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Mathematics and Software for Coordinated Planning Using Aggregated Linear Volume-time Models of Discrete Manufacturing Systems

Alexander Pavlov*
National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Kyiv 03056, Ukraine
E-mail: pavlov.fiot@gmail.com
ORCID iD: https://orcid.org/0000-0002-6524-6410
*Corresponding author

Kateryna Lishchuk
National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Kyiv 03056, Ukraine
E-mail: lishchuk_kpi@ukr.net
ORCID iD: https://orcid.org/0000-0002-9902-0065

Oleg Melnikov
National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Kyiv 03056, Ukraine
E-mail: oleg.v.melnikov72@gmail.com
ORCID iD: https://orcid.org/0000-0002-2651-2739

Mykyta Kyselov
National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Kyiv 03056, Ukraine
E-mail: zeusmobilenick@gmail.com
ORCID iD: https://orcid.org/0009-0005-3686-3419

Cennuo Hu
Department of Computer Science, College of Science, Purdue University, West Lafayette, IN 47907, USA
E-mail: hu945@purdue.edu
ORCID iD: https://orcid.org/0009-0008-9589-9253

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Abstract: The problems of managing modern complex organizational and manufacturing systems, such as international production corporations, regional economies, sectoral ministries, etc., in conditions of fierce competition are primarily related to the need to consider the activity of organizational and manufacturing objects that make up a multi-level manufacturing system, that is, the ability to efficiently solve the problem of coordinating interests. This problem cannot be solved efficiently without the use of modern scientific achievements and appropriate software. As an example, we can cite the active systems theory pioneered by Prof. V. M. Burkov and his students, which successfully claims to be a constructive implementation of the idea of coordinated planning. This paper proposes new models and methods of coordinated planning of two-level organizational and manufacturing systems. Our models and methods use original compromise criteria and the corresponding constructive algorithms. The original aggregated volume-time models are used as models of organizational and manufacturing objects. We present a well-founded software structure for the proposed methods of coordinated planning. It contains an intelligent interface for using the presented results in solving applied problems.

Index Terms: Coordinated Planning, Multi-objective Linear Programming, Compromise Criterion, Aggregated Linear Volume-time Model of Discrete Manufacture.

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1. Introduction

1.1. Related Works Review Substantiating the Theoretical and Practical Relevance of the Results that we Present

As of today, models and methods of active systems theory are used the most widely in implementing the strategy of coordinated planning of complex organizational and manufacturing systems. Examples of the successful use of this theory are given in [1–12]. However, a certain drawback of this theory is the fact that the compromise between the efficiency of a complex organizational system functioning and the interests of its elements is achieved based on given analytical expressions for the elements' functions of penalties for non-fulfillment of plans. The compromise, even for the simplest models of organizational and manufacturing objects (OMOs, hereinafter referred to as elements), only in some cases reduces to solving standard constructive problems of mathematical programming.

Despite the current significant achievements in active system theory, existing approaches, particularly those described in the works of Belov and Novikov on network active systems [13], often encounter difficulties in direct application to complex discrete manufacturing systems because they require rather complex mechanisms for stimulating and coordinating plans in conditions of network interaction. Therefore, the creation of constructive models and methods of coordinated planning based on the paradigms of active systems theory is a relevant problem both in theoretical and practical terms. The approach proposed in this article based on aggregated linear volume-time models (ALVTMs) and specially developed compromise criteria offers a more constructive solution. It allows not only to avoid the explicit analytical specification of penalty functions in the coordinating algorithms (which is a weak point of many classical models of active systems), but also to directly consider both the volume and time aspects of planning reducing the problem to efficiently solved linear programming problems (LPPs). This provides better adaptability to the realities of discrete manufacturing and increases the practical value of coordinated planning methods. In this work, the solution to the problem of dependence of the elements' functioning is entrusted to the center by distributing common manufacturing resources between the elements of the organizational and manufacturing system and introducing appropriate additional linear constraints in the ALVTMs, which implement the necessary dependence between the parameters of the components of aggregated product vectors for the system elements.

1.2. The General Methodology that Formed the Basis of the Proposed Methods

The general methodology of the obtained results is as follows. As in active system theory, we consider a two-level organizational and manufacturing system (TLOMS) consisting of a center for planning the work of elements (first level) and elements managed by the center (second level). If they function independently, the coordinated planning problem is automatically decomposed into separate subproblems of finding coordinated plans for each element. The quality functionals of the center and elements are given as linear. In an explicit form, the functions of penalties for non-fulfillment of plans are not introduced in the coordinated planning algorithm. It is proposed to take penalties for non-fulfillment of coordinated plans out of the scope of models and methods, focusing on such constructive algorithms for developing the center's plans for the elements, which in an explicit form allow direct considering the interests of elements by a straight or iterative procedure. We chose, as element models, the original linear models, which, unlike the known ones, simultaneously consider aggregated volume and time constraints [14]. This allows the use of linear optimization models, which are LPPs. As shown in [15], LPPs constructively solve high-dimensional problems.

The compromise criteria and methods presented in [10] were chosen as the methodological basis of the constructive algorithms for coordinated planning.

1.3. The Structure of the Paper

In Section 2, we present an aggregated linear volume-time model (ALVTM) of OMOs (two-level system elements). Section 3 presents the original constructive algorithms of coordinated planning for independently functioning elements (OMOs). In Section 4, we substantiate the originality of ALVTM and indicate how it differs from existing aggregated models of manufacturing. We give a classification of the proposed compromise criteria for the coordinated planning algorithms and formulate the complexity of implementing the proposed original algorithms of coordinated planning at a qualitative level. Section 5 presents well-founded software architecture for the proposed methods of coordinated planning. Section 6 contains the conclusions of the work.

2. The Model of a Two-level Organizational and Manufacturing System (TLOMS)

2.1. The Aggregated Linear Volume-time Model (ALVTM)

This material is taken from [14]. Unlike the set of models from [14], it is the only ALVTM that contains all the properties of these models. The ALVTM consists of two parts. The first one specifies linear volumetric aggregated constraints

$$A^l y^l \le b^l,\ y^l_j \ge \bar{0},\ j=\overline{1,n^l},\ l=\overline{1,k} \tag{1}$$

where $y^l = \left(y_1^l,...,y_{n^l}^l\right)^{\mathrm{T}}$; y_j^l are aggregated components of the production plan for the *l*th OMO, an element of TLOMS in terms of active system theory. The positive components of the vector $b^l = \left(b_1^l,...,b_{m^l}^l\right)^{\mathrm{T}}$ are resource constraints. The components of the matrix A^l (of the dimensions $m^l \times n^l$) are resource costs per unit of the corresponding component of the aggregated plan.

The second part of the model sets aggregated linear constraints on time parameters. We will use two linear models of the completion time of the *j*th aggregated product in the quantity y_j^l, $j = \overline{1, n_1^l}$, $n_1^l \le n^l$. The first linear model:

$$T_i^l = t_{0,i}^l + \alpha_i^l y_i^l,\ i = \overline{1, n_1^l},\ n_1^l \le n^l \tag{2}$$

where $\alpha_i^l > 0$ are expert coefficients, $t_{0,i}$ is the start time of production of the *i*th aggregated product. Formula (2) is used when aggregated products are produced independently of each other.

The second linear model corresponds to the case when there is equipment on which operations of the same type are performed for each aggregated product. In this case, the following restrictions are imposed on the start times of production of the aggregated products:

$$t_{0,i_j}^l \ge t_{0,i_1}^l + \sum_{t=1}^{j-1} \hat{\alpha}_{i_t}^l y_{i_t}^l,\ j = \overline{2, n_1^l},\ n_1^l \le n^l \tag{3}$$

where $i_1,...,i_{n_1^l}$ is the specified order of operations execution for aggregated products on common equipment; $\forall \hat{\alpha}_{i_j} > 0$ are expert coefficients.

Remark 1. $t_{0,i}^l$ may be unknowns of the corresponding LPP, then the following inequalities are added to constraints (2), (3):

$$t_{0,i}^l \ge 0,\ i = \overline{1, n_1^l},\ n_1^l \le n^l. \tag{4}$$

Remark 2. If the processing sequence of aggregated products can be arbitrary, then it is given by non-increasing priorities

$$\frac{\alpha_j^l y_j^{\text{given},l}}{D_i^l},\ j = \overline{1, n_1^l},\ n_1^l \le n^l \tag{5}$$

where D_i^l, $j = \overline{1, n_1^l}$, are deadlines for the aggregated products in a given minimum possible quantity $y_j^{\text{given},l}$, $j = \overline{1, n_1}$. These requirements are given by the constraints

$$y_j^l \ge y_j^{\text{given},l},\ j = \overline{1, n_1^l},\ n_1^l \le n^l. \tag{6}$$

The element's quality functional has the form

$$\max\left[\left(c^l\right)^{\mathrm{T}} y^l - \sum_{j=1}^{n_1^l} f_j^l z_j^l\right]. \tag{7}$$

where c^l, f^l are positive vectors;

$$z_j^l = \max\left(0, T_j^l - D_j^l\right),\ j = \overline{1, n_2^l},\ n_2^l \le n_1^l; \tag{8}$$

$$z_j^l = \left|D_j^l - T_j^l\right|,\ j = \overline{n_2^l + 1, n_1^l},\ n_1^l \le n^l. \tag{9}$$

Remark 3. The conditions of Remark 2 apply only to penalties for violation of deadlines (8). If the model simultaneously contains penalties (8), (9), then in the absence of technological restrictions, it is only possible to propose an enumeration of all cases of processing sequences of aggregated products. For each of the sequences the compromise

criteria and models presented below are implemented.

Remark 4. The proposed models use linear functionals of penalty for deadlines violation. They are efficient when the deadlines violation stays within acceptable limits. If the planning result does not satisfy these constraints, the following additional constraints can be added to ALVTM: $z_j^l \le Z_{1,j}^l$ if the penalty is given by formula (8) or $z_j^l \le Z_{2,j}^l$ if the penalty is given by formula (9) where $Z_{1,j}^l, Z_{2,j}^l$ are expert constraints.

Thus, the proposed ALVTM, unlike other linear aggregated manufacturing models, aggregates not only the volume indicators of manufacturing, but also the time characteristics of order fulfillment. This makes the model more adequate for real manufacturing. Also, unlike other coordinated planning algorithms, ALVTM allows (remaining in the class of linear models) to explicitly consider in the optimality criteria of the center and elements both the profit from the manufactured products and the penalties for violation of the deadlines agreed in advance. The adequacy of ALVTM for a real manufacturing system depends both on the choice of the aggregation method for the manufactured products and on the accuracy of the empirical coefficients that specify the time parameters of the aggregated products vector and, thus, depend on the professionalism of specialists who create mathematical models for each specific manufacturing (element).

2.2. A Model of the Center (the First Level of TLOMS)

The model is given by the plans

$$y^l,\ l=\overline{1,k}, \tag{10}$$

which the center sets to the elements for processing. The TLOMS quality functional has the form

$$\max \sum_{l=1}^{k}\left[\left(d^l\right)^{\mathrm{T}} y^l - \sum_{j=1}^{n_1^l} f_j^l z_j^l\right] \tag{11}$$

where d^l is a positive vector; $\forall f_j^l > 0$ are the given coefficients that define the linear function of penalties for violation of deadlines.

3. Compromise Criteria and Corresponding Algorithms for Coordinated Planning of the Center for a TLOMS with Independent OMOs (Elements)

Let us consider the algorithmic procedure for assigning a compromise plan by the center to the lth element ($l=\overline{1,k}$) in accordance with eight compromise criteria proposed by the authors.

The first compromise criterion. The center assigns to the lth element a plan y_1^l that satisfies the following conditions:

$$y_1^l = \arg\max_{y^l} \sum_{l=1}^{k}\left[\left(d^l\right)^{\mathrm{T}} y^l - \sum_{j=1}^{n_2^l} f_j^l z_j^{l,+} - \sum_{j=n_2^l+1}^{n_1^l} f_j^l\left(z_j^{l,+}+z_j^{l,-}\right)\right]; \tag{12}$$

$$A^l y^l \le b^l,\ y_j^l \ge y_j^{\text{given},l},\ j=\overline{1,n_1^l},\ \forall y_j^l \ge 0; \tag{13}$$

$$T_i^l = t_{0,i}^l + \alpha_i^l y_i^l,\ i=\overline{1,n_1^l}; \tag{14}$$

$$t_{0,i_j}^l = t_{0,i_1}^l + \sum_{t=1}^{j-1} \hat{\alpha}_{i_t}^l y_{i_t}^l,\ j=\overline{2,n_1^l},\ n_1^l \le n^l; \tag{15}$$

$$T_j^l - D_j^l = z_j^{l,+} - z_j^{l,-},\ z_j^{l,+} \ge 0,\ z_j^{l,-} \ge 0,\ j=\overline{1,n_1^l}; \tag{16}$$

$$y_1^l = \arg\max_{\{y_1^l\}} \sum_{l=1}^{k}\left[\left(c^l\right)^{\mathrm{T}} y^l - \sum_{j=1}^{n_2^l} f_j^l z_j^{l,+} - \sum_{j=n_2^l+1}^{n_1^l} f_j^l\left(z_j^{l,+}+z_j^{l,-}\right)\right]. \tag{17}$$

Variables of the combinatorial optimization problem (12)–(17) are the non-negative variables y_j^l, $z_j^{l,+}$, $z_j^{l,-}$, and for any fixed l, j we have $z_j^{l,+} \cdot z_j^{l,-} = 0$.

Remark 5. If aggregated products can be processed independently of each other, then all coefficients $\hat{\alpha}_j^l = 0$.

Remark 6. If the parameter $n_2^l = 0$, then this means that there are no penalties (8). If $n_2^l = n_1^l$, then there are no penalties (9).

$\left\{y_1^l\right\}$ is the set of solutions of the following LPP:

$$f_{1,\text{opt}}^l = \max_{y^l} \sum_{l=1}^{k} \left[\left(d^l\right)^{\mathrm{T}} y^l - \sum_{j=1}^{n_2^l} f_j^l z_j^{l,+} - \sum_{j=n_2^l+1}^{n_1^l} f_j^l \left(z_j^{l,+} + z_j^{l,-}\right) \right] \tag{18}$$

with linear constraints (13)–(16). y_j^l, $j = \overline{1, n_1^l}$; $z_j^{l,+}$, $z_j^{l,-}$, $j = \overline{1, n_1^l}$ are variables of the LPP (12)–(16), $t_j^l = 0$ for the case of independent processing of aggregated products y_j, $j = \overline{1, n_1^l}$, otherwise only $t_{0,i_1}^l = 0$.

Then the plan for the lth element of the TLOMS according to the first compromise criterion is the solution of the following LPP:

$$y_{1,\text{comp}}^l = \arg\max_{y^l} \sum_{l=1}^{k} \left[\left(c^l\right)^{\mathrm{T}} y^l - \sum_{j=1}^{n_2^l} f_j^l z_j^{l,+} - \sum_{j=n_2^l+1}^{n_1^l} f_j^l \left(z_j^{l,+} + z_j^{l,-}\right) \right] \tag{19}$$

with linear constraints (13)–(16) and additional constraint

$$\left(d^l\right)^{\mathrm{T}} y^l - \sum_{j=1}^{n_2^l} f_j^l z_j^{l,+} - \sum_{j=n_2^l+1}^{n_1^l} f_j^l \left(z_j^{l,+} + z_j^{l,-}\right) = f_{1,\text{opt}}^l . \tag{20}$$

Remark 7. LPP (12)–(16) and (19), (13)–(16) and all LPPs given below can be solved only by the simplex method. In this case, for any j both $z_j^{l,+}$ and $z_j^{l,-}$ cannot be greater than zero at the same time.

Remark 8. Coefficients of the penalties $f_j^l > 0$, $j = \overline{n_2^l + 1, n_1^l}$, can be replaced by $f_j^{l,+} > 0$, $f_j^{l,-} > 0$, and the corresponding penalty component in all LPPs can have the form

$$-\sum_{j=1}^{n_2^l} f_j^l z_j^{l,+} - \sum_{j=n_2^l+1}^{n_1^l} \left(f_j^{l,+} z_j^{l,+} + f_j^{l,-} z_j^{l,-}\right). \tag{21}$$

The second compromise criterion. Let $f_{2,\text{opt}}^l$ be the optimal value of the LPP functional (19), (13)–(16). Then $y_{2,\text{comp}}^l$ is the solution of the following LPP:

$$y_{2,\text{comp}}^l = \arg\max_{y^l} \sum_{l=1}^{k} \left[\left(d^l\right)^{\mathrm{T}} y^l - \sum_{j=1}^{n_2^l} f_j^l z_j^{l,+} - \sum_{j=n_2^l+1}^{n_1^l} f_j^l \left(z_j^{l,+} + z_j^{l,-}\right) \right] \tag{22}$$

with linear constraints (13)–(16) and additional constraint

$$\left(c^l\right)^{\mathrm{T}} y^l - \sum_{j=1}^{n_2^l} f_j^l z_j^{l,+} - \sum_{j=n_2^l+1}^{n_1^l} f_j^l \left(z_j^{l,+} + z_j^{l,-}\right) \geq f_{2,\text{opt}}^l - \Delta^l \tag{23}$$

where $\Delta^l > 0$ is set by the center as a percentage of the value of $f_{2,\text{opt}}^l$, e.g., $\Delta^l = 2\%$ of $f_{2,\text{opt}}^l$.

The third compromise criterion is used by the center when, for a given Δ^l (see the second compromise criterion), the optimal functional value of the LPP (22), (13)–(16), (23) does not satisfy the center. In this case, the LPP (22), (13)–(16), (23) is solved sequentially for the given $\Delta_1^l < \Delta_2^l < \Delta_3^l < \ldots$. $y_{3,\text{comp}}^l$ is a solution of the LPP, for which the

compromise between the values of $f^l_{1,\text{opt}} - \left(d^l\right)^{\mathrm{T}} y^l_{2,\text{comp}} + \sum\limits_{j=1}^{n^l_2} f^l_j z^{l,+}_j \left(y^l_{2,\text{comp}}\right) + \sum\limits_{j=n^l_2+1}^{n^l_1} f^l_j \left[z^{l,+}_j \left(y^l_{2,\text{comp}}\right) + z^{l,-}_j \left(y^l_{2,\text{comp}}\right)\right]$ and the corresponding current value of Δ^l_i, $i \geq 2$, is acceptable for the center.

The fourth compromise criterion. All compromise criteria, starting from the fourth one, are used for the case when $y^l = y^l_1 + y^l_2$ where y^l_1 belongs to the TLOMS (the center), y^l_2 is a private property of the *l*th element ($y^l_1 \geq \bar{0}$, $y^l_2 \geq \bar{0}$). $y^l_{4,\text{comp}}$ meets the fourth compromise criterion if the following conditions are met. Let

$$f^l_{3,\text{opt}} = \max_{y^l} \left[\left(d^l\right)^{\mathrm{T}} y^l_1 - \sum_{j=1}^{n^l_2} f^l_j z^{l,+}_j - \sum_{j=n^l_2+1}^{n^l_1} f^l_j \left(z^{l,+}_j + z^{l,-}_j\right)\right]; \tag{24}$$

$$A^l \left(y^l_1 + y^l_2\right) \leq b^l,\ y^l_{2,j} \geq y^{l,\min}_{2,j} \geq 0,\ j = \overline{1,n^l},\ y^l_{1,j} \geq y^{\text{given},l}_{1,j},\ j = \overline{1,n^l_1}; \tag{25}$$

$$T^l_i = t^l_{0,i} + \hat{\alpha}^l_i y^l_{1,i},\ i = \overline{1,n^l_1}; \tag{26}$$

$$t^l_{0,i_j} = t^l_{0,i_1} = t_{0,i_1} + \sum_{t=1}^{j-1} \hat{\alpha}^l_{i_t} y^l_{1,i_t},\ j = \overline{2,n^l_1},\ n^l_1 \leq n^l, \tag{27}$$

and the linear constraint (16).

Remark 9. It is obvious that the vector y^l_2 in the LPP (24)–(27), (16) must be replaced by the vector $y^{l,\min}_2 = \text{const}$. Then the LPP (24)–(27), (16) is simplified.

Then the compromise plan $y^l_{4,\text{comp}}$ for the *l*th element is the solution of the following LPP:

$$y^l_{4,\text{comp}} = \left(y^l_1 + y^l_2\right) = \arg\max_{y^l_1} \sum_{l=1}^{k} \left[\left(d^l\right)^{\mathrm{T}} y^l_1 - \sum_{j=1}^{n^l_2} f^l_j z^{l,+}_j - \sum_{j=n^l_2+1}^{n^l_1} f^l_j \left(z^{l,+}_j + z^{l,-}_j\right)\right] \tag{28}$$

with linear constraints (25)–(27), (16) and additional constraint

$$\left(c^l\right)^{\mathrm{T}} y^l_2 \geq f^l_{4,\text{opt}} - \Delta^l \tag{29}$$

where $f^l_{4,\text{opt}} = \max\left(c^l\right)^{\mathrm{T}} y^l_2$ at linear constraints (25)–(27), (16), which include instead of the components of the vector y^l_1 the components of the vector

$$y^{\text{given},l} = \left(y^{\text{given},l}_{1,1}, \ldots, y^{\text{given},l}_{1,n^l_1}, 0, \ldots, 0\right)^{\mathrm{T}} \tag{30}$$

and the inequality $y^l_{1,j} \geq y^{\text{given},l}_{1,j}$, $j = \overline{1,n^l_1}$, is excluded. The variables of the LPP (28), (25)–(27), (16), (29) are the components of the vectors $y^l_{1,j}$, $y^l_{2,j}$, and $z^{l,+}_j$, $z^{l,-}_j$. The constant $\Delta^l > 0$ is set by the center as a percentage of the value of $f^l_{4,\text{opt}}$. If, for a given Δ^l, the difference

$$f^l_{3,\text{opt}} - \left[\left(d^l\right)^{\mathrm{T}} y^l_1 - \sum_{j=1}^{n^l_2} f^l_j z^{l,+}_j - \sum_{j=n^l_2+1}^{n^l_1} f^l_j \left(z^{l,+}_j + z^{l,-}_j\right)\right] \tag{31}$$

where y^l_1 is the optimal solution of the LPP (28), (25)–(27), (16), (29), does not satisfy the center, then we implement the fifth compromise criterion, which is a complete analogy of the third compromise criterion, that is, the LPP (28), (25)–(27), (16), (29) is sequentially solved for $\Delta^l_1 < \Delta^l_2 < \ldots$.

The sixth compromise criterion. Let

$$f_{5,\text{opt}}^{l} = \max_{y_1^l} \left(d^{1,l} \right)^{\mathrm{T}} y_1^l ; \tag{32}$$

$$A^l \left(y^{\text{given},l} + y_1^l + y_2^l \right) \le b^l , \ \forall y_{1,j}^l \ge 0, \ j = \overline{1, n^l} \tag{33}$$

where $d^{1,l}$ is a n^l-dimensional vector with positive components.

Then $y_{6,\text{comp}}^l$ satisfies the sixth compromise criterion if $y_{6,\text{comp}}^l$ is a solution of the following LPP:

$$y_{6,\text{comp}}^l = \arg\max_{y_2^l} \left(c^l \right)^{\mathrm{T}} y_2^l ; \tag{34}$$

$$A^l \left(y^{\text{given},l} + y_1^l + y_2^l \right) \le b^l , \ \forall y_{1,j}^l, y_{2,j}^l \ge 0, \ j = \overline{1, n^l} ; \tag{35}$$

$$y_2^l \ge y_2^{l,\min} , \ j = \overline{1, n^l} , \tag{36}$$

with the additional constraint

$$\left(d^{1,l} \right)^{\mathrm{T}} y_1^l = f_{5,\text{opt}}^l . \tag{37}$$

The seventh compromise criterion $y_{7,\text{comp}}^l$**.** Let $f_{6,\text{opt}}^l$ is the optimal functional value of the following LPP:

$$f_{6,\text{opt}}^l = \max \left(c^l \right)^{\mathrm{T}} y_2^l ; \tag{38}$$

$$A^l \left(y^{\text{given},l} + y_2^l \right) \le b^l , \ \forall y_{2,j}^l \ge 0 . \tag{39}$$

Then y_1^l, y_2^l satisfy the seventh compromise criterion if it is a solution to the following LPP:

$$\max_{y_1^l} \left(d^{1,l} \right)^{\mathrm{T}} y_1^l ; \tag{40}$$

$$A^l \left(y^{\text{given},l} + y_1^l + y_2^l \right) \le b^l , \ \forall y_{1,j}^l, y_{2,j}^l \ge 0, \ j = \overline{1, n^l} , \tag{41}$$

with the additional constraint

$$\left(c^l \right)^{\mathrm{T}} y_2^l \ge f_{6,\text{opt}}^l - \Delta^l \tag{42}$$

where $\Delta^l > 0$ is set by the center as a percentage of the value of $f_{6,\text{opt}}^l$.

If the difference $f_{5,\text{opt}}^l - \left(d^{1,l} \right)^{\mathrm{T}} y_1^l$ does not satisfy the center, where y_1^l is the optimal solution of the LPP (40)–(42), then we introduce the *eighth compromise criterion* when the LPP (40)–(42) is successively solved for $\Delta_1^l < \Delta_2^l < \ldots$ until a compromise solution is found that satisfies the center.

Remark 10. All components of the vector $y^{\text{given},l}$, starting from $n_1^l + 1$, are zero. The profit of the center for the processing of $y^{\text{given},l}$ taking into account penalties for deadlines violation is fixed and is unambiguously given by the sequence $i_1, \ldots, i_{n_1}$ of processing the aggregated products $y_{i_1}^{\text{given},l}, \ldots, y_{i_{n_1}}^{\text{given},l}$. Therefore, this profit and restrictions (14)–(16) are absent in the models (32)–(42). Time restrictions are not imposed on the processing of y_1^l, y_2^l.

4. Characteristics of the Given Compromise Criteria and the Complexity of the Proposed Algorithms for Their Satisfaction

4.1. The Meaning of the Compromise Criteria

We presented the compromise criteria for two types of ALVTM:

- The first type, when manufacturing of a vector of aggregated products by an element belongs to a two-level system.
- The second one, when this vector is the sum of two vectors: a vector belonging to a two-level system and a vector that is a private property of the element, the characteristics of the latter are the object of compromise between the center and the element.

The proposed compromise criteria are conditionally divided into three groups according to their qualitative sense.

- The first group includes the first and sixth compromise criteria. They build a plan for each element that is optimal for the center, but among all such plans, the one that is most beneficial for the element is selected.
- The second group includes the second, fourth, and seventh compromise criteria. Each of them builds a compromise solution that is conditionally optimal for the center, i.e., optimal under the condition that the value of the element's functional differs from the optimal one by no more than the value specified by the center.
- The third group includes the third, fifth, and eighth compromise criteria, each of which implements an iterative procedure, where each iteration is an implementation of the corresponding criterion from the second group with a different numerical parameter of the compromise specified by the center. From the generated set of plans for the element, the center selects the one that corresponds to its conception of the acceptable losses of the system and the element.

4.2. Acceptability of Using Linear Penalties for Deadlines Violation

Linear penalties are efficient when the deadlines violation in the planning results is within the acceptable range. Therefore, the restrictions given in Remark 4 (Subsection 2.1) can be added to all proposed linear models. The introduction of additional restrictions is logical because a significant violation of the deadlines cannot be acceptable when we require efficient and stable functioning of two-level organizational and manufacturing systems.

4.3. The Complexity of the Compromise Criteria

The complexity of each algorithm consists of the complexity of solving each LPP by the simplex method for the criteria of the first and second groups. For the criteria of the third group, the complexity of the algorithm also depends on the number of its iterations that completely depends on the center. The real complexity of the simplex method is acceptable because it allows you to obtain optimal solutions for problems with thousands and tens of thousands of variables.

5. Software Architecture

This section is devoted to the analysis of the proposed software architecture for coordinated planning of the TLOMS.

The library is implemented as a monolith. This simplifies its deployment and use and reduces the number of external dependencies. This approach was chosen because the main goal is to create a single tool for solving coordinated planning problems rather than a set of independent modules. Monolithic architecture allows for closer integration of system components, which improves performance and usability.

The library is cross-platform due to the use of the Python programming language and the OR-Tools library. This provides support for work on various operating systems, such as Windows, macOS, and Linux. Users can use the library on any computer without the need to install additional software or change the code.

The architecture consists of several macro components: *tlops*, *models*, *solvers*, *utils*. As shown in Fig. 1, *tlops* is the entry point to the library and is responsible for coordinating the work of other components. *models* encapsulates the logic of presenting data about the system, in particular, data structures for production elements and the control center. *solvers* contains implementations of optimization algorithms for both individual elements and the control center. *utils* provides auxiliary functions and utilities necessary for the work of other components.

This approach is optimal because it provides modularity and separation of concerns. This simplifies the development, testing, and support of the software. Each component performs a clearly defined function, which allows you to change one component without affecting the others. This also facilitates testing, since the components can be tested independently. The approach allows you to expand the functionality by adding new or modifying existing components. The relationships between the diagrams reflect the data flow and dependencies between different parts of the system, from data presentation to their processing and obtaining results.

The functional interaction of the key software components that provide the coordinated planning process is organized as follows. The coordinated planning process can be initialized both through the developed intelligent interface (described in detail in subsection 5.6) and programmatically, by directly calling the library functions, this provides flexibility in integration and automatization. In the second case, the system configuration parameters (number of elements, their structure, selected criterion) are passed through programmatically generated objects. The *models* component is used to structure these parameters. Similarly, detailed numerical data (functional coefficients, resource constraints, planned indicators, penalties) are provided programmatically, forming the corresponding model objects (*CenterData* and structures for elements) using the *models* component.

After complete data preparation, the optimization process begins by transferring the formed data model objects to the *solvers* component. The *solvers* component contains implementations of optimization algorithms. Based on the selected criterion, the corresponding solver for the central level is activated, which coordinates the overall coordination process. This central solver can sequentially or in parallel call solvers for individual elements in order to find their local optimal plans or reactions to the control actions from the center. Each solver at the corresponding level of the hierarchy (center or element) performs standard stages of mathematical programming: determining optimization variables, forming a system of mathematical constraints and constructing an objective function. These operations are done based on data provided by the *models* component and can involve auxiliary functions from the *utils* component for tasks such as validating input data, performing specific mathematical calculations or formatting intermediate results. The final results of the optimization process, including agreed manufacturing plans, functional values, and calculated compromise indicators obtained by the *solvers* component, are passed on for further program analysis or saving.

5.1. Description of the Component Diagram "General Architecture of the TLOMS Library's Top Level"

Fig. 1 shows the general architecture of the top level of the TLOMS library.

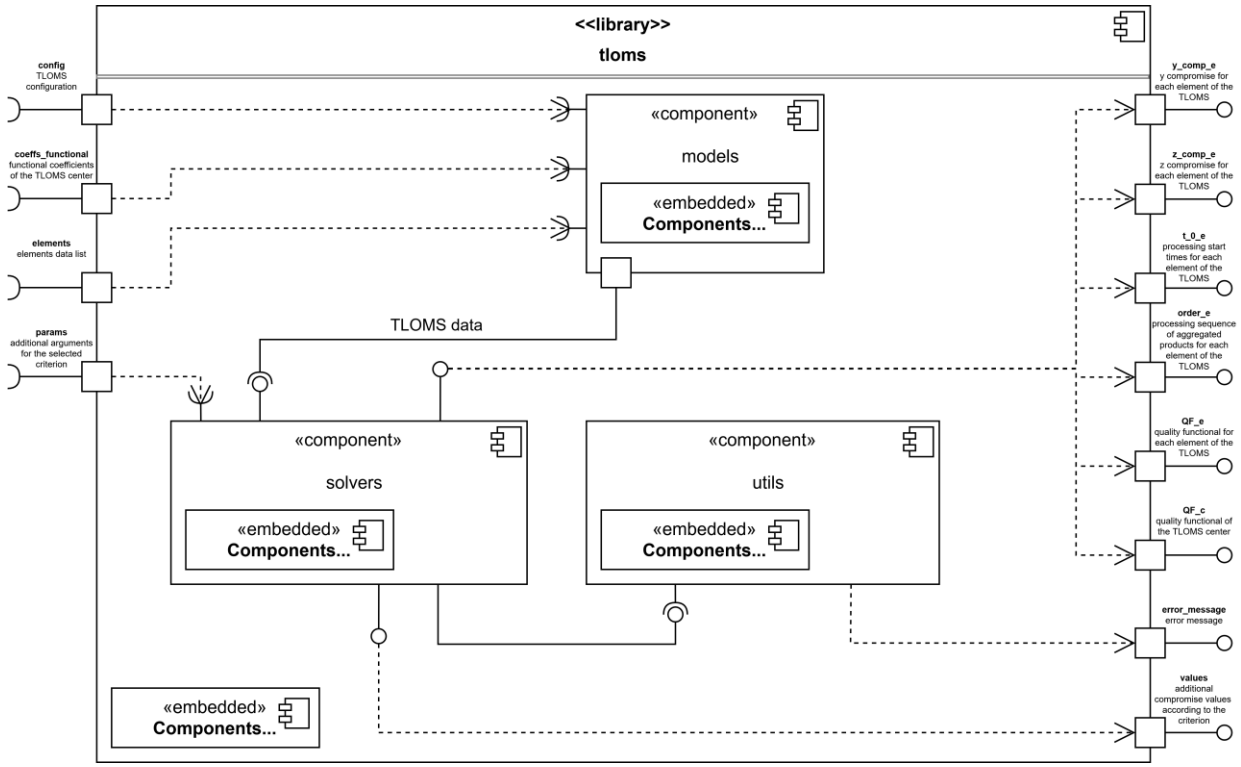


Fig.1. The general architecture of the TLOMS library's top level

- *models*: the component contains the logic for building TLOMS models (Sect. 2). Creates model objects (Subsections 2.1, 2.2) necessary for solving optimization problems in accordance with the theoretical provisions of this article.
- *solvers*: the component implements algorithms for solving optimization problems, takes model data and finds optimal solutions that meet the compromise criteria from the article, as described in the Subsection 5.3.
- *utils*: the component contains auxiliary functions. Provides support for other components, which makes the architecture more flexible.

5.2. Description of the Component Diagram "Detailed Architecture of the TLOMS Model"

Fig. 2 shows the detailed architecture of the TLOMS model.

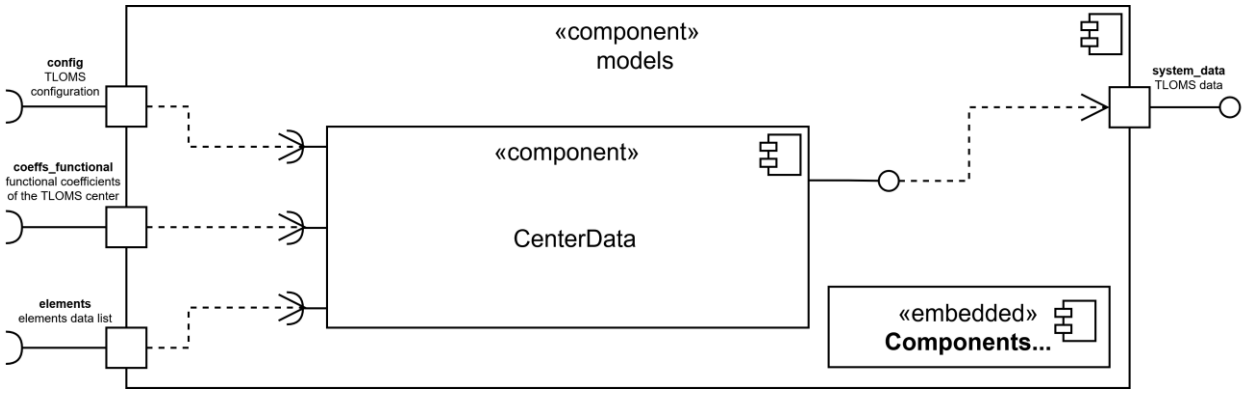


Fig.2. The detailed architecture of the TLOMS model

- *CenterData*: the component stores and aggregates data of the TLOMS center: configuration (number of elements, variables, products, constraints), functional coefficients of the center and parameters of the elements (functional coefficients, resource constraints, processing times, penalties for deadlines violation). Provides data transfer between the elements and the center.

5.3. Description of the Component Diagram "Detailed Architecture of the TLOMS Solver"

Fig. 3 shows the detailed architecture of the TLOMS solver.

- *element*: the component implements a solver for a separate element of the TLOMS to optimize taking into account local constraints and objective functions (13)–(16).
- *center*: the component implements a solver for the TLOMS center coordinating the interaction between elements. It provides planning that meets the compromise criteria: for the third criterion, if for a given Δ^l the optimal value of the functional (22), (13)–(16), (23) does not satisfy the center, the problem (12)–(16), (23) is solved sequentially for $\Delta_1^l < \Delta_2^l < \Delta_3^l < \ldots$. The *center* component changes Δ^l passing it to the *element* component, which solves the optimization problem for its element and returns the updated value of the quality functional. *center* analyzes the results and continues the process until a solution is found that meets all requirements.

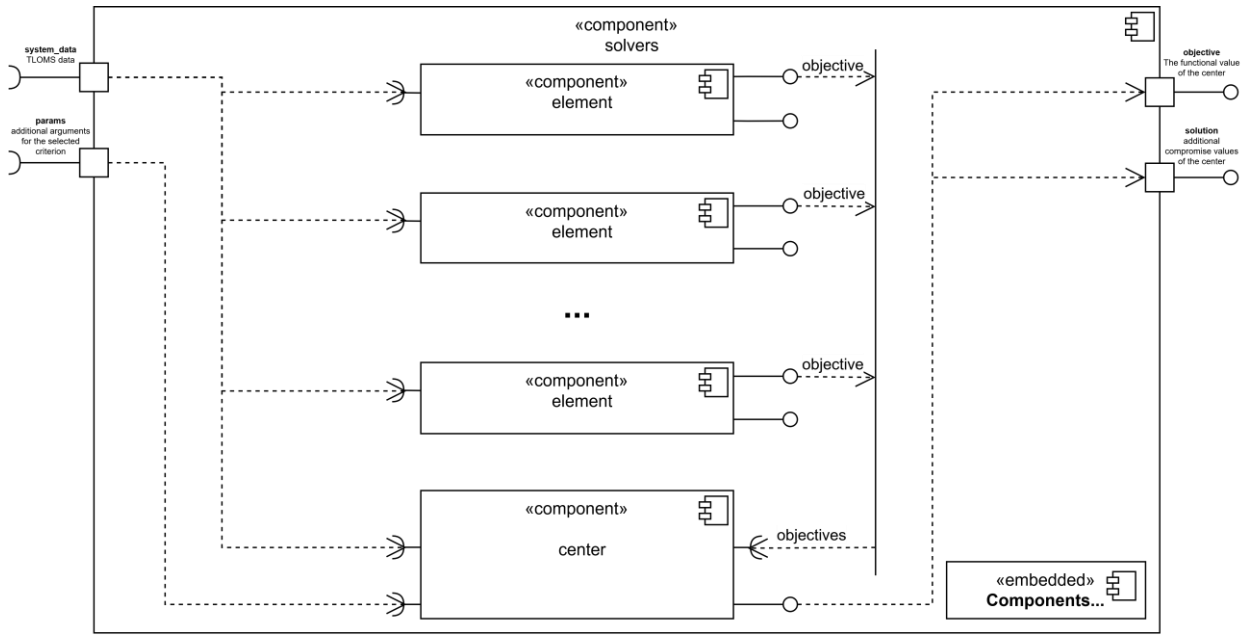


Fig.3. The detailed architecture of the TLOMS solver

5.4. Description of the Component Diagram "Detailed Architecture of the TLOMS Center"

Fig. 4 shows the detailed architecture of the TLOMS center.

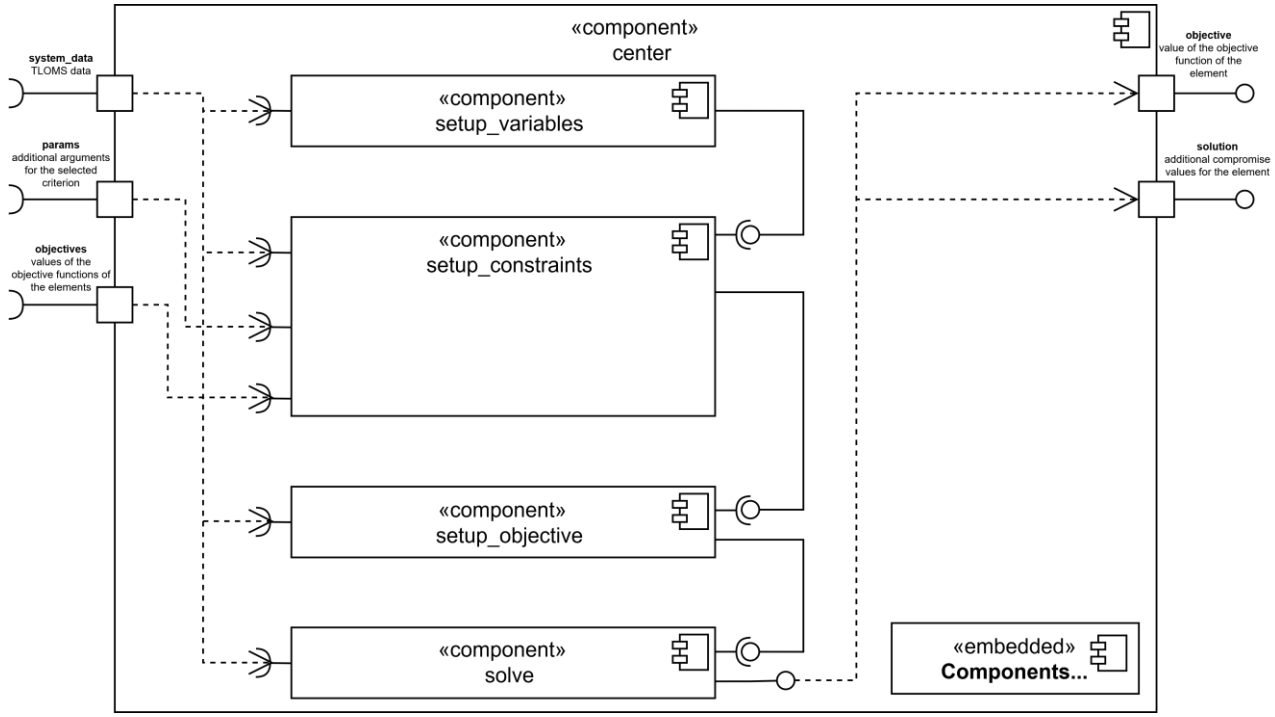


Fig.4. The detailed architecture of the TLOMS center

- *setup_variables*: the component initializes variables for the center optimization considering element data.
- *setup_constraints*: the component generates constraints for the center considering element constraints.
- *setup_objective*: the component builds an objective function for the center integrating the elements' objective functions, maximizes the system efficiency according to functional (11).
- *solve*: the component finds the optimal solution for the center using the variables, the constraints, and the objective function.

5.5. *Description of the Component Diagram "Detailed Architecture of an Element of the TLOMS"*

Fig. 5 illustrates the detailed architecture of an element of the TLOMS.

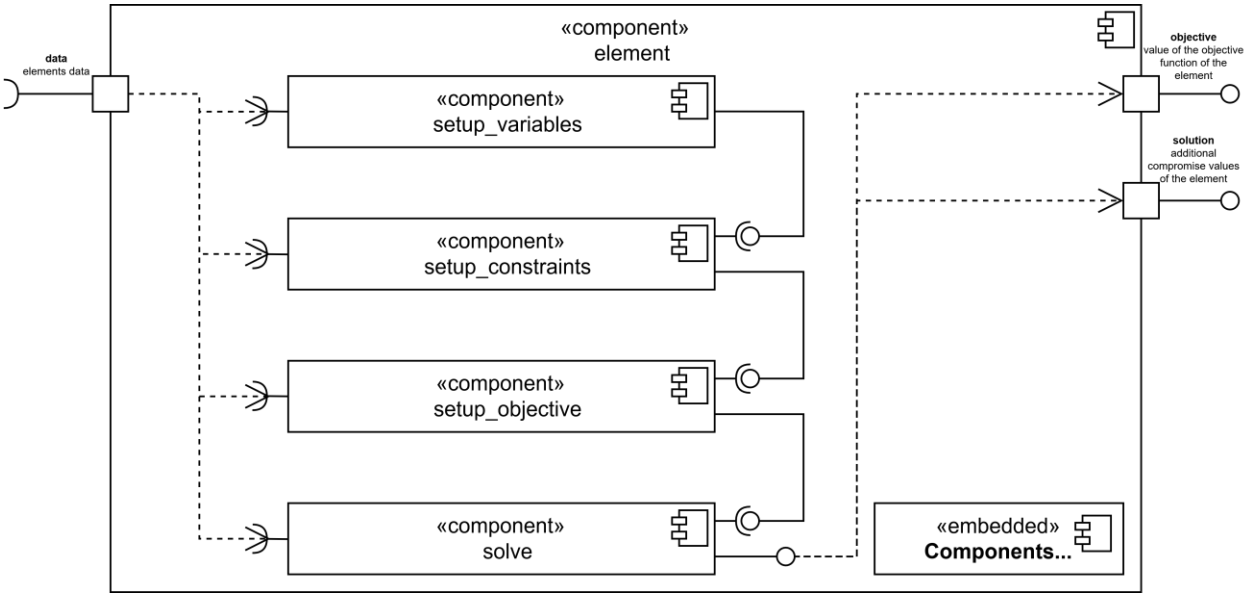


Fig.5. The detailed architecture of an element of the TLOMS

- *setup_variables*: the component initializes variables for optimization, namely the number of products and the processing start time. These variables correspond to the model parameters (2) and (3) where time constraints and resources are defined.
- *setup_constraints*: the component forms optimization constraints, in particular (1)–(6), ensuring their compliance with the theoretical requirements of the model.

- *setup_objective*: the component creates an objective function based on (7) maximizing production efficiency taking into account penalties (8)–(9).
- *solve*: the component finds the optimal solution to the optimization problem using the variables, the constraints, and the objective function. Implements the algorithms described in this paper to find compromise solutions for all criteria.

5.6. Intelligent Interface

The developed interface provides a structured framework for implementing the coordinated planning methodology in a TLOMS with ALVTM. It operationalizes the theoretical constructs of compromise criteria and algorithms through an intuitive multi-stage process that corresponds to the hierarchical planning from Sect. 2 and 3 of the paper.

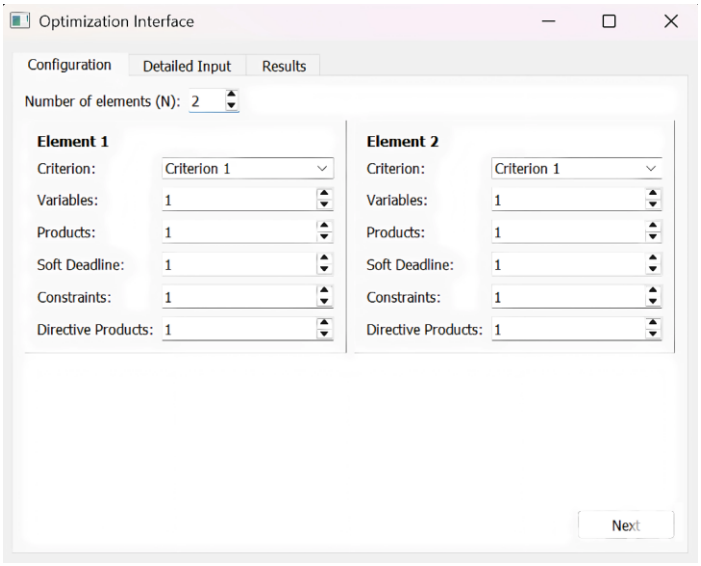


Fig.6. An example of configuring parameters in the configuration tab

The interface starts with the configuration phase (Fig. 6) where users define the system elements and their parameters: the number of elements, decision variables, aggregate/directive products, constraints, and choose one of eight compromise criteria (subsection 3.3). The configuration reflects the mathematical formulation of the TLOMS allowing to set resource limits, deadlines, and deviation penalties, as defined in conditions (1)–(11). The selection of criteria is associated with iterative optimization procedures, in particular, balancing the goals of the center and elements with an LPP.

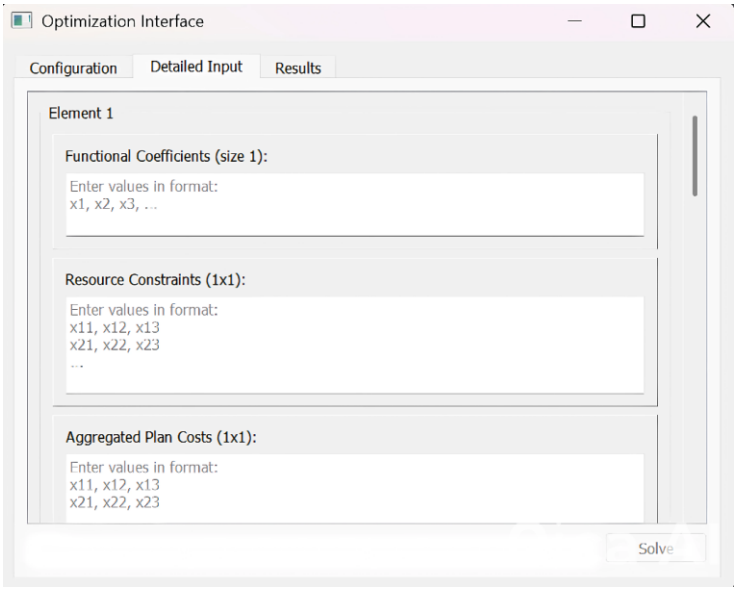


Fig.7. An example of entering detailed data in the data entry tab

In the detailed input stage (Fig. 7), users provide specific data via matrices and vectors, according to the ALVTM. The data includes functional coefficients, resource constraint matrices, planned costs/times, and directive conditions corresponding to (7)–(9) and (24)–(27). A test ensures dimensional consistency, compliance with linear constraints and time models, it emphasizes the separation of volume and time constraints with the integration of penalties for deadlines violation.

The results stage (Fig. 8) displays the optimization data: the value of the objective function, the planned volumes and time deviations obtained by the algorithms for the compromise criteria. This corresponds to the focus of the paper on the quantitative estimation of compromises between the central efficiency and the autonomy of the elements. The results can be exported for further analysis, which ensures the reproducibility and practical application of the methodology. The interface combines theoretical models with real implementation, allowing you to explore the algorithms of the paper while maintaining mathematical rigor and the hierarchical planning paradigm. An example is shown below in Fig. 8.

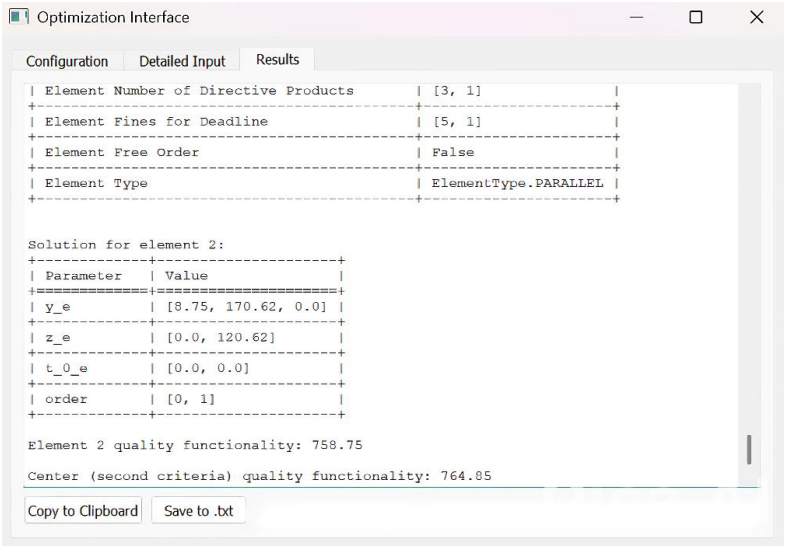


Fig.8. An example of optimization results in the results tab

6. Conclusions

We proposed an original two-level model of a discrete organizational and manufacturing system in terms of active system theory.

We developed a model of an element of a two-level organizational and manufacturing system in the form of an original aggregated linear volume-time model that allows (remaining in the class of linear models) to explicitly consider in the optimality criteria of the center and elements both the profit from the manufactured products and the penalties for violation of the deadlines agreed in advance.

We proposed eight new compromise criteria and, accordingly, eight constructive algorithms implementing the planning of a two-level organizational and manufacturing system, considering the interests of each element. The originality of the proposed algorithms and their difference from existing ones lies in the fact that the center of a two-level organizational and manufacturing system explicitly gets the opportunity to find a compromise solution that is conditionally optimal for the center, i.e., optimal under the condition that the element's functional value differs from the optimal one by no more than the value set by the center. The center can also implement an iterative procedure to find a compromise solution that corresponds to its conception of acceptable losses of the system and the element in relation to the plans that are optimal for the center and the element, respectively.

We presented an efficient software structure that implements the procedure of coordinated planning of a two-level organizational and manufacturing system using the proposed models, compromise criteria, and algorithms.

The proposed algorithms of coordinated planning can be used in any complex organizational and manufacturing systems, which allow efficient use of the proposed aggregated linear volume-time models of the system elements in case of conflicts of their interests with the interests of the center (the system as a whole). The independence condition used when the center builds compromise plans for the elements is natural if the interdependence of the elements' functioning is unambiguously given by the constraints on manufacturing resources, which the center distributes between the elements. Algorithms of distribution and redistribution of manufacturing resources by the center to each element are not considered in this work and are the goal of the authors' future scientific research.

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Authors' Profiles



Alexander Pavlov is a professor of the Department of Computer Science and Software Engineering in Igor Sikorsky Kyiv Polytechnic Institute, Ukraine. He got a PhD degree in 1970 and Doct. of Science degree in 1977. Associate professor of Technical Cybernetics Dept. since 1974. Professor (since 1979) and head (since 1981) of the Dept. of Computer-Aided Management Systems (renamed to Dept. of Computer-Aided Management and Data Processing Systems in 1994). Dean of the Faculty of Informatics and Computer Engineering in 1995–2018. Honored Worker of Science and Technology of Ukraine (1998). Winner of the State Prize of Ukraine in Science and Technology (2009). Academician in the Academy of Sciences of the Higher School of Ukraine. His research interests include combinatorial optimization, discrete production systems management, decision making under uncertainty, and regression analysis. Scopus ID: 57202701815



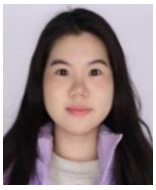
Kateryna Lishchuk is an Associate Professor of the Department of Computer Science and Software Engineering in Igor Sikorsky Kyiv Polytechnic Institute, Ukraine, and connected with it his further scientific activity. In 2001, she graduated with honors from Igor Sikorsky KPI with a degree in Information Control Systems and Technologies. Master of Computer Science. In March 2012, she defended her PhD thesis for the academic title of Candidate of Technical Sciences. Her research interests include human-machine interface, discrete structures, algorithmization and programming, object-oriented programming, discrete mathematics, functional and convex analysis, pattern recognition theory, and classification in artificial intelligence. Scopus ID: 57208345238



Oleg Melnikov is a scientific researcher and lead engineer of the Department of Computer Science and Software Engineering in Igor Sikorsky Kyiv Polytechnic Institute, Ukraine. He graduated with honors from Igor Sikorsky KPI in 1995 and connected with it his further scientific activity. He got a PhD degree in 2013 and is now a researcher in the above-mentioned department. His research interests include intractable scheduling problems, models and methods of planning and control, APS software. Scopus ID: 57202213883



Mykyta Kyselov is a Bachelor of Computer Science and Software Engineering Department of Igor Sikorsky Kyiv Polytechnic Institute, Ukraine. His scientific interests include software engineering, discrete optimization, and planning in systems with network representation of discrete technological processes.



Cennuo Hu was born on September 16, 2005. Now she is a student in the Department of Computer Science of the College of Science at Purdue University, West Lafayette, IN 47907, USA. Her main research interests are software engineering and computer security. She is an IEEE student member and has three invention patents. Scopus ID: 58871058700

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