

Application of Cauchy (Lipschitz) Criterion for Obtaining Theoretical Models of Atmosphere Striking Overvoltages

Slobodan N. Bjelić

Faculty of Technical Sciences, Kosovska Mitrovica, Serbia
E-mail: slobodanbjelic49@yahoo.com

Nenad A. Marković

Higher Technical School of Professional Studies Urosevac, in Leposavic, Serbia
E-mail: nen.mark74@yahoo.com

Zorica S. Bogićević

Higher Technical Professional School, Zvečan, Serbia
E-mail: zocabogi@gmail.com

Igor S. Bjelić

The National Archaeological Institute of Serbia, Belgrade, Serbia
E-mail: igor_bjelic@yahoo.com

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Abstract—In investigation of consequences of atmosphere and commutating striking voltages, for simulation of the overvoltage are used the models of generators whose RC circuits have standard passive parameters of the elements upon which the form of striking overvoltage depends.

According to IEC 62 305-1 standard, these formulas in the theoretical model serve for dimensioning the RC circuit of the generator of striking voltages although the definitions of time constants and passive parameters have only axiomatic character. Related to classical solution, this paper presents the model formed by mathematical procedure the solutions of which give sufficiently accurate values of time constants and essential parameters of RC circuit as well as the shape of striking voltage wave. The formulas for voltages and currents in model contain parameters of passive elements, and their accuracy has been confirmed by diagrams obtained in simulation by means of adapted psbttrnsrg.mdl part of MATLAB program. Theoretical model is suitable for simulation of standard wave forms of striking atmospheric and commutating overvoltages which replace laboratory testing.

Index Terms—Atmospheric voltage, Cauchy (Lipschitz) criterion, RC generator, Simulation, Theoretical model.

I. INTRODUCTION

There is no universal method for calculation of transient processes of switching and atmospheric

overvoltages. Approximate solutions can be obtained by applying methods from the theory of electrical circuits, that is method of projection of one modification (Finite Element Method-FEM, of Ric, Galerkin [1]). The method is based on testing approximations of solutions of differential equations in order to determine “projection”, that is an approximate solution defined in spatial and time domain. The striking voltages which are many times higher than the values of drive voltages are short-time high voltages occurring due to atmospheric influence (external) or due to commutations in electricity networks (internal) [2,3].

This paper presents the model formed by mathematical procedure the solutions of which give sufficiently accurate values and shape of striking voltage wave.

The paper has been organized in the following manner: Section 2 gives a review of the literature of related works in the field of atmosphere and commutating striking voltages. Section 3 gives amounts and forms of overvoltage as well as classical solution and correction of the classical solutions according to Cauchy-Lipchitz criterion. In Section 4 equations are derived for voltages and currents in RC circuit of striking voltage generator with sparking, depending upon the number of elements and their structure. Approximate solution of the form of striking voltage wave is given in Section 5. Section 6 shows the results of simulation and discussion of the model suggested for obtaining transient overvoltages. Finally, section 7 gives some concluding considerations.

II. RELATED WORKS

Testing the effects of shock voltages is not a new phenomenon. Many studies have been conducted on this subject and a number of papers have dealt with this issue [2,4].

Wilson [5] claims that the amount of electricity in storm clouds is $(10-50)C$, while intensity $\vec{E} = -grad\varphi$ rapidly decreases in the zone of maximum potential gradient.

The claim is also that the leader comprises less than 2 km of active zone.

Simpson, [6] claims to have measured intensities of fields close to the ground of $200\text{ kV}/m$ and that the voltages before the lightning flash were $V \approx 10^6\text{ kV}$. Majority of the energy was spent on ionization of the path of movement of the leader. Potentials of the body at strokes of atmospheric discharges are $V \approx (1-10)MV$, but polarity of the voltage is disputable. The currents between the cloud and the Earth were measured to be at the value of $I \approx 150kA$.

Simpson [6] also considers “the positive discharges” to be 1-10 times more frequent than negative; the authors of USA detect higher number of occurrence of negative discharges. Contradiction is apparent since negative strokes reach the Earth at one point through the leader; therefore they are stronger than positive strokes whose path to the ground goes through streamers which transfer only a part of the charge. Model of the process is LC circuit, and time of the process and aperiodic character determine parameter values. This theory is rejected because air can be more or less conductive if ions and electrons accidentally appear in it, and cloud is a perfect isolated ambient - H_2O molecules bind free ions from the surrounding atmosphere.

In the opinion of N. Marković et al. [7] the theory also did not explain the phenomenon of the path of leader tracer of ghost lightning depending upon some source of radiation in the universe. New investigations show that the lightning strikes at the points of geological discontinuity which are ionized due to action of radioactive masses.

III. FORMS OF OVERVOLTAGE, CHARACTERISTIC AMOUNTS AND CLASSICAL SOLUTIONS BY CAUCHY-LIPSCHITZ CRITERION

The amounts and form of atmospheric striking voltage are defined by IEC standard, Fig. 1,a,b,c [8,9,10,11]. Standard voltage value is described by the relation $\tau_f / \tau_t = T_1 / T_2 = 1.2 / 50\mu s$, duration time of the front $T_1 = 1.2\mu s (\pm 30\%)$ and achieving $1/2$ of amplitude value of the back for $T_2 = 50\mu s (\pm 20\%)$. According to IEC standard [8] T_1 is effective time of wavefront between two relative voltage values denoted by point A (30% amplitude) and point B (90% amplitude), Fig. 1.b,

magnified 1.67 times, since duration time of the wavefront is by that amount longer than that corresponding to the voltages at points A and B. T_2 is effective time within voltage is reduced down to $1/2$ value of amplitude.

Overvoltage is any voltage between a phase conductor and the Earth of phase conductors whose amplitude is in accordance with the definition of overvoltage [12]. Atmospheric and switching overvoltage-transient overvoltage is short-time overvoltage of the order (ms) or shorter that oscillates or does not oscillate, and it is usually strongly damped and achieves the full value at various times [13].

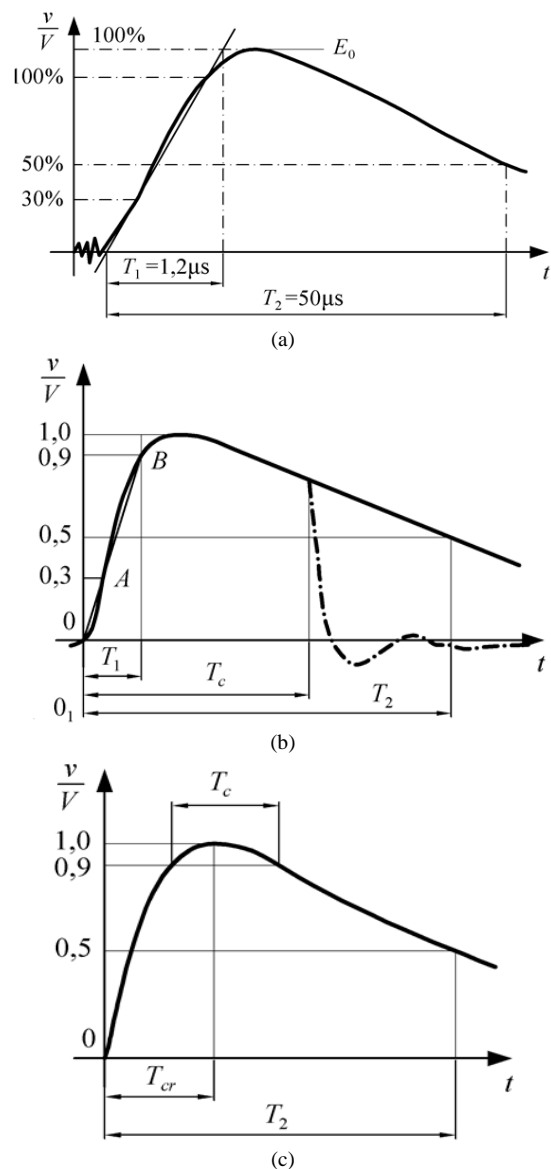


Fig.1. Real and general form: a) striking voltage, b) striking voltage of atmospheric origin, c) switching overvoltage.

Transient overvoltages are of the same polarity and are categorized as follows [14]:

- a) slower front growth: front time t_f from $20\mu s < t_f < 5000\mu s$ and back time from $t_t < 20ms$;

- b) rapid front growth: front time t_f from $0.1\mu s < t_f < 20\mu s$, and back duration from $t_t < 300\mu s$;
- c) very rapid front growth (overvoltages of one polarity with front time $t_f < 0.1\mu s$, duration $T < 3ms$).

They have shorter time of voltage growth T_1 to amplitude value in relation to the time of achieving $1/2$ of amplitude value at the back T_2 . Cutting of striking voltage occurs due to breakdown and is described by time of cutting voltage T_c . Time to amplitude of voltage T_{cr} with switching striking overvoltages is the time between initial condition (0.0) and V_m . In addition to back time T_2 for achieving $1/2$ of amplitude value, the characteristic quantity is also the duration time of maximum T_d within which the voltage exceeds 90% of amplitude value V_m . According to references [8,10,15,16] the standard form of atmospheric overvoltage impulse is $1.2/50\mu s$ while of switching it is $100/1000\mu s$. For both impulses, duration time of voltage growth to amplitude value T_1 greatly exceeds τ_f .

Fig. 2 shows the basic characteristics of overvoltage.

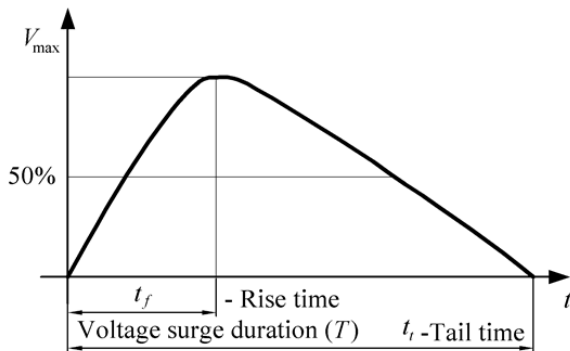


Fig.2. Basic characteristics of overvoltage.

The parameters given in Fig. 2 are: amplitude $V_{max}(kV)$, front time $t_f(\mu s)$, back time $T, t_t(\mu s)$, and their approximate values are: $t_f \approx (1-10)\mu s$, $t_t \approx 20\mu s$ and sometimes also $t_t \approx (200-300)\mu s$.

Standard waveform of induced atmospheric voltage wave is defined by hypotheses in the references [4,10] where it is stated that the wave is the difference of two exponential functions $v(t)=V_m(e^{-\alpha t} - e^{-\beta t})$. Parameter β is joined to the time of wave front growth t_f and parameter α is joined to the time of wave back t_t . By choosing the values α and β the given waveform is obtained with two exponential members and relation t_f / t_t [3,11,12]. Such a presentation does not correspond to the facts because:

- the solution $v(t)=V_m(e^{-\alpha t} - e^{-\beta t})$ is not mathematically correct and does not have an adequate physical explanation and it does not satisfy the condition of equivalency of original of the function and its first derivative,

- analytical method can provide formulas for shaping transient atmospheric of switching striking voltage in RC circuit of n - order with minimal number of elements of the second order which will be the basis for designing the generator of assigned form of voltage/current wave (the example is given in the paper).

Cauchy condition is satisfied if the function $y = f(x, y)$ has partial differential df / dy in the region, and, if it is not, then Lipschitz condition is used [17,18]. For an isolated singular point (0.0) Cauchy conditions are disturbed if they are satisfied at any other point. Course of the curve around the singular point depends upon the root of the characteristic equation. For continuous function $f(x, y)$ around the point $M(x_0, y_0)$, $|x - x_0| < a$ and $|y - y_0| < b$ there is at least one solution for the equation $y' = f(x, y)$ which is defined and continuous in the interval around the point $x = x_0$ which is also the solution $y = y_0$.

Lipshitz condition $|f(x, y) - f(x_0, y_0)| < N(y_1 - y_2)$ is fulfilled if N does not depend upon x, y_1, y_2 , and the solution is continuous function of $y = y_0$. If through the point $M(x, y)$ goes the graph of the solution $y' = f(x, y)$, of the equation $y' = f(x, y)$, coefficient of the direction of tangent upon the graph at that point determines the expression dy / dx .

All straight lines in the field in the plane, parallel to that tangent on the graph at the set point have equal directions (other straight lines apart from it are not tangent). Other directions belonging to remaining straight lines do not satisfy the condition to be the first derivatives of the graph at the set point since Cauchy conditions for linear homogenous differential equations of the second order with constant coefficients have not been fulfilled:

$$\Leftrightarrow ay'' + by' + cy = 0, \quad a\lambda^2 + b\lambda + c = 0. \quad (1)$$

Solution of the previous equation has two exponential components $y_1(x) = Ae^{\lambda_1 x}$ and $y_2(x) = Be^{\lambda_2 x}$ and it depends upon coefficients, that is the root value of the characteristic equation. Three cases can be distinguished:

- a) (λ_1) and (λ_2) are real and different, then $y = y_1(x) + y_2(x) = Ae^{\lambda_1 x} + Be^{\lambda_2 x}$. If due to initial conditions $B = -A$ then $y = y_1(x) + y_2(x) = A(e^{\lambda_1 x} - e^{\lambda_2 x})$;

b) (λ_1) and (λ_2) are real and equal, then:

$$y = y_1(x) + y_2(x) = Ae^{\lambda_1 x} + xBe^{\lambda_2 x};$$

c) $(\lambda_1 = \lambda_{Re} + i \cdot \lambda_{Im})$ and $(\lambda_2 = \lambda_{Re} - i \cdot \lambda_{Im})$ are conjugated complex numbers with two real and equal parts, then:

$$y(x) = y_1(x) + y_2(x) = Ae^{\lambda_{Re} x} \cos \lambda_{Im} + xBe^{\lambda_{Re} x} \sin \lambda_{Im} x.$$

Cauchy-Lipschitz theorem about the existence of solution for differential equation of the first order and the principle of equivalency of function and its first derivative states: Function is equivalent to its derivative only if it can be obtained by integration from that derivative [3]. The analysis is valid for both exponential components. For case a) it is true: λ_1 and λ_2 if and are real and different, the first derivative of the first component at a point x is $y_1'(x) = dy/dx = A\lambda_1 e^{\lambda_1 x}$.

Differential equation determines at each point the direction of tangent on the graph of solution. That direction is identical to directions of all straight lines parallel to belonging tangent on the graph at the specified point within a field in the plane.

Other straight lines of the same directions are not tangent because they do not touch the straight line, and other directions of remaining straight lines do not fulfill the condition to be the first derivatives of the graph at the set points (Cauchy conditions). A set of directions creates a field of directions, and one point and direction at that point is only one element of direction of the field. Integration of the equation involves connecting geometrical location of the points of integral curve, and direction of the tangent at each point of the curve coincides with direction of the field.

In some equations for solving processes of technical matters there are vertical directions, i.e. $f(x, y) \rightarrow \infty$. Then, in solving the roles of dependent and independent variables can be interchanged. Operation of differentiation is used, principle of equivalency introduced and it is considered that $dx/dy = f^{-1}(x, y)$ is equivalent to $f(x, y)$ (equations are equivalent if they have the same roots [3]).

In the domain where Cauchy conditions for equations under a) and b) are met through each point $M(x_0, y_0)$ passes only one integral curve. The set of curves depends upon one parameters and the equation of family of curves-integral of equation of the first order has one liberally chosen constant C . Particular integral $y = \varphi(x)$ which satisfies the condition $y_0 = \varphi(x_0)$ obtained is from the integral $F(x, y, C) = 0$ if C is determined from $F(x_0, y_0, C) = 0$.

Variable x in the equation (1) is replaced by the time $x \equiv t$ quantity and dependent variable y by voltage $y \equiv v$:

$$\begin{aligned} a \frac{d^2 v(t)}{dt^2} + b \frac{dv(t)}{dt} + cv(t) &= 0 \\ \frac{d^2 v(t)}{dt^2} + \frac{b}{a} \frac{dv(t)}{dt} + \frac{c}{a} v(t) &= 0 \end{aligned} \quad (2)$$

Classical solution for the variable $x = t$ is [3]:

$$v = v_1(t) + v_2(t) = A(e^{\lambda_1 t} - e^{\lambda_2 t}). \quad (3)$$

If $v_1(t) = Ae^{\lambda_1 t}$ and $v_2(t) = Be^{\lambda_2 t}$ are continuous functions within the rectangle P , if elements of the set \mathfrak{R} are real numbers, then functions $v_1(t)$ and $v_2(t)$ belong to the field (domain) \mathfrak{R}^2 , and s corresponds to the maximum $\mu > 0$.

A. Classical solution

When the functions $v_1(t)$ and $v_2(t)$ satisfy Lipschitz condition [3], in accordance with theorem about existence of η numbers as a unit within the set \mathfrak{R} , we obtain:

$$\begin{aligned} |\eta| &< \frac{1}{\mu(\chi - \xi)}, \exists \eta \in \mathfrak{R} \\ |v(t, s_1) - v(t, s_2)| &\leq \eta |s_1 - s_2|, (t, s_1), (t, s_2) \in P \end{aligned} \quad (4)$$

If $\xi_1 < \min\left\{\xi, \frac{\chi}{\mu}, \frac{1}{\eta}\right\}$, it is proved that there is only one differential function $v = v(t)$ in the domain $P = [t_0 - \xi, t_0 + \xi] \times [v_0 - \chi, v_0 + \chi]$ which is the solution of the differential equation $\frac{dv}{dt} = f(v, t)$ (*), at the segments $[t_0 - \xi_1, t_0 + \xi_1]$ and satisfies the initial condition $v(t=0+) = v_0$. Solutions of the equation (*), with the initial condition $v_{t=0+} = v_0$ are continuous solutions of the equation [3]:

$$v(t) = v_0 + \int_{t_0}^t f(\tau, v(\tau)) d\tau, \tau \in [t_0 - \xi_1, t_0 + \xi_1]. \quad (5)$$

B. Correction of classical solution

The solution where the first derivative and original equivalent is obtained by Cauchy-Lipschitz criterion makes it easier to determine voltage constants because the time constants define passive RC parameters:

$$v_I(t) = \int_0^t \frac{dv_1(t)}{dt} - v_1(t)_{t=0+} = A(1 - e^{\lambda_1 t}). \quad (6)$$

$$v_{II}(t) = \int_0^t \frac{dv_2(t)}{dt} - v_2(t)_{t=0+} = B(1 - e^{-\lambda_2 t}). \quad (7)$$

For $B = -A$, $v(t)|_{t=0} = 0$ and $e^{\lambda_1 t_0}|_{t_0=0} = e^{\lambda_2 t_0}|_{t_0=0} = 1$ voltage values are: $v_I(t) = A(1 - e^{-\lambda_1 t})$ and $v_{II}(t) = A(1 - e^{-\lambda_2 t})$. Then the new solution is obtained very similar to the known classical solution $v(t) = Ae^{\lambda_1 t} - Ae^{\lambda_2 t}$:

$$v(t) = A(1 - e^{-\lambda_1 t}) - A(1 - e^{-\lambda_2 t}). \quad (8)$$

The solution with $B = -A$ and two components $v(t) = v_I(t) + v_{II}(t)$ should satisfy the equation and equivalency condition of the first derivative and original. Also, by analytical method, integration of the first derivative gives:

$$v_I(t)|_{t=0} = -A \left(e^{-\frac{t}{\tau_1}} - e^{-\frac{t_0}{\tau_1}} \right) = A \left(1 - e^{-\frac{t}{\tau_1}} \right). \quad (9)$$

$$v_{II}(t)|_{t=0} = -B \left(e^{-\frac{t}{\tau_2}} - e^{-\frac{t_0}{\tau_2}} \right) = B \left(1 - e^{-\frac{t}{\tau_2}} \right). \quad (10)$$

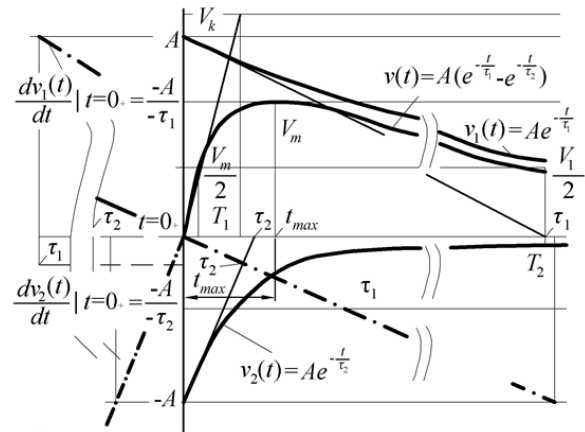
$$v(t) = v_I(t) + v_{II}(t) = A \left[\left(1 - e^{-\frac{t}{\tau_1}} \right) - \left(1 - e^{-\frac{t}{\tau_2}} \right) \right]. \quad (11)$$

$$\frac{dv(t)}{dt} = A \left(\frac{1}{\tau_1} e^{-\frac{t}{\tau_1}} - \frac{1}{\tau_2} e^{-\frac{t}{\tau_2}} \right). \quad (12)$$

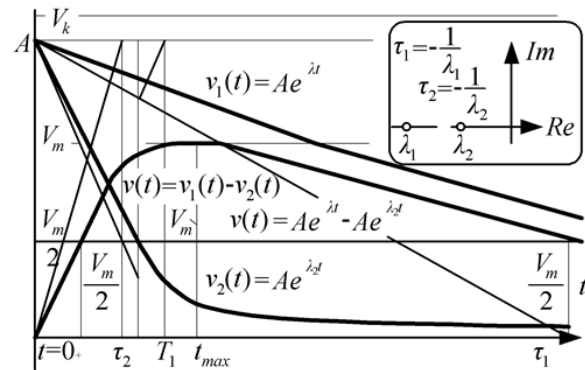
$$\frac{dv(t)}{dt} \Big|_{t=0} = A \frac{\tau_2 - \tau_1}{\tau_1 \tau_2} = A(\tau_2 - \tau_1) \frac{c}{a} = \frac{V_k}{a} \quad (13)$$

$$A = \frac{V_k}{\tau_2 - \tau_1} \frac{1}{c}, A = \frac{RC_k}{\tau_2 - \tau_1} V_k$$

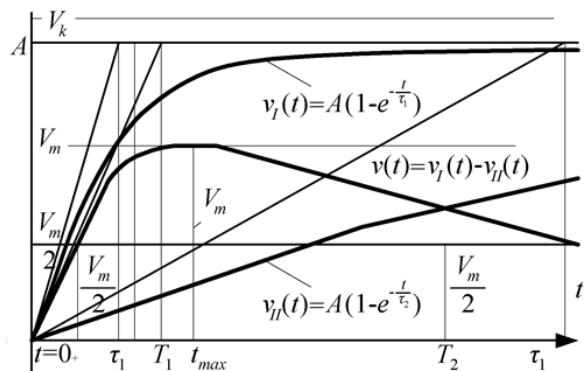
Solution of the equation (13) satisfies homogeneous differential equation of the second order and can be used for determining parameters for the generator for simulation of atmospheric and switching striking overvoltages. Graph shapes of the components are different classical solution, Fig.3,a,b. They also differ for the corrected solution, Fig. 3.c, for voltages $v(t)$ in RC circuit of the second order, which in accordance with IEC standard [8] has specified monotonous and aperiodic course.



(a)



(b)



(c)

Fig.3. Striking overvoltage $v(t)$ in RC circuit of the second order for: a), b) classical solution, c) corrected solution [8].

From the moment $t = 0_+$ the shape asymptotically approaches the course of stationary state $v(t) = v_I(t) + v_{II}(t)$.

Classical solutions are disputable because for both components the first derivatives are not tangents but straight lines parallel to tangents. Per abscissa these are curves of variables displaced by t_1 and t_2 which leads to an error in determining time constants of voltage forms defined by the parameters of RC circuit. The new solution is determined by analytical method in which Cauchy and Lipschitz condition for the first derivative on the function graph is met for all abscissa values $0_+ < t < \infty$.

IV. GENERATOR OF STRIKING OVERVOLTAGE WITH PASSIVE PARAMETERS OF ELEMENTS AND SPARKING

Generator of striking voltages with sparking, depending upon number of elements and structure is made with RC circuit of the first second and higher n order-Marks generator with multiple RC circuits. Amplitude V_m and constants $\tau_1, \tau_2, t_m, C, C_k$ are adjusted according to the requirement of IEC standards [8]. r_f, r_T, R are determined depending upon number and values of parameters.

Breakthrough of sparking that influences waveform is defined by Streamers theory of channel and Paschen's law. Multiple Streamers occur in homogenous fields and further ionization creates the channel of high temperature plasma-leader.

Breakthrough time $t = t_0$ ($t \approx t_0 = 10 \div 20 ns \approx 0_+$), is the time from the moment voltage occurs to breakthrough.

In Fig. 4.a the circuit has at input the capacitor of capacitance C_k under voltage by circuit V_k . After breakthrough of sparking through a combination of damping resistor r_i and resistor r_f (wave-front), capacitance C of the object tested is loaded and at the same time discharged through discharge resistor R . Capacitance C value is often not known when determining shape of the voltage. In the diagram of RC circuit of striking voltage in Fig. 4.a the currents are:

$$i = -C_k \frac{dv_k}{dt}, i_2 = C \frac{dv}{dt}. \quad (14)$$

The wave form of striking voltage is obtained as follows:

$$\frac{a}{c} \frac{d^2v(t)}{dt^2} + \frac{b}{c} \frac{dv(t)}{dt} + v(t) = 0. \quad (15)$$

$$\tau_1 \tau_2 \frac{d^2v(t)}{dt^2} + (\tau_1 + \tau_2) \frac{dv(t)}{dt} + v(t) = 0. \quad (16)$$

The same shape is obtained also in analog way:

$$\frac{d^2v(t)}{dt^2} + \frac{b}{a} \frac{dv(t)}{dt} + \frac{c}{a} v(t) = 0 \quad (17)$$

$$\frac{d^2v(t)}{dt^2} + \frac{\tau_1 + \tau_2}{\tau_1 \tau_2} \frac{dv(t)}{dt} + \frac{1}{\tau_1 \tau_2} v(t) = 0$$

where is the equation:

$$\tau_{1,2} = \frac{b \mp \sqrt{b^2 - 4ac}}{2c}, \tau_1 \tau_2 = \frac{a}{c} \quad (18)$$

$$\tau_2 + \tau_1 = \frac{b}{c}, \tau_2 - \tau_1 = \frac{\sqrt{b^2 - 4ac}}{c}$$

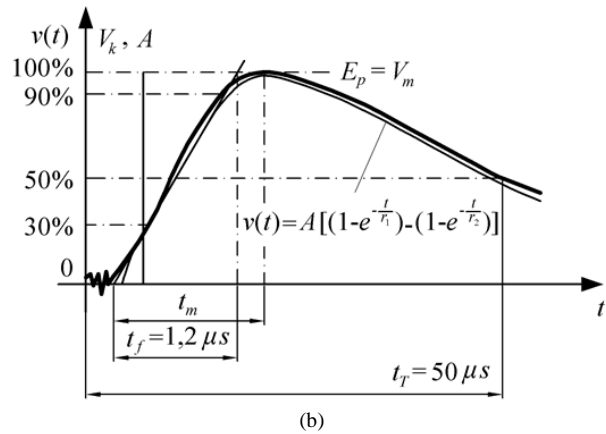
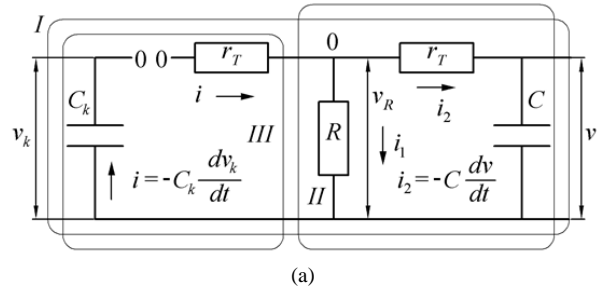


Fig.4. a) RC circuit and generator sparking, b) standard shape of voltage wave [8].

The solution with two components in equations (9-13), for $B = -A$ should satisfy value of parameters (18) and equivalency condition of the first derivative and original. Differentiation of the equation (12) gives:

$$\frac{dv(t)}{dt} \Big|_{t=0} = A \left(\frac{1}{\tau_1} e^{-\frac{t}{\tau_1}} - \frac{1}{\tau_2} e^{-\frac{t}{\tau_2}} \right) = A \frac{\tau_2 - \tau_1}{\tau_1 \tau_2}. \quad (19)$$

The expression $v(t) = A \left[(1 - e^{-t/\tau_1}) - (1 - e^{-t/\tau_2}) \right]$ satisfies the equation and can be used for choosing parameters in simulation of atmospheric and switching striking overvoltages.

IEC standard determines parameters of RC circuit of the model through time constants T_1 and T_2 :

- wave front time $T_1 = t_f$ defined by time difference at the points with values $0.3V_m$ and $0.9V_m$,

- back time $T_2 = \tau_r$ as the time at which voltage value at the back reduces to $(1/2)V_m$.

Wave form theoretical model is defined by mathematically determined time constants τ_1 and τ_2 , which are in different correlations in regards to standard values t_f, t_t . When choosing the shape, possible solution variants are:

- set value $\tau_1 \leq T_1$ or $\tau_1 \geq T_1$ and then can be determined and $k = V_m / A = f(\tau_1)$ and $\tau_2 = f(T_2, k)$,
- set value $\tau_2 \leq T_2$ or $\tau_2 \geq T_2$ then can be determined and $k = V_m / A = f(\tau_2)$ and $\tau_1 = f(T_1, k)$,
- after determining τ_1, τ_2, k and adopting necessary values C_k, R, C according to derived equations, parameters of RC circuit, r_f, r_T are determined.

V. APPROXIMATE SOLUTION OF STRIKING VOLTAGE WAVEFORM

If there is no data about capacitance of tested object C , in order to calculate parameters of RC it is necessary to adopt values C, C_k and to determine relation of the value r_T, R . To obtain expected waveform of the striking voltage, all parameter values should be in accordance with specified constants V_m, τ_1, τ_2, k .

According to IEC standard the value [8]

$$\frac{\tau_2}{\tau_2 - \tau_1} \Big|_{\tau_2 \gg \tau_1} \cong 1 \text{ hence follows the expression:}$$

$$\frac{dv(t)}{dt} \Big|_{t=0} = A \frac{\tau_2 - \tau_1}{\tau_1 \tau_2} \cong \frac{A}{\tau_1} . \tag{20}$$

Constant A which defines time constants is determined from V_m and the values $k < 1, V_m = kA$ [8]. Atmospheric wave is characterized by amplitude V_m and relation T_1 / T_2 :

$$\tau_f = T_1 = 1.67(t_2 - t_1), \quad t_2 - t_1 = T_1 / 1.67 . \tag{21}$$

Since $t_1, t_2 < (\tau_f = T_1) \ll (\tau_t = T_2) < \tau_2$, both exponential members are: $e^{-t_1 / \tau_2} \cong 1, e^{-t_2 / \tau_2} \cong 1$.

Value τ_1 of striking voltage of the generator $v(t) = A \left[-e^{-t / \tau_1} + e^{-t / \tau_2} \right]$ per IEC standard [8] and axiomatic definitions $e^{-t_1 / \tau_2} \cong 1, e^{-t_2 / \tau_2} \cong 1$ is determined from:

$$v(t_1)_{0.3V_m} = A \left[-e^{-\frac{t_1}{\tau_1}} + e^{-\frac{t_1}{\tau_2}} \right] = 0.3V_m \tag{22}$$

$$v_{t1} \cong A \left[-e^{-\frac{t_1}{\tau_1}} + 1 \right] = 0.3V_m \Rightarrow -e^{-\frac{t_1}{\tau_1}} = 0.3k - 1$$

$$v(t_2)_{0.9V_m} = A \left[-e^{-\frac{t_2}{\tau_1}} + e^{-\frac{t_2}{\tau_2}} \right] = 0.9V_m \tag{23}$$

$$v_{t2} \cong A \left[-e^{-\frac{t_2}{\tau_1}} + 1 \right] = 0.9V_m \Rightarrow -e^{-\frac{t_2}{\tau_1}} = 0.9k - 1$$

$$\begin{aligned} e^{-\frac{t_1}{\tau_1}} = 1 - 0.3k, e^{-\frac{t_2}{\tau_1}} = 1 - 0.9k &\Leftrightarrow e^{-\frac{-t_1 + \tau_2}{\tau_1}} = \frac{1 - 0.3k}{1 - 0.9k} \Leftrightarrow \\ \Leftrightarrow \frac{t_2 - t_1}{\tau_1} = \ln \frac{1 - 0.3k}{1 - 0.9k} = \frac{T_1}{1.67} \frac{1}{\tau_1}, \tau_1 = \frac{T_1}{1.67} \frac{1}{\ln \frac{1 - 0.3k}{1 - 0.9k}} \end{aligned} \tag{24}$$

Back time τ_2 , is determined from equation (23)

$$v(t) \Big|_{T_2} = \frac{V_m}{2} = A \left[-e^{-\frac{T_2}{\tau_1}} + e^{-\frac{T_2}{\tau_2}} \right], \quad \text{so if}$$

$$e^{-T_2 / \tau_1} \cong 10^{-4} \rightarrow 0 \quad \text{then} \quad v(t) \Big|_{T_2} = \frac{V_m}{2} \cong A \left[e^{-\frac{T_2}{\tau_2}} \right],$$

$$\frac{k}{2} \cong e^{-\frac{T_2}{\tau_2}}, \quad \tau_2 \cong \frac{T_2}{\ln(2/k)} \quad \text{and} \quad t_m = \frac{\tau_1 \tau_2}{\tau_2 - \tau_1} \ln \frac{\tau_2}{\tau_1} .$$

VI. CALCULATION, SIMULATION AND ANALYSIS OF TRANSIENT PROCESS

Solution for $v_k(t) = V_{ck}$ is determined from state within the circuit of the simulation model, Fig. 5 when the switch is closed and sparking open (there is no breakthrough of the sparking)

$$v_k = V_{ck} = r_f C \left[\frac{r_T}{r_f} + 1 + \frac{r_T}{R} \right] \cdot \frac{dv}{dt}, \quad \text{and solution for}$$

$$v(t) = V_{c.surge} \quad \text{and} \quad i_2(t) = I_{c.surge} \quad \text{from the state within}$$

the circuit when breakthrough of the sparking occurs

$$v(t) = A \left[-e^{-\frac{t}{\tau_1}} + e^{-\frac{t}{\tau_2}} \right], \quad i_2 = I_{c.surge} = C \frac{dv}{dt}.$$

In order to check derived equations of transient process simulation was made in the part of the program MATLAB psbtransient.mdl Power Sym package titled psbtrsg.mdl, Fig. 5 [19]. The diagrams obtained, Fig. 6, confirm that derived relations are in accordance with the set values in Simulation model.

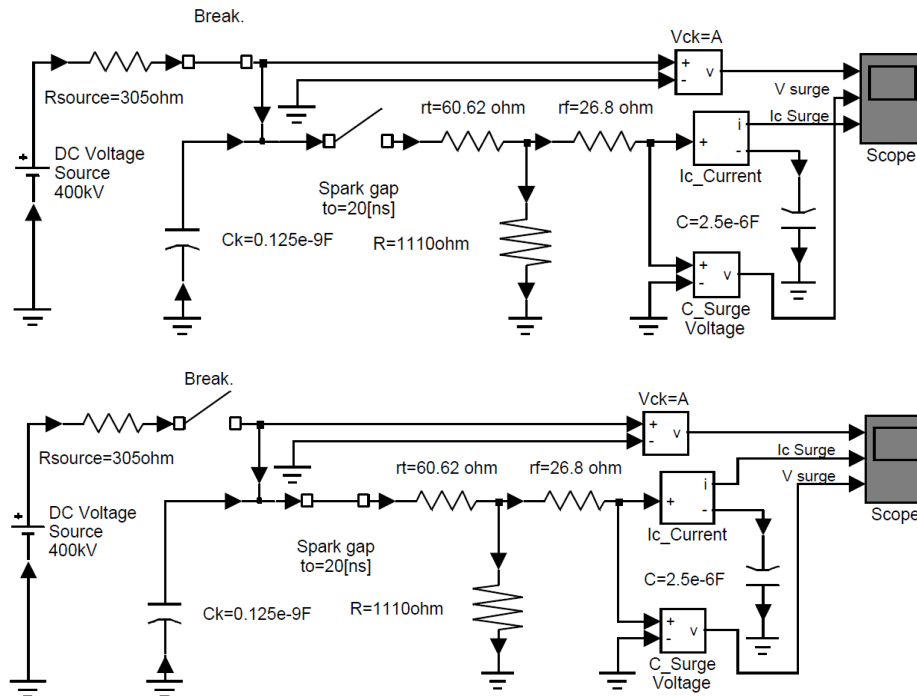


Fig.5. Diagram of simulation model for transient process in striking voltage generator: a) before breakthrough of sparking, b) after breakthrough.

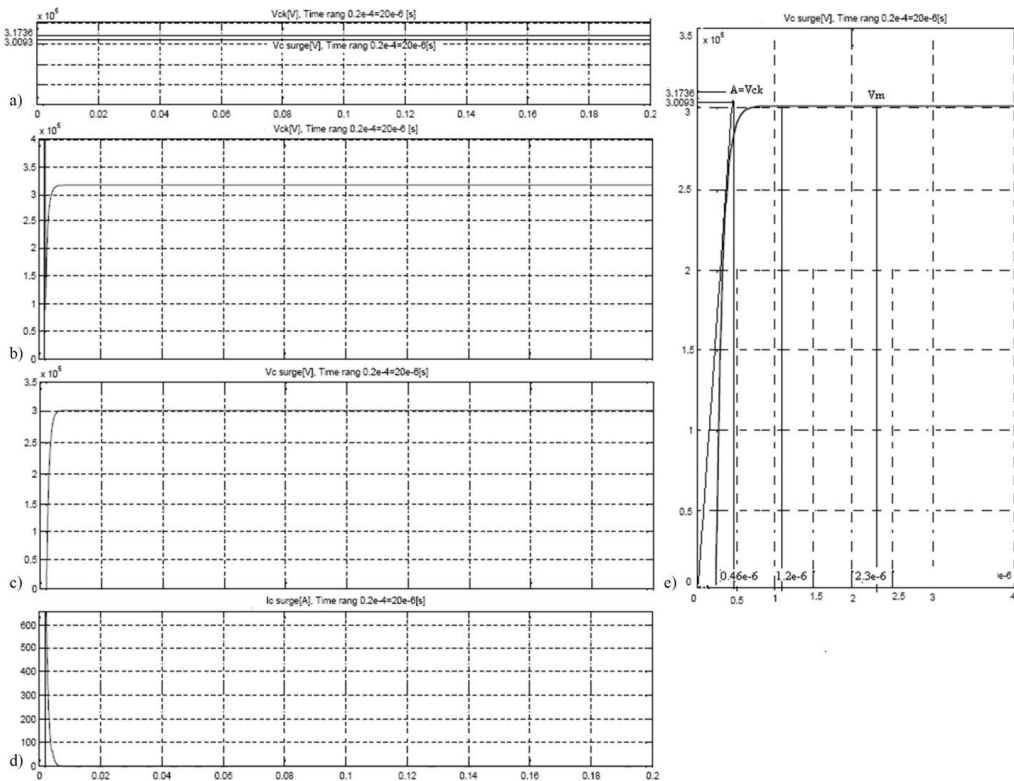


Fig.6. Transient processes in linear RC circuit of generator current voltage: a) Values A and Vck, b) diagram Vck, c) diagram Vc, d) diagrams of current and Ic and e) detail of striking voltage wavefront.

Results and diagrams obtained, Fig. 6, show variations of currents and voltages and indicate possibility of applying the model of RC circuit in the generator to obtain expected striking voltage waveforms.

For RC circuit of striking voltage generator with specified $V_{cm,surge}$ (kV), given shape; front and back times $T_1/T_2 = 1.2\mu s/50\mu s = 41.7$ and adopted values $k = \frac{V_m}{A}$; $C_k, C(\mu F)$, it is necessary to determine $V_{ck} = A$, mathematical time constants τ_2, τ_1 and resistance values $R, r_f, r_T(\Omega)$.

As mentioned earlier in the paper, this is an example where formulas for shaping transient atmospheric or switching striking voltage within RC circuit can be obtained by analytical method.

Example:

$$V_{cm,surge} = 300.94kV$$

$$k = 0.948 \cong 0.95$$

$$A = \frac{V_m}{0.948}$$

$$A = 317.36kV$$

$$V_{ck} = A = 317.36kV$$

$$\tau_1 = \frac{T_1}{1.67} \frac{1}{\ln \frac{1-0.3k}{1-0.9k}} = \frac{1.2}{1.67} \frac{1}{\ln \frac{1-0.3 \cdot 0.95}{1-0.9 \cdot 0.95}} \cong 0.46\mu s$$

$$\frac{\tau_1 \tau_2}{\tau_2 - \tau_1} = \frac{30.82}{66.54} = 0.46318$$

$$\tau_2 \cong \frac{T_2}{\ln(2/k)} = \frac{50}{\ln(2/0.948)} = 66.99 \cong 67\mu s$$

$$\tau_2 \tau_1 = 30.82(\mu s)^2$$

$$\tau_2 - \tau_1 = 66.54\mu s$$

$$\tau_2 + \tau_1 = 67.46\mu s$$

$$t_m = \frac{\tau_1 \tau_2}{\tau_2 - \tau_1} \ln \frac{\tau_2}{\tau_1} = \frac{0.46 \cdot 67.0}{67.16 - 0.46} \ln \frac{67.0}{0.46} = 2.3\mu s$$

Checking:

$$V_m = A \left[-e^{-\frac{t_m}{\tau_1}} + e^{-\frac{t_m}{\tau_2}} \right] = 317.33 \left[-e^{-\frac{2.3}{0.46}} + e^{-\frac{2.3}{67}} \right] =$$

$$= 317.36[-0.00674 + 0.966] = 317.33 \cdot 0.959 \cong$$

$$\cong 304.3kV \cong 300.03kV \Leftrightarrow 300.94kV$$

From specified parameter values of input branch of RC_k , electric circuit $C_k = 0.125 \cdot 10^{-3} \mu F$ and $R = 1110\Omega$ is determined time constant RC_k which amounts to:

$$RC_k = 1110\Omega \cdot 0.125 \cdot 10^{-9} F \cong 0.13875\mu s$$

From adopted value $C = 2.5\mu F$ and time constants of

the back $\tau_2 = 67\mu s$ we obtain:

$$r_f = \frac{67 \cdot 10^{-6}}{2.5 \cdot 10^{-6}} = 26.8\Omega$$

Then, value r_T is calculated as:

$$r_T = \frac{r_f R}{r_f + R} \left(\frac{\tau_1}{RC_k} - 1 \right) =$$

$$= \frac{26.8 \cdot 1110}{26.8 + 1110} \left(\frac{0.46}{0.13875} - 1 \right) = 60.62\Omega$$

Checking:

$$\tau_1 = RC_k \left[\frac{r_T}{r_f} + \left(1 + \frac{r_T}{R} \right) \right] =$$

$$= 0.13875 \left[\frac{60.62}{26.8} + 1 + \frac{60.62}{1100} \right] =$$

$$= 0.13875 \cdot 3.322621 = 0.4608\mu s$$

For analysis of transient process with specified waveform of striking voltage is used diagram of the model that contains specified values: $V_{source} = 400kV$, $k = 0.95$, $V_m = 300kV$, $T_1/T_2(\mu s/\mu s) = 41.7$, switch, sparking and passive elements: $C_k = 0.125 \cdot 10^{-9} F$, $C \cong 2.5 \cdot 10^{-6} F$, $R = 1110\Omega$.

Resistances calculated are $r_T = 60.62\Omega$ and $r_f = 26.8\Omega$. Opening and closing the switch and sparking start the simulation and activate program psbtrsrg.mdl, with chosen circuit parameters. Measured currents and voltages are:

$$i_2 = I_{c,surge} = C \frac{dv}{dt} = C \cdot A \left(\frac{1}{\tau_1} e^{-\frac{t}{\tau_1}} - \frac{1}{\tau_2} e^{-\frac{t}{\tau_2}} \right)$$

$$v_k = V_{ck} = r_f C \left[\frac{r_T}{r_f} + \left(1 + \frac{r_T}{R} \right) \right] \cdot \frac{dv}{dt}$$

$$v(t) = V_c = A \left[-e^{-\frac{t}{\tau_1}} + e^{-\frac{t}{\tau_2}} \right]$$

VII. CONCLUSION

Established equations and expressions can be used to determine values of standard parameters of striking voltage shapes and time constants in the generator models with electric RC circuits. The example shown and its checks show that the errors occurring in calculation of parameter values and constants are very small.

Advantages of MATLAB Simulink are a great number of possible Simulations and the results of these simulations, forms and characteristic values of obtained wave diagrams verify the method suggested for obtaining the parameters of voltages and currents of striking voltages waves.

Ready-made MATLAB Simulink programs simulate changes of frequencies pretty well, but own development both of models and programs gives special advantages, such as detailed insight into all components of the models and programs and introduction of various alterations that otherwise could not be introduced into available program package.

Application of auxiliary time constants τ_1 , τ_2 which are in correlation with constants T_1 , T_2 gives correct parameter values and that is confirmed also by diagrams obtained by simulation using the innovated part of the program psbtrsrg.mdl MATLAB. The results of the simulation also confirm that the solutions obtained by application of the analytical method shown are correct and applicable. A high correlation can be the result of the third variable influence, which caused the appearance of their interdependence. This phenomenon is left to the authors to examine and explain in some of the following work.

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Authors' Profiles



Slobodan N. Bjelić: Full-time professor at Faculty of Technical Sciences in Kosovska Mitrovica, University of Pristina. He received his Ph.D. degree in 1982, field of interest: Electrical Engineering-Computer Science-Protective Relays-Electrical Installation and Illumination Engineerings-Electrical Power Converters. He is author or coauthor of 10 textbooks and over 200 scientific and professional papers published in international and Serbian journals.



Nenad A. Marković: Assistant Director for Education at High Technical School of Professional Studies Urosevac with temporary seat in Leposavic., and professor at study programs Mechanical engineering and Road Traffic in mentioned school. He is received his Ph.D. degree from Faculty of technical Sciences in Cacak-University of Kragujevac and is author or coauthor of many textbooks, exercises and over 120 papers published in international and Serbian journals.



Zorica S. Bogićević: Professor of vocational study in Higher Technical Professional School in Zvečan, Serbia. She received her Ph.D. degree in 2016, field of interest: Electrical Engineering-Computer Science. Her dissertation thesis was: "Development of algorithms and simulation of transitional process in transformer windings with computer supports". She is author of textbook Electric drivers and over 60 scientific and professional papers published in international and Serbian journals.



Igor S. Bjelić: Her dissertation thesis was: "Typology and structural characteristics of arches and vaults of sacral buildings in the medieval Serbia", Faculty of Civil Engineering and Architecture, University of Nis, September 2016. She is author of 36 scientific and professional papers published in international and Serbian journals.

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