)| \le \mu_f |x-y|, \ |f(x)| \le M_f$.

To the best of our knowledge, few researchers have considersed SICNNs without (T_0). Thus, it is worthwhile to continue to investigate system (1.1).

In this paper, we will establish new sufficient conditions ensuring the existence, uniqueness and exponential stability of anti-periodic solutions of system (1.1) without (T_0). Moreover, an example is provided to illustrate the effectiveness of our results.

Let $u(t): R \to R$ be continuous in t. $u(t)$ is

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said to be T -anti-periodic on R if

$$u(t+T) = -u(t) \quad \text{for all } t \in R .$$

Throughout this paper, for $i = 1,2,\cdots,m$, $j = 1, 2,\cdots,n$, it will be assumed that $a_{ij}, C_{ij} : R \to [0,+\infty)$, $L_{ij} : R \to R$ are continuous functions and $\tau(t)$: $R \to [0,\tau]$, and

$$a_{ij}(t+T) = a_{ij}(t) , \; C_{ij}(t+T) = C_{ij}(t)$$
$$f(-u) = f(u) , \qquad \tau(t+T) = \tau(t),$$
$$L_{ij}(t+T) = -L_{ij}(t) , \forall t,u \in R . \qquad (1.2)$$

Set

$$\{x_{ij}(t)\} = (x_{11}(t),\cdots,x_{1n}(t),\cdots,x_{m1}(t),\cdots,x_{mn}(t))^T \in R^{m\times n}.$$

For $\forall\, x(t) = \{x_{ij}(t)\} \in R^{m\times n}$, we define the norm $\|x(t)\| = \max\limits_{(i,j)} \{|x_{ij}(t)|\}$.

The initial conditions associated with system (1.1) are of the form

$$x_{ij}(s) = \varphi_{ij}(s) , \; s \in [-\tau,0] , \qquad (1.3)$$

where $ij = 11,12,\cdots,mn$ and $\varphi_{ij}(\cdot)$ denotes real-valued continuous function defined on $[-\tau,0]$.

We also assume that the following conditions hold.

(T_1) There exist continuous functions $\mu : R^+ \to R^+$ such that for each D

$$|f(u) - f(v)| \le \mu(D)|u - v| , \; |u|,|v| \le D . \qquad (1.4)$$

(T_2) There exist constants $\lambda > 0, D_1 \ge D_0 > 0$, and $ij = 11,12,\cdots,mn$, such that

$$\underline{a}_{ij}D_0 - \sum_{C_{kl}\in N_r(i,j)} \overline{C}_{ij}^{kl}[|f(0)| + \mu(D_0)D_0]D_0 - L_{ij}^+ > 0,$$
$$(\lambda - \underline{a}_{ij}) + \sum_{C_{kl}\in N_r(i,j)} \overline{C}_{ij}^{kl}[\mu(D_1)D_1e^{\lambda\tau} + |f(0)| + \mu(D_0)D_0] < 0, \qquad (1.5)$$

where

$$\underline{a}_{ij} = \inf_{t\in R} a_{ij}(t) , \qquad \overline{C}_{ij}^{kl} = \sup_{t\in R} C_{ij}^{kl}(t) ,$$
$$L_{ij}^+ > \sup_{t\in R} L_{ij}(t) .$$

Definition 1. Let $x^*(t) = \{x_{ij}^*(t)\}$ be an anti-periodic solution of system (1.1) with initial value $\varphi^*(t) = \{\varphi_{ij}^*(t)\}$. If there exist constants $\lambda > 0$ and $M > 1$ such that for every solution $x(t) = \{x_{ij}(t)\}$ of system (1.1) with initial value $\varphi(t) = \{\varphi_{ij}(t)\}$,

$$|x_{ij}(t) - x_{ij}^*(t)| \le M\|\varphi - \varphi^*\|_1 e^{-\lambda t}, \forall t > 0, ij = 11,12,\cdots,mn.$$

where $\|\varphi - \varphi^*\|_1 = \sup\limits_{-\tau\le s\le 0}\{\max\limits_{(i,j)} |\varphi_{ij}(s) - \varphi_{ij}^*(s)|\}$ .Then $x^*(t)$ is said to be globally exponentially stable.

The remaining parts of this Letter are organized as follows. In Section 2, sufficient conditions are derived for the boundedness of solution of (1.1). In Section 3, we present new sufficient conditions for the existence , uniqueness and exponential stability of anti-periodic solutionof (1.1) . In Section 4 , an illustrative example is given to show the effectiveness of the proposed theory and method . In Section 5, we give several remarks.

Ⅱ. Preliminary results

Lemma 2.1. Let (T_1) and (T_2) hold. Suppose that $x^*(t) = \{x_{ij}^*(t)\}$ is a solution of system (1.1) with initial Conditions

$$x_{ij}^*(s) = \varphi_{ij}^*(s) , \; |\varphi_{ij}^*(s)| < D_0 , \; s \in [-\tau,0] . \qquad (2.1)$$

Then

$$|x_{ij}^*(t)| < D_0 , \; t \ge 0 , \; ij = 11,12,\cdots,mn . \qquad (2.2)$$

Proof. Assume, by way of contradiction, that (2.2) does not hold. Then , there exist $ij \in \{11,12,\cdots,mn\}$ and $\delta > 0$ such that

$$|x_{ij}^*(\delta)| = D_0 , \text{ and } |x_{\overline{ij}}^*(t)| < D_0 , \; \forall t \in [-\tau,\delta] , \qquad (2.3)$$

where $\overline{ij} \in \{11,12,\cdots,mn\}$. Calculating the upper right derivative of $|x_{ij}^*(\delta)|$, together with (T1) , (T2) and (2.3) , we can obtain

$$0 \le D^+(|x_{ij}^*(\delta)|) = \mathrm{sgn}(x_{ij}^*(\delta))D^+(x_{ij}^*(\delta))$$
$$= -a_{ij}(\delta)\,\mathrm{sgn}(x_{ij}^*(\delta))x_{ij}^*(\delta) - \mathrm{sgn}(x_{ij}^*)[\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(\delta)$$
$$f(x_{ij}^*(\delta - \tau(\delta)))x_{ij}^*(\delta) - L_{ij}(\delta)]$$
$$\le -a_{ij}(\delta)|x_{ij}^*(\delta)| + \left|\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(\delta) f(x_{ij}^*(\delta - \tau(\delta)))x_{ij}^*(\delta)\right|$$
$$+ |L_{ij}(\delta)|$$
$$\le -a_{ij}(\delta)|x_{ij}^*(\delta)| + \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(\delta)|f(x_{ij}^*(\delta - \tau(\delta)))||x_{ij}^*(\delta)|$$
$$+ |L_{ij}(\delta)|$$
$$\le -\underline{a}_{ij}D_0 + \sum_{C_{kl}\in N_r(i,j)} \overline{C}_{ij}^{kl}[|f(0)| + \mu(D_0)|x_{ij}^*(\delta - \tau(\delta)|]D_0 + L_{ij}^+$$
$$\le -\underline{a}_{ij}D_0 + \sum_{C_{kl}\in N_r(i,j)} \overline{C}_{ij}^{kl}[|f(0)| + \mu(D_0)D_0]D_0 + L_{ij}^+$$
$$< 0$$

This is a contradiction and hence (2.2) holds. This completes the proof.

Remark 2.1. In view of the boundedness of this solution, from the theory of functional differential equations in [18], it follows that $x^*(t)$ can be defined on $[-\tau,+\infty)$.

Lemma 2.2. Suppose that (T_1) and (T_2) are satisfied. Let $x^*(t) = (x_{11}^*(t), x_{12}^*(t),\cdots,x_{mn}^*(t))^T$ be the solution of system (1.1) with initial value (2.1) , and $x(t) = (x_{11}(t), x_{12}(t),\cdots,x_{mn}(t))^T$ be the solution of system (1.1) with initial value $\varphi(t) = (\varphi_{11}(t), \varphi_{12}(t),\cdots\varphi_{mn}(t))^T$ and

$\left|\varphi_{ij}(s)\right| < D_1$ for $s \in [-\tau, 0]$. Then there exist constants $\lambda > 0$ and $M > 1$ such that

$\left|x_{ij}(t) - x_{ij}^*(t)\right| \le M\|\varphi - \varphi^*\|_1 e^{-\lambda t}, \forall t > 0, ij = 11, 12, \cdots, mn$.

Proof. Set $y(t) = \{y_{ij}(t)\} = \{x_{ij}(t) - x_{ij}^*(t)\}$ Then

$$y_{ij}'(t) = -a_{ij}(t)y_{ij}(t) - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)[f(x_{kl}(t-\tau(t)))x_{ij}(t) - f(x_{kl}^*(t-\tau(t)))x_{ij}^*(t)], \quad (2.4)$$

where $ij \in \{11, 12, \cdots, mn\}$.

We consider the Lyapunov functional

$$V_{ij}(t) = \left|y_{ij}(t)\right| e^{\lambda t}, ij = 11, 12, \cdots mn. \quad (2.5)$$

Let $M > 1$ denotes an arbitrary real number and set

$$\|\varphi - \varphi^*\|_1 = \sup_{-\tau\le s\le 0}\{\max_{(i,j)}\left|\varphi_{ij}(s) - \varphi_{ij}^*(s)\right|\} > 0.$$

It follows from (2.5) that

$$V_{ij}(t) = \left|y_{ij}(t)\right|e^{\lambda t} < M\|\varphi - \varphi^*\|_1, t \in [-\tau, 0],$$

where $ij \in \{11, 12, \cdots, mn\}$. We claim that

$$V_{ij}(t) = \left|y_{ij}(t)\right|e^{\lambda t} < M\|\varphi - \varphi^*\|_1, t > 0, ij = 11, 12, \cdots mn. \quad (2.6)$$

Contrarily, there must exist $ij \in \{11, 12, \cdots, mn\}$ and $t_{ij} > 0$ such that

$$V_{ij}(t_{ij}) = M\|\varphi - \varphi^*\|_1, \text{and } V_{\overline{ij}}(t) < M\|\varphi - \varphi^*\|_1, t \in [-\tau, t_{ij}). \quad (2.7)$$

where $\overline{ij} \in \{11, 12, \cdots, mn\}$. It follows from (2.7) that

$$V_{ij}(t_{ij}) - M\|\varphi - \varphi^*\|_1 = 0, \; V_{\overline{ij}}(t) - M\|\varphi - \varphi^*\|_1 < 0, t \in [-\tau, t_{ij}). \quad (2.8)$$

Calculating the upper right derivative of $V_{ij}(t_{ij})$ along the solution $y(t_{ij}) = \{y_{ij}(t_{ij})\}$ of system (2.4) with the initial value $\overline{\varphi} = \varphi - \varphi^*$ and with (2.2) and (2.8), we obtain

$$\begin{aligned}
0 &\le D^+(V_{ij}(t_{ij}) - M\|\varphi - \varphi^*\|_1) = D^+(V_{ij}(t_{ij})) \\
&= \operatorname{sgn}(y_{ij}(t_{ij}))D^+(y_{ij}(t_{ij}))e^{\lambda t_{ij}} + \lambda\left|y_{ij}(t_{ij})\right|e^{\lambda t_{ij}} \\
&= \operatorname{sgn}(y_{ij}(t_{ij}))(D^+(x_{ij}(t_{ij})) - D^+(x_{ij}^*(t_{ij})))e^{\lambda\tau_{ij}} \\
&\quad + \lambda\left|y_{ij}(t_{ij})\right|e^{\lambda\tau_{ij}} \\
&= -a_{ij}(t_{ij})\operatorname{sgn}(y_{ij}(t_{ij}))[x_{ij}(t_{ij}) - x_{ij}^*(t_{ij})]e^{\lambda\tau_{ij}} - \operatorname{sgn}(y_{ij}(t_{ij})) \\
&\quad \{\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t_{ij})[f(x_{kl}(t_{ij}-\tau(t_{ij})))x_{ij}(t_{ij}) - f(x_{ij}^*(t_{ij}-\tau(t_{ij})))x_{ij}^*(t_{ij})]\} \\
&\quad e^{\lambda\tau_{ij}} + \lambda\left|y_{ij}(t_{ij})\right|e^{\lambda\tau_{ij}} \\
&\le -a_{ij}(t_{ij})\left|y_{ij}(t_{ij})\right|e^{\lambda t_{ij}} + \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t_{ij}) \\
&\quad \left|f(x_{kl}(t_{ij}-\tau(t_{ij})))x_{ij}(t_{ij}) - f(x_{kl}^*(t_{ij}-\tau(t_{ij})))x_{ij}^*(t_{ij})\right| \\
&\quad e^{\lambda t_{ij}} + \lambda\left|y_{ij}(t_{ij})\right|e^{\lambda t_{ij}} \\
&\le (\lambda - \underline{a}_{ij})\left|y_{ij}(t_{ij})\right|e^{\lambda t_{ij}} + \sum_{C_{kl}\in N_r(i,j)} \overline{C}_{ij}^{kl} e^{\lambda t_{ij}} \\
&\quad [\left|f(x_{kl}(t_{ij}-\tau(t_{ij})))x_{ij}(t_{ij}) - f(x_{kl}^*(t_{ij}-\tau(t_{ij})))x_{ij}(t_{ij})\right| \\
&\quad + \left|f(x_{kl}^*(t_{ij}-\tau(t_{ij})))x_{ij}(t_{ij}) - f(x_{kl}^*(t_{ij}-\tau(t_{ij})))x_{ij}^*(t_{ij})\right|] \\
&\le (\lambda - \underline{a}_{ij})\left|y_{ij}(t_{ij})\right|e^{\lambda t_{ij}} + \sum_{C_{kl}\in N_r(i,j)} \overline{C}_{ij}^{kl} e^{\lambda t_{ij}} \\
&\quad [\mu(D_1)\left|x_{kl}(t_{ij}-\tau(t_{ij})) - x_{kl}^*(t_{ij}-\tau(t_{ij}))\right|\left|x_{ij}(t_{ij})\right| \\
&\quad + \left|f(x_{kl}^*(t_{ij}-\tau(t_{ij}))) - f(0)\right|\left|x_{ij}(t_{ij}) - x_{ij}^*(t_{ij})\right|] \\
&\le (\lambda - \underline{a}_{ij})\left|y_{ij}(t_{ij})\right|e^{\lambda t_{ij}} + \sum_{C_{kl}\in N_r(i,j)} \overline{C}_{ij}^{kl}[\mu(D_1)D_1 \\
&\quad \left|y_{kl}(t_{ij}-\tau(t_{ij}))\right|e^{\lambda(t_{ij}-\tau(t_{ij}))}e^{\lambda\tau(t_{ij})} \\
&\quad + (\left|f(0)\right| + \mu(D_0)D_0)\left|y_{ij}(t_{ij})\right|e^{\lambda t_{ij}}] \\
&\le \{(\lambda - \underline{a}_{ij}) + \sum_{C_{kl}\in N_r(i,j)} \overline{C}_{ij}^{kl}[\mu(D_1)D_1 e^{\lambda\tau} + \left|f(0)\right| \\
&\quad + \mu(D_0)D_0]\}M\|\varphi - \varphi^*\|_1.
\end{aligned} \quad (2.9)$$

Thus,

$$0 \le (\lambda - \underline{a}_{ij}) + \sum_{C_{kl}\in N_r(i,j)} \overline{C}_{ij}^{kl}[\mu(D_1)D_1 e^{\lambda\tau} + \left|f(0)\right| + \mu(D_0)D_0],$$

which contradicts (T_2) Hence, (2.6) holds. It follows that

$$\left|y_{ij}(t)\right| < M\|\varphi - \varphi^*\|_1 e^{-\lambda t}, t > 0, ij = 11, 12, \cdots mn. \quad (2.10)$$

This completes the proof. The proof of Lemma 2.2 is completed.

Remark 2.2. If $x^*(t) = (x_{11}^*(t), x_{12}^*(t), \cdots, x_{mn}^*(t))^T$ is the T-anti-periodic solution of system (1.1), it follows from Lemma 2.2 and Definition 1 that $x^*(t)$ is globally exponentially stable.

Ⅲ. Main results

The following is our main result.

Theorem 3.1. Suppose that (T_1) and (T_2) are satisfied. Then system (1.1) has exactly one T-anti-periodic solution $x^*(t)$. Moreover, $x^*(t)$ is globally exponentially stable.

Proof. Let $x(t) = (x_{11}(t), x_{12}(t), \cdots, x_{mn}(t))^T$ be a solution of system (1.1) with initial conditions

$$x_{ij}(s)=\varphi_{ij}(s), |\varphi_{ij}(s)| < D_0, s\in[-\tau,0], ij=11,12,\cdots,mn. \quad (3.1)$$

By Lemma 2.1, the solution $x(t)$ is bounded and

$$|x_{ij}(s)| < D_0, t\in[-\tau,+\infty), ij=11,12,\cdots,mn. \quad (3.2)$$

From (1.1) and (1.2), we have

$$((-1)^{k+1}x_{ij}(t+(k+1)T))' = (-1)^{k+1}x'_{ij}(t+(k+1)T)$$

$$= (-1)^{k+1}\{-a_{ij}(t+(k+1)T)x_{ij}(t+(k+1)T) - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t+(k+1)T)f(x_{kl}(t+(k+1)T-\tau(t+(k+1)T)))$$

$$x_{ij}(t+(k+1)T)+L_{ij}(t+(k+1)T)\}$$

$$= (-1)^{k+1}\{-a_{ij}(t)x_{ij}(t+(k+1)T) - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)$$

$$f(x_{kl}(t+(k+1)T-\tau(t)))x_{ij}(t+(k+1)T)+(-1)^{k+1}L_{ij}(t)\}$$

$$= -a_{ij}(t)(-1)^{k+1}x_{ij}(t+(k+1)T) - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)$$

$$f((-1)^{k+1}x_{kl}(t+(k+1)T-\tau(t)))(-1)^{k+1}x_{ij}(t+(k+1)T))$$

$$+L_{ij}(t), \quad (3.3)$$

where $ij\in\{11,12,\cdots,mn\}$. Thus, for any natural number k, $(-1)^{k+1}x(t+(k+1)T)$ are the solutions of system (1.1). Then, by Lemma 2.2, there exists a constant $M>0$ such that

$$|(-1)^{k+1}x_{ij}(t+(k+1)T)) - (-1)^k x_{ij}(t+kT)|$$

$$= |x_{ij}(t+kt+T)) + x_{ij}(t+kT)|$$

$$\le Me^{-\lambda(t+kT)} \sup_{-\tau\le s\le 0}\{\max_{(i,j)}|x_{ij}(s+T)+x_{ij}(s)|\}$$

$$\le 2MD_0e^{-\lambda t}(e^{-\lambda T})^k, \forall t+kT>0, \quad (3.4)$$

Thus, we can choose a sufficiently large constant $N>0$ and a positive constant α such that

$$|(-1)^{k+1}x_{ij}(t+(k+1)T)) - (-1)^k x_{ij}(t+kT)|$$

$$\le \alpha(e^{-\lambda T})^k, \quad \forall k>N, \quad (3.5)$$

on any compact set of R. For any natural number p, we obtain

$$(-1)^{p+1}x_{ij}(t+(p+1)T)$$

$$= x_{ij}(t) + \sum_{k=0}^{p}[(-1)^{k+1}x_{ij}(t+(k+1)T)$$

$$-(-1)^k x_{ij}(t+kT)]. \quad (3.6)$$

Then

$$|(-1)^{p+1}x_{ij}(t+(p+1)T)|$$

$$\le |x_{ij}(t)| + \sum_{k=0}^{p}|(-1)^{k+1}x_{ij}(t+(k+1)T) - (-1)^k x_{ij}(t+kT)|, \quad (3.7)$$

where $ij=11,12,\cdots,mn$. It follows from (3.5) and (3.7) that $\{(-1)^p x(t+pT)\}$ uniformly converges to a continuous function $x^*(t)=(x_{11}^*(t),x_{12}^*(t),\cdots,x_{mn}^*(t))^T$ on any compact set of R.

Now we will show that $x^*(t)$ is the T-anti-periodic solution of system (1.1). First, $x^*(t)$ is T-anti-periodic, since

$$x^*(t+T) = \lim_{p\to\infty}(-1)^p x(t+T+pT)$$

$$= -\lim_{p+1\to\infty}(-1)^{p+1}x(t+(p+1)T) = -x^*(t).$$

Next, we prove that $x^*(t)$ is a solution of (1.1). In fact, together with the continuity of the right side of (1.1), (3.3) implies that $\{((-1)^p x(t+pT))'\}$ uniformly converges to a continuous function on any compact set of R. Thus, letting $p\to\infty$, we obtain

$$\frac{d}{dt}\{x_{il}^*(t)\} = -a_{ij}(t)x_{ij}^*(t) - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)$$

$$f(x_{ij}^*(t-\tau(t)))\; x_{ij}^*(t) + L_{ij}(t). \quad (3.8)$$

Therefore, $x^*(t)$ is a solution of (1.1).

Finally, by Lemma 2.2, we can prove that $x^*(t)$ is globally exponentially stable. This completes the proof.

If we take $a_{ij}(t)=a_{ij}, C_{ij}(t)=C_{ij}$, then the system (1.1) can be modified to the following form:

$$x'_{ij}(t) = -a_{ij}x_{ij}(t) - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl} f(x_{kl}(t-\tau(t)))$$

$$x_{ij}(t)+L_{ij}(t), \quad (3.9)$$

We can make the following conclusion:

Corollary 3.2 Assume that (T_1) and $(T_2)'$ hold, where $(T_2)'$:There exist constants $\lambda>0, D_1\ge D_0>0$, and $ij=11,12,\cdots,mn$, such that

$$a_{ij}D_0 - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}[|f(0)|+\mu(D_0)D_0]D_0 - L_{ij}^+ > 0,$$

$$(\lambda - a_{ij}) + \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}[\mu(D_1)D_1e^{\lambda\tau} + |f(0)| + \mu(D_0)D_0] < 0.$$

then system (3.9) has one T-anti-periodic solution. Moreover, the T-anti-periodic solution is globally exponentially stable.

In [14], J.Shao also discussed existence and exponential stability of anti-periodic solutions of system (3.9) and The following result was proved.

Theorem A (J.Shao[14]) Assume that

(T_1^*) there exist constants μ_f and M_f such that for all $u,v\in R$

$$|f(u)-f(v)|\le \mu_f|u-v|, \quad |f(u)|\le M_f.$$

(T_2^*) there exist constants $\delta_{ij} > 0, \eta > 0$ and $\lambda > 0$, $ij = 11, 12, \cdots, mn$, such that

$$\delta_{ij} = a_{ij} - \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl} M_f,$$

$$(\lambda - a_{ij}) + \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl} M_f + \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl} \mu_f \frac{L_{ij}^+}{\delta_{ij}} e^{\lambda \bar{\tau}} < -\eta < 0,$$

where $L_{ij}^+ > \max_{t \in R} |L_{ij}(t)|, \bar{\tau} = \max_{t \in R} \{\tau(t)\}$. then system (3.9) has one T-anti-periodic solution. Moreover, the T-anti-periodic solution is globally exponentially stable.

Remark 3.1 The Corollary 3.2 is different with Theorem **A** because we do not require that the activity function f(·) is assumed to be global Lipschitz continuous and bounded.Our criteria is more general than the one in [14].

Ⅳ. An example

In this section , we give an example to demonstrate the results obtained in previous sections.

Example 4.1. Consider the following SICNNs with continuously distributed delays:

$$x_{ij}'(t) = -a_{ij}(t)x_{ij}(t) - \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl}(t) f(x_{kl}(t - \tau(t))) x_{ij}(t) + L_{ij}(t), \quad (4.1)$$

$$\begin{pmatrix} a_{11}(t) & a_{12}(t) & a_{13}(t) & a_{14}(t) \\ a_{21}(t) & a_{22}(t) & a_{23}(t) & a_{24}(t) \\ a_{31}(t) & a_{32}(t) & a_{33}(t) & a_{34}(t) \\ a_{41}(t) & a_{42}(t) & a_{43}(t) & a_{44}(t) \end{pmatrix}$$

$$= \begin{pmatrix} 2+|\sin t| & 2+|\sin t| & 2+|\cos t| & 3+|\cos t| \\ 3+|\cos t| & 2+|\cos t| & 3+|\sin t| & 3+|\sin t| \\ 3+|\sin t| & 2+|\cos t| & 2+|\sin t| & 2+|\sin t| \\ 3+|\cos t| & 2+|\sin t| & 3+|\cos t| & 2+|\sin t| \end{pmatrix},$$

$$\begin{pmatrix} C_{11}(t) & C_{12}(t) & C_{13}(t) & C_{14}(t) \\ C_{21}(t) & C_{22}(t) & C_{23}(t) & C_{24}(t) \\ C_{31}(t) & C_{32}(t) & C_{33}(t) & C_{34}(t) \\ C_{41}(t) & C_{42}(t) & C_{43}(t) & C_{44}(t) \end{pmatrix}$$

$$= \begin{pmatrix} 0.07 & 0.05 & 0.04 & 0.02 \\ 0.13 & 0 & 0.15 & 0.05 \\ 0.01 & 0.12 & 0 & 0.14 \\ 0.03 & 0.14 & 0.05 & 0.01 \end{pmatrix} +$$

$$\begin{pmatrix} 0.03|\sin t| & 0.15|\sin t| & 0.06|\cos t| & 0.08|\cos t| \\ 0.07|\cos t| & 0 & 0.05|\sin t| & 0.05|\sin t| \\ 0.09|\sin t| & 0.08|\cos t| & 0 & 0.06|\sin t| \\ 0.07|\cos t| & 0.06|\sin t| & 0.05|\cos t| & 0.09|\sin t| \end{pmatrix}$$

$$\begin{pmatrix} L_{11}(t) & L_{12}(t) & L_{13}(t) & L_{14}(t) \\ L_{21}(t) & L_{22}(t) & L_{23}(t) & L_{24}(t) \\ L_{31}(t) & L_{32}(t) & L_{33}(t) & L_{34}(t) \\ L_{41}(t) & L_{42}(t) & L_{43}(t) & L_{44}(t) \end{pmatrix}$$

$$= \begin{pmatrix} 0.4\sin t & 0.4\cos t & 0.1\sin t & 0.2\cos t \\ 0.3\cos t & 0.1\sin t & 0.2\cos t & 0.3\sin t \\ 0.2\cos t & 0.4\sin t & 0.1\cos t & 0.2\sin t \\ 0.1\sin t & 0.1\cos t & 0.2\sin t & 0.1\cos t \end{pmatrix}.$$

Set $r = 1, \tau(t) = \cos^2 t$, and $f(x) = \frac{x^2+1}{5}$, clearly, $f'(x) = \frac{2}{5}x$, then $\mu(D) = \frac{2}{5}D$,

$\sum_{C_{kl} \in N_1(1,1)} \bar{C}_{11}^{kl} = 0.5,\quad \sum_{C_{kl} \in N_1(1,2)} \bar{C}_{12}^{kl} = 0.8,$

$\sum_{C_{kl} \in N_1(1,3)} \bar{C}_{13}^{kl} = 0.7,\quad \sum_{C_{kl} \in N_1(1,4)} \bar{C}_{14}^{kl} = 0.5,$

$\sum_{C_{kl} \in N_1(2,1)} \bar{C}_{21}^{kl} = 0.8,\quad \sum_{C_{kl} \in N_1(2,2)} \bar{C}_{22}^{kl} = 1.1,$

$\sum_{C_{kl} \in N_1(2,3)} \bar{C}_{23}^{kl} = 1.0,\quad \sum_{C_{kl} \in N_1(2,4)} \bar{C}_{24}^{kl} = 0.7,$

$\sum_{C_{kl} \in N_1(3,1)} \bar{C}_{31}^{kl} = 0.8,\quad \sum_{C_{kl} \in N_1(3,2)} \bar{C}_{32}^{kl} = 1.1,$

$\sum_{C_{kl} \in N_1(3,3)} \bar{C}_{33}^{kl} = 1.1,\quad \sum_{C_{kl} \in N_1(3,4)} \bar{C}_{34}^{kl} = 0.7,$

$\sum_{C_{kl} \in N_1(4,1)} \bar{C}_{41}^{kl} = 0.6,\quad \sum_{C_{kl} \in N_1(4,2)} \bar{C}_{42}^{kl} = 0.7,$

$\sum_{C_{kl} \in N_1(4,3)} \bar{C}_{43}^{kl} = 0.8,\quad \sum_{C_{kl} \in N_1(4,4)} \bar{C}_{44}^{kl} = 0.4,$

$$\begin{pmatrix} \underline{a}_{11} & \underline{a}_{12} & \underline{a}_{13} & \underline{a}_{14} \\ \underline{a}_{21} & \underline{a}_{22} & \underline{a}_{23} & \underline{a}_{24} \\ \underline{a}_{31} & \underline{a}_{32} & \underline{a}_{33} & \underline{a}_{34} \\ \underline{a}_{41} & \underline{a}_{42} & \underline{a}_{43} & \underline{a}_{44} \end{pmatrix} = \begin{pmatrix} 2 & 2 & 2 & 3 \\ 3 & 2 & 3 & 3 \\ 3 & 2 & 2 & 2 \\ 3 & 2 & 3 & 2 \end{pmatrix},$$

$$\begin{pmatrix} L_{11}^+ & L_{12}^+ & L_{13}^+ & L_{14}^+ \\ L_{21}^+ & L_{22}^+ & L_{23}^+ & L_{24}^+ \\ L_{31}^+ & L_{32}^+ & L_{33}^+ & L_{34}^+ \\ L_{41}^+ & L_{42}^+ & L_{43}^+ & L_{44}^+ \end{pmatrix} > \begin{pmatrix} 0.5 & 0.5 & 0.2 & 0.3 \\ 0.4 & 0.2 & 0.3 & 0.4 \\ 0.3 & 0.5 & 0.2 & 0.3 \\ 0.2 & 0.2 & 0.3 & 0.2 \end{pmatrix},$$

Take $D_0 = D_1 = 1$ and Define continuous function $F_{ij}(\omega)$ by setting

$$F_{ij}(\omega) = (\omega - \underline{a}_{ij}) + \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} [\mu(D_1) D_1 e^{\omega\tau} + |f(0)|$$

$+\mu(D_0)D_0]$,

where $\omega\in[0,1]$, $ij = 11,12,\cdots,mn$. Then, we obtain

$$\max_{(i,j)}\{F_{ij}(0)\} = \max_{(i,j)}\{-\underline{a}_{ij} + \sum_{C_{kl}\in N_r(i,j)} \overline{C}_{ij}^{kl}[\mu(D_1)D_1$$

$$+\left|f(0)\right| + \mu(D_0)D_0]\} \le -0.9 .$$

Thus, there exists $\lambda > 0$ such that $F_{ij}(\lambda) < 0$ for $ij = 11,12,\cdots,mn$ and

$$\min_{(i,j)}\{\underline{a}_{ij}D_0 - \sum_{C_{kl}\in N_r(i,j)} \overline{C}_{ij}^{kl}[\left|f(0)\right| + \mu(D_0)D_0]D_0 - L_{ij}^{+}\} > \frac{21}{25},$$

So (T_2) holds. By Theorem 3.1, all the solution of system (4.1) with initial value $\left|\varphi_{ij}(s)\right| < D_1$ for $s\in[-\tau,0]$ converge exponentially to one π-anti-periodic solution.

If we take $a_{ij}(t) = \underline{a}_{ij}$,

$$\begin{pmatrix} C_{11}(t) & C_{12}(t) & C_{13}(t) & C_{14}(t) \\ C_{21}(t) & C_{22}(t) & C_{23}(t) & C_{24}(t) \\ C_{31}(t) & C_{32}(t) & C_{33}(t) & C_{34}(t) \\ C_{41}(t) & C_{42}(t) & C_{43}(t) & C_{44}(t) \end{pmatrix} = \begin{pmatrix} 0.1 & 0.2 & 0.1 & 0.1 \\ 0.2 & 0 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0 & 0.2 \\ 0.1 & 0.2 & 0.1 & 0.1 \end{pmatrix},$$

and other conditions of the above example also hold, then, by Corollary 3.2**,** all the solution of system (4.1) with initial value $\left|\varphi_{ij}(s)\right| < D_1$ for $s\in[-\tau,0]$ converge exponentially to one π-anti-periodic solution.

Remark 4.1. The function $f(\cdot)$ in Example 4.1 does not satisfy the conditions of Theorem A (J.Shao[14]) . Thus , the results in Theorem A (J.Shao[14]) can not be applied to Example 4.1. This implies that the results of this paper are essentially new.

Ⅴ**. Conclusion**

In this paper , shunting inhibitory cellular neural networks with continuously distributed delays have been studied. New Sufficient conditions for the existence and global exponential stability of anti-periodic solutions have been established , which complement previously known results. Moreover, an example is given to illustrate the effectiveness of our results.

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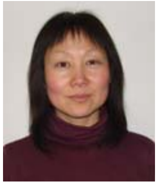

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