

# Criterion for Ranking Interval Alternatives in a Decision-Making Task

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Received: 08 September, 2023; Revised: 21 October, 2023; Accepted: 14 February, 2024; Published: 08 April, 2024

**Abstract:** The study solves the problem of improving the methodological and algorithmic support of the decision-making process by developing a model of the preference criterion for interval evaluations of alternatives. The aim of the study is to improve the efficiency of decision-making based on interval expert data under conditions of uncertainty and risk by developing a criterion for the preferences of interval evaluations of overlapping alternatives. The object of the study is the decision-making process based on the classical efficiency matrix with interval elements, the subject is the

model of decision maker's (DMP) preference criteria for interval evaluations of alternatives. The relevance of the task is stipulated by the urgency of the problem of adapting classical decision-making methods and models to practical problems of gray analysis, in particular, with interval uncertainty of primary expert data. A multifactorial model of the normalized preference criterion for interval evaluations of alternatives is proposed. Due to the additional consideration of the degree of preference of the DMP for the width of interval estimates, it allows ranking interval estimates of alternatives that overlap and are considered classically incomparable. A single analytical form of the normalized criterion model for ranking interval, weighted interval and point estimates makes it possible to increase the degree of automation of processing interval expert estimates in the decision-making process. Recommendations for the practical application of the proposed model are formulated. The developed model and corresponding algorithms can be used in automated expert decision support systems.

**Index Terms:** Decision-making, preference criterion, interval analysis, expert evaluation, confidence probability.

## 1. Introduction

The decision-making task is one of the main ones that arise in the management of organizational and technical systems [1]. This determines the importance and relevance of developing effective models, methods and tools for decision support [2–4].

In the traditional formulation of the decision-making task under conditions of complete uncertainty or risk [5], the decision maker (DMP) needs to choose the best alternative according to a certain criterion. At the same time, the initial data may contain uncertainty of various forms, for example, interval [6,7], fuzzy [8] or stochastic [9]. This uncertainty is usually due to the manifestation of different types of non-factors [10], which may arise in the process of obtaining and processing expert knowledge (Table 1). In practice, there may be situations when ignorance manifests itself simultaneously in several forms.

Table 1. The main NON-factors that manifest themselves in the process of expert evaluation and their characteristics

Title	Definition, characterization	Examples in the process of expert evaluation
Incompleteness	Some data required to solve the task are unknown, but all available information is complete and correct	Experts are provided with instructions on how to form assessments, but they do not take into account the full range of properties, characteristics or parameters of the object of expertise The expert does not have enough information (facts, knowledge) necessary to solve the task, and the only way to reduce the scope of this ignorance is to replenish the missing information (data, knowledge)
Uncertainty	The situation when there is no information (knowledge) about the subject of examination, characterized by two extreme levels of uncertainty: complete confidence and absolute ignorance	When analyzing expert opinions, the expert does not have a priori information about the competence of the experts
Fuzziness	Information the reliability of which is beyond doubt is available only in a fuzzy form	The information received from experts is reliable, but expressed in the form of a function of belonging of the objects of expertise or their parameters to a certain fuzzy set
Imprecision	The value of the parameter of the object of examination can be obtained with an accuracy that does not exceed a certain threshold, objectively determined by the nature of the corresponding parameter	The values of some parameters are measured with an a priori known error, and it is this error that generates inaccuracy
Heterogeneity	In the studied data samples, there are observations that stand out from the original population to some extent, which makes the sample heterogeneous	The appearance of heterogeneous data in samples causes distortion of strict parametric models described by known laws of probability distribution. This leads to the appearance of so-called "tails" and "hills" characterized by pronounced skewness and kurtosis.

*The aim of the study* is to improve the efficiency of decision-making based on interval expert data under conditions of uncertainty and risk by developing a criterion for the preferences of interval evaluations of overlapping alternatives.

*The object of the study* is the decision-making process based on the classical efficiency matrix with interval elements.

*The subject of the study* is the models of preference criteria of the DMP for interval evaluations of alternatives in the conditions of their classical incomparability.

*Study tasks are as follows:*

- 1) to develop a model of the DMP's preference criterion for interval evaluations of overlapping alternatives;
- 2) to assess the limits of the model's adequacy and develop recommendations for its practical application.

## 2. Literature Review

The deterministic decision-making problem under conditions of complete uncertainty or risk is a classic one and is formalized, for example, in [12] as an efficiency matrix (Table 2).

Table 2. Matrix of decision efficiency

	$F_1$	$F_2$	...	$F_m$
$A_1$	$E_{11}$	$E_{12}$	...	$E_{1m}$
$A_2$	$E_{21}$	$E_{22}$	...	$E_{2m}$
...	...	...	...	...
$A_n$	$E_{n1}$	$E_{n2}$	...	$E_{nm}$

The rows  $A_i$  of the matrix correspond to alternative decision options or strategies, and the columns  $F_j$  correspond to possible states (conditions) of the environment. At their intersection there are performance indicators (sometimes called payments)  $E_{ij}$  that characterize the implementation of strategies  $A_i$  in the relevant conditions  $F_j$ .

To find the optimal strategy under conditions of complete uncertainty and risk, criteria are used, for example, classical and derived criteria are described in [13]. Modified criteria described, for example, in [14,15] are also used (Table 3).

Table 3. Key criteria for decision-making under conditions of complete uncertainty and risk

Name of the criterion	Formula for calculating the value of the criterion
maximin (Wald)	$Z_{MM} = \max_i \left( \min_j (E_{ij}) \right)$
Bayesian-Laplace	$Z_{BL} = \max_i \left( \sum_{j=1}^m E_{ij} q_j \right)$
Savage	$Z_S = \min_i \left( \max_j \left( \max_i (E_{ij}) - E_{ij} \right) \right)$
extended maximum	$Z_{ME} = \max_p \left( \min_q \left( \sum_{i=1}^n \sum_{j=1}^m E_{ij} p_i q_j \right) \right)$
of a gambler	$Z_{AG} = \max_i \left( \max_j (E_{ij}) \right)$
Hurwitz	$Z_{HW} = \max_i \left( \lambda \cdot \min_j (E_{ij}) + (1 - \lambda) \cdot \max_j (E_{ij}) \right), \lambda \in [0, 1]$
Hodge-Lehman	$Z_{HL} = \max_i \left( \nu \cdot \sum_{j=1}^n E_{ij} q_j + (1 - \nu) \cdot \min_j (E_{ij}) \right), \nu \in [0, 1]$
Hermeyer	$Z_G = \max_i \left( \min_j (E_{ij} q_j) \right)$
BL(MM)-criterion	$I_1 := \left\{ i \mid i \{1, \dots, m\} \& E_{i0j0} - \min_j (E_{ij0}) \leq \varepsilon_{oon} \right\},$ $I_2 := \left\{ i \mid i \{1, \dots, m\} \& \max_j (E_{ij}) - \max_j (E_{i0j}) \geq e_{i0j0} - \min_j (e_{ij0}) \right\},$ $Z_{BL(MM)} = \max_{I_1 \cap I_2} \left( \sum_{j=1}^m E_{ij} q_j \right)$
products	$Z_p = \max_i \left( \prod_{j=1}^m E_{ij} q_j \right)$

Let's consider the case when some elements of the efficiency matrix contain interval uncertainty [16,17], i.e., are represented by interval numbers

$$[E_{ij}] = [\underline{E}_{ij}, \overline{E}_{ij}] = \{E_{ij} \mid \underline{E}_{ij} \leq E_{ij} \leq \overline{E}_{ij}\}, \quad (1)$$

where  $\underline{E}_{ij}$ ,  $\overline{E}_{ij}$  are respectively the lower (left) and upper (right) boundaries of the interval, respectively. The other elements of the matrix are expressed as real numbers, i.e., a point estimate of their values is available.

For the purposes of this study, we will use the equivalent form of representation of intervals in the center-radius format [18]:

$$[X] = [\underline{x}, \bar{x}] = \langle x, r \rangle, \quad (2)$$

where  $x = \frac{1}{2}(\underline{x} + \bar{x})$  is the center, and  $r = \frac{1}{2}(\bar{x} - \underline{x})$  – radius of the interval. For it, the rules for performing arithmetic operations are formulated and algorithms for calculating the values of elementary functions are proposed [19].

Since the choice and use of a specific criterion from Table 3 actually determines a specific type of dependence  $Z_i(E_{ij})$ , the final values of the criterion to be ranked will also be expressed in interval form:

$$[Z_i] = \langle z_i, r_i \rangle. \quad (3)$$

The problem of ranking interval values has a strict solution only if the intervals do not overlap. The best alternative is the one with the maximum values of the lower and upper bounds [20]:

$$[Z_k] \leq [Z_l] \Leftrightarrow ((\forall Z_k \in [Z_k])(\forall Z_l \in [Z_l])(Z_k \leq Z_l)). \quad (4)$$

Otherwise, there is a "weak" inequality:

$$[Z_k] \leq [Z_l] \Leftrightarrow ((\exists Z_k \in [Z_k])(\exists Z_l \in [Z_l])(Z_k \leq Z_l)), \quad (5)$$

that is, overlapping intervals are considered incomparable within the classical paradigm of interval analysis [21]. The choice of the best of these alternatives requires further research of the preference systems of the DMP using special methods, or negotiations in the "analyst – DMP" format in order to reduce the conflict in the preference system by finding certain compromises [20].

Thus, an urgent scientific and practical task is to develop models of preference criteria for interval evaluations of alternatives, in particular for situations of their classical incomparability.

### 3. Methodology Section

Let the interval values of the efficiency criterion of alternatives are known  $[Z_i] = \langle z_i, r_i \rangle$ ,  $i = 1, \dots, N$ . In [11], it is proposed to use a monotonically increasing function that is integral on the entire real axis as a preference function  $u(z)$ . Using the criterion of the form

$$u_i^*(z_i, r_i) = \frac{1}{2r_i} \int_{z_i - r_i}^{z_i + r_i} u(z) dz, \quad (6)$$

it is possible to assess the degree of superiority of one interval estimate over another.

The use of criterion (6) to rank the advantages of interval values of the criterion  $[Z_i]$  only partially removes the uncertainty in the case of mutual overlap of intervals. In particular, it is easy to see that the transformation (6) is not mutually unambiguous, i.e., a particular value of the criterion  $u^*$  corresponds to an unlimited number of intervals for which it is valid.

For example, in fig. 1, the intervals  $[Z_1] = \langle z_1, r_1 \rangle$  and  $[Z_2] = \langle z_2, r_2 \rangle$  are incomparable within the paradigm of classical interval analysis and at the same time equivalent according to criterion (6).

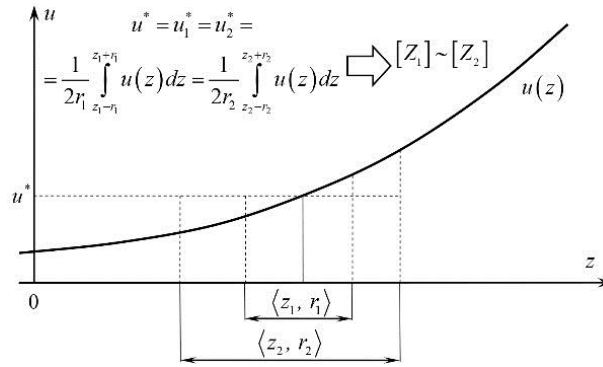


Fig. 1. Graphical illustration of the ambiguity of assessing the degree of preference of interval estimates of alternatives according to criterion (6)

To overcome this ambiguity, it is proposed to modify criterion (6) to take into account the width of the interval estimates.

We will assume that the initial interval assessments  $\langle e_{ij}, r_{ij} \rangle$  are obtained by the expert way, so their width  $\text{wid}[E_{ij}] = 2r_{ij}$  reflects the subjective degree of uncertainty of the expert assessment. In this case, the natural desire of the DMP will be to reasonably narrow the uncertainty intervals, down to point values  $e_{ij}$ .

Let us define the "gain" from narrowing the uncertainty intervals by a normalized piecewise linear function of the following form:

$$y(z, r) = \begin{cases} 1 - \frac{r/|z|}{(r/|z|)_{\max}}, & 0 \leq r/|z| \leq (r/|z|)_{\max}, \\ 0, & r/|z| > (r/|z|)_{\max} \end{cases}, \quad (7)$$

where  $(r/|z|)_{\max} = \max_i (r_i/|z_i|)$ ,  $i = 1, \dots, N$  – the maximum value of the ratio of the interval radius ( $r \geq 0$ ) to the module of its middle from the entire set  $[Z_i(e_{ij})] = \langle z_i, r_i \rangle$ .

Note that  $y(z, r)$  is determined for all  $r \geq 0$  (fig. 2), and for all point estimates  $y(z, r) = 1$  is true.

Interval  $0 \leq r/|z| \leq (r/|z|)_{\max}$  defines the working area of the model. It is within its boundaries that all alternatives from the set  $[Z_i]$ .

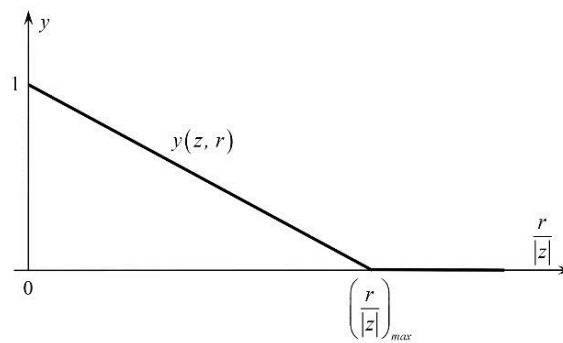


Fig. 2. Graph of the normalized piecewise linear function of the "gain" from narrowing the interval uncertainty of expert opinions

To preserve the common normalized form of the criteria, we will use a piecewise linear function of the following form as a preference function:

$$u(z) = \begin{cases} 0, & z < z_{\min} \\ \frac{z - z_{\min}}{z_{\max} - z_{\min}}, & z_{\min} \leq z \leq z_{\max} \\ 1, & z > z_{\max} \end{cases}, \quad (8)$$

where  $z_{\min} = \min_i (z_i - r_i)$  – the smallest, and  $z_{\max} = \max_i (z_i + r_i)$  – is the largest value of the variable  $z$ , belonging to at least one of the intervals  $[Z_i]$ .

In this case, within the boundaries of the model's working area (fig. 3), i.e. for the entire set  $[Z_i]$ , it is true

$$u_i^*(z_i, r_i) = \frac{1}{2r_i} \int_{z_i - r_i}^{z_i + r_i} u(z) dz = u(z_i). \quad (9)$$

Thus, the general form of the normalized multiplicative criterion of the DMP's preferences for interval evaluations of alternatives can be formalized in the following way:

$$K(z, r) = u^*(z, r) \cdot y(z, r) = u(z) \cdot y(z, r). \quad (10)$$

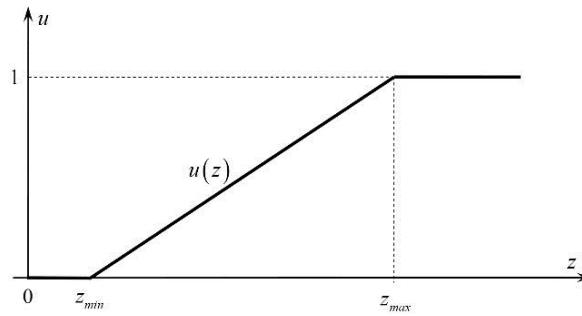


Fig. 3. Preference function graph with workspace  $(z_{\min}, z_{\max})$

In some cases, primary expert data may be available in a weighted interval format [21-23]:

$$[E_{ij}] = \langle e_{ij}, r_{ij} \rangle, \alpha_{ij} = P(E_{ij} \in [E_{ij}]), \quad i = 1, \dots, N, \quad (11)$$

where  $\alpha_{ij}$  – confidence probabilities assigned to the intervals  $\langle e_{ij}, r_{ij} \rangle$ , which are also interpreted as the degree of confidence of the expert in finding the parameter within the corresponding interval [21].

Taking into account the fact that the values of the criterion of the alternatives effectiveness  $[Z_i]$  in this case will be expressed in the form of weighted intervals, criterion (10) can be transformed as follows:

$$\left\{ \begin{array}{l} K(z, r, \alpha) = \alpha \cdot u(z) \cdot y(z, r) \\ u(z) = \begin{cases} 0, & z < z_{\min} \\ \frac{z - z_{\min}}{z_{\max} - z_{\min}}, & z_{\min} \leq z \leq z_{\max} \\ 1, & z > z_{\max} \end{cases} \\ y(z, r) = \begin{cases} 1 - \frac{r/|z|}{(r/|z|)_{\max}}, & 0 \leq r/|z| \leq (r/|z|)_{\max} \\ 0, & r/|z| > (r/|z|)_{\max} \end{cases} \end{array} \right. \quad (12)$$

Thus, an analytical multiplicative multifactorial model of the normalized criterion of preferences of the DMP for interval or weighted interval estimates of alternatives is proposed, which makes it possible to rank them, in particular in the case of their mutual intersection.

#### 4. Example of Ranking Interval and Weighted Interval Estimates of Alternatives

The efficiency matrix with interval elements is presented in Table 4.

Table 4. Example of a decision efficiency matrix with interval elements

	$F_1$	$F_2$	$F_3$	$F_4$
$A_1$	[5, 5.5]	[15, 16.5]	[1, 1.1]	[5, 5.5]
$A_2$	[6, 6.6]	[12, 13.2]	[19, 20.9]	[2, 2.2]
$A_3$	1.5	4	[7, 9]	[2, 3]
$A_4$	[5, 6]	2	6	[3, 5]
$A_5$	[4, 6]	[3, 4]	2.3	7
$A_6$	[10, 11]	[14, 15.4]	[0, 0.5]	[6, 6.6]
$A_7$	[1, 1.1]	[15, 16.5]	[4, 4.4]	[6, 6.6]
$A_8$	[12, 13.2]	[1.9, 2.3]	[5, 5.5]	[16, 17.6]

The scores of the alternatives according to the maximum criterion, as well as the result of their ranking according to the criterion proposed above (10) are presented in Table 5.

Table 5. Results of ranking the interval estimates of alternatives

	$Z$	$z$	$r$	$r/z$	$u(z)$	$y(z, r)$	$K(z, r)$	$rank$
$A_1$	[1, 1.1]	1.05	0.05	0.048	0.457	0.952	0.435	6–7
$A_2$	[2, 2.2]	2.1	0.1	0.048	0.913	0.952	0.870	2–3
$A_3$	1.5	1.5	0	0	0.652	1	0.652	5
$A_4$	2	2	0	0	0.870	1	0.870	2–3
$A_5$	2.3	2.3	0	0	1	1	1	1
$A_6$	[0, 0.5]	0.25	0.25	1	0.109	0	0	8
$A_7$	[1, 1.1]	1.05	0.05	0.048	0.457	0.952	0.435	6–7
$A_8$	[1.9, 2.3]	2.1	0.2	0.095	0.913	0.905	0.826	4
$A_5 \succ A_2 \sim A_4 \succ A_8 \succ A_3 \succ A_1 \sim A_7 \succ A_6$								

Table 6 presents the results of the ranking of alternatives expressed in weighted intervals.

Table 6. Ranking results of weighted interval estimates of alternatives

	$Z$	$\alpha$	$z$	$r$	$r/z$	$u(z)$	$y(z, r)$	$K(z, r, \alpha)$	$rank_\alpha$
$A_1$	[1, 1.1]	0.7	1.05	0.05	0.048	0.457	0.952	0.304	6
$A_2$	[2, 2.2]	0.5	2.1	0.1	0.048	0.913	0.952	0.435	4
$A_3$	1.5	1	1.5	0	0	0.652	1	0.652	3
$A_4$	2	0.8	2	0	0	0.870	1	0.696	2
$A_5$	2.3	0.9	2.3	0	0	1	1	0.9	1
$A_6$	[0, 0.5]	1	0.25	0.25	1	0.109	0	0	8
$A_7$	[1, 1.1]	0.3	1.05	0.05	0.048	0.457	0.952	0.130	7
$A_8$	[1.9, 2.3]	0.4	2.1	0.2	0.095	0.913	0.905	0.330	5
$A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_8 \succ A_1 \succ A_7 \succ A_6$									

## 5. Discussion of the Study Results

As can be seen from Table 5, among the set of interval estimates of alternatives  $[Z_1] - [Z_8]$ , there are groups of estimates that cannot be compared within the paradigm of classical interval analysis, for example,  $[Z_1] \sim [Z_7]$ ,  $[Z_2] \sim [Z_8]$ ,  $[Z_2] \sim [Z_4]$ ,  $[Z_4] \sim [Z_8]$ . The use of criterion (10) makes it possible, if necessary, to reduce the level of uncertainty when ranking alternatives to two pairs of equivalent strategies  $A_2 \sim A_4$  and  $A_1 \sim A_7$ , arranged all indicators of alternatives by the rank.

In the case of weighted estimates (Table 6), ranking by criterion (12) makes it possible to completely remove uncertainty in the interval estimates of alternatives.

Let us identify some important features of model (12).

1. Criteria (10), (12) are analytical and normalized, i.e., it is a specific set  $[Z_i]$  that determines the working areas of the model, limited to the range of values  $[0, 1]$ .
2. Within the workspace, the models are linear, i.e. scaling its arguments  $(z, r, \alpha)$  leads to scaling the criterion itself.
3. The coincidence of interval estimates of alternatives by rank in no way makes the interval estimates "equal". The proposed criterion model allows the DMP to formalize its own preferences regarding the interval uncertainty of the primary expert assessments. This may be required by the high risks of unreasonable decisions and an "optimistic" attitude to uncertainty as such.
4. Initial interval estimates of experts should meet the requirements of the terms of reference for the expertise, which in turn are determined by the properties of the subject matter of the expertise. These requirements should limit the permissible size of intervals, for example, within 20% of the median values, which is usually sufficient for most practical situations. This should prevent the interval model from degenerating into an analytical model with full parametric uncertainty [21–24].
5. The proposed model can be included in the methodological support of automated systems for processing expert information (fig. 4).

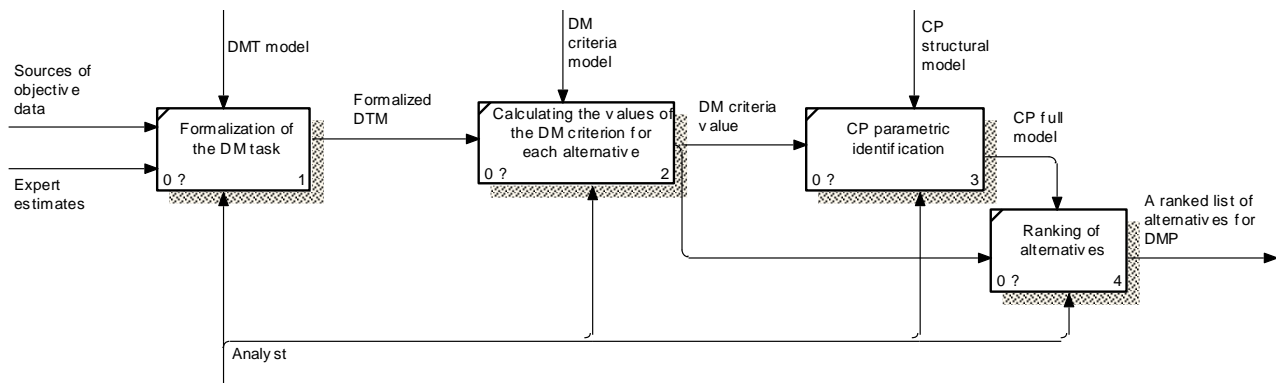


Fig. 4. Functional model of the decision support subsystem: DM – decision-making; DMT – decision-making task; DMP – decision-maker; CP – preference criterion

The advantages of the developed model include the following:

1. The proposed model of criterion (12) makes it possible to process interval, weighted interval and point estimates. Thus, it is invariant to the type of input data.
2. The proposed analytical form of criterion (12) does not require parametric adjustment by the analyst or the DMP. Normalization of the criterion is based only on the properties of the set of interval estimates of alternatives and uniquely determines the parameters of the criterion model.
3. The software implementation of algorithms that reproduce the proposed model is not difficult and can be done even in a spreadsheet editor. The results of the ranking can be included in graph-analytical models that accompany the decision-making process and provide visibility of the results of the examination.

The practical significance of the developed model lies in obtaining an analytical apparatus for ranking interval estimates of alternatives for situations when the DMP is forced to make decisions on alternatives whose estimates overlap. The peculiarity of this study is the possibility of taking into account additional expert information expressed in the form of weights assigned to interval estimates.

The main limitation of the proposed model is the rigid choice of linear workspaces in the function of preferences and "gains" from narrowing the interval uncertainty of expert opinions. This is due to the hypothesis of linear perception of uncertainty by the DMP as a whole, although the real picture may be much more complicated. In any case, the proposed analytical multiplicative multifactorial model of the normalized criterion of DMP preferences for interval or weighted interval estimates of alternatives should be used as an auxiliary tool for forcing uncertainty removal in specific practical situations. It should be noted that an alternative in this process is the forced narrowing of the initial expert interval estimates, which may not correspond to the nature and properties of the subject matter of the expert evaluation.



## 6. Conclusion

1. A multifactorial model of the normalized criterion of preferences for interval estimates of alternatives is proposed. Due to the additional consideration of the degree of preference of the DMP for the width of interval estimates, it allows to rank interval estimates of alternatives that overlap and are considered classically incomparable. A single analytical form of the normalized criterion model for ranking interval, weighted interval and point estimates makes it possible to increase the degree of automation of processing interval expert estimates in the decision-making process.

2. Recommendations for the practical application of the proposed model are formulated. The developed model and corresponding algorithms can be used in automated expert decision support systems. An example illustrates the use of the proposed model for ranking interval and weighted interval evaluations of alternatives. In this case, it can be seen that the DMP has additional normalized numerical estimates for alternatives that were considered incomparable at the stage of preliminary analysis. This made it possible to reduce the number of non-comparable alternatives (by 50% in the case of interval estimates and by 100% in the case of weighted interval estimates).

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**How to cite this paper:** Yuri Romanenkov, Vadym Mukhin, Viktor Kosenko, Daniil Revenko, Olena Lobach, Natalia Kosenko, Alla Yakovleva, "Criterion for Ranking Interval Alternatives in a Decision-Making Task", International Journal of Modern Education and Computer Science(IJMECS), Vol.16, No.2, pp. 72-82, 2024. DOI:10.5815/ijmecs.2024.02.06