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Bayesian Conditional Volatility Models of Skewed Generalized Error Distribution via Mode as the Stable Location Parameter

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Abstract: In this article, novel mixture of conditional volatility models of Generalized Autoregressive Conditional Heteroscedasticity (GARCH); Exponential GARCH (EGARCH); Glosten, Jagannathan, and Runkle GARCH (GJR-GARCH); and dependent variable-GARCH (TGARCH) were thoroughly expounded in a Bayesian paradigm. Expectation-Maximization (EM) algorithm was employed as the parameter estimation technique to work-out posterior distributions of the involved hyper-parameters after setting-up their corresponding prior distributions. Mode was considered as the stable location parameter instead of the mean, because it could robustly adapt to symmetric, skewedness, heteroscedasticity and multimodality effects simulteanously needed to redefine switching conditional variance processes conceived as mixture components based on shifting number of modes in the marginal density of Skewed Generalized Error Distribution (SGED) set as the prior random noise.

In application to ten (10) most used cryptocurrency coins and tokens via their daily open, high, low, close and volume converted and transacted in USD from the same date of inception. Binance Coin (BNB) via its daily lower units transacted in USD (that is, low-BNB), yielded the most reduced Deviance Information Criteria (DIC) of 3651.1935. The low-BNB process yielded a two-regime process of TGARCH, that is, Mixture dependent variable-GARCH (TGARCH (2: 2, 2)) with stable probabilities of 33% and 66% respectively. The first regime was attributed with low unconditional volatility of 16.96664, while the second regime was traded with high unconditional volatility of 585.6190. In summary, Binance Coin (BNB) was a mixture of tranquil market conditions and stormy market conditions. Implicatively, this implies that the first regime of the low-BNB was characterized with strong fluctuating reaction to past negative daily returns of low-BNB converted to USD, while the second regime was attributed with weak fluctuating reaction. Additionally, the first regime was attributed with low repetitive volatility process, while the second regime was characterized with high persistence fluctuating process. For financial and economic decision-making, crypocurrency users and financial bodies should look-out for financial and economic sabotage agents, like war, exchange rate instability, political crises, inflation, browsing network fluctuation etc. that arose, declined or fluctuated doing the ten (10) years to study of the coins and tokens to ascertain which of this/these agent(s) contributed to the volatility process.

Mixture models from a Bayesian perspective were of interest because; some of the classical (traditional) models cannot accommodate and absolve regime-switching traits, and as well contain prior information known about cryptocurrency coins and tokens. In light of model performance, DIC values were compared on the basis of most performed to less perform via lower to higher values of DICs.

Index Terms: Bayesian, Conditional Volatility Models, Deviance Information Criteria (DIC), Expectation Maximization-Algorithm, Generalized Autoregressive Conditional Heteroscedasticity (GARCH), Skewed Generalized Error Distribution (SGED).

1. Introduction

One of the important issues in financial times series is the changes in the process mean that usually directly or indirectly affect the uncertainty of the considered process [1]. It was this important issue that brought-up stochastic volatility modeling that makes it feasible to incorporate correlated conditional variance and mean into time-variant processes. However, it was notably inferred that stochastic volatility found it difficult to directly evaluate past effects of the mean process in the conditional variance (See [2]). It was because of this lacuna that a well-known future volatile process regarded as Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model was propounded by [3] based on conditional volatility. The variance based-GARCH model is a function of the past values and residuals with a vital assumption that the variance process is independent of the shocks in the mean of the process. The merit of GARCH-type models over stochastic volatility model is that it allows direct accessing of the changes in the mean effect in the dynamicity of the conditional variance process understudy (See [4]).

The stylized fact that GARCH-types capture conditional variance in reaction to positive asymmetric shocks versus negative shocks (that is, characterized large shocks versus small shocks that can either be positive or either). According to [5] and [6], conditional variance processes might follow different regimes in line with its size, associated heteroskedasticities, asymmetry effects, and jumps (multimodality) that contributed to shocks. Three (3) variant processes of the GARCH model, as well as the GARCH process itself were regarded as the conditional volatility models. The three other GARCH-variants are Exponential GARCH (EGARCH), Glosten, Jagannathan, and Runkle GARCH-type, usually refer to as the GJR-GARCH and dependent variable GARCH (TGARCH). Each of these processes is attributed with either large (or small) negative shocks or large (or small) positive shocks. Notably, financial returns with such characterization is the cryptocurrency tokens and coins; their exchange rates; stock prices, exchange rates etc. (See [7 - 9]). In other words, both the negative and positive effects of some of these mentioned financial returns fall under the scope of asymmetric impact of the these stated conditional volatility models. For instance, large negative returns might affect future volatility in different way when compare to positive returns with the same magnitude. To curb this effect of asymmetric issue, an all-for-all skewed distributional random noise could be adopted to drive these asymmetric and non-asymmetric conditional volatility models in a Bayesian paradigm (See [10]).

Informatively, recent studies affirmed that volatile GARCH-type processes failed to capture true variation in volatile contained data points in the case of regime-switching in a volatized dynamics. Possible solution to the dynamicity of volatilized regime-switching processes is to allow involved parameters to vary over regime-switching according to latent Markov process in a Bayesian paradigm. This approach could be regarded as Bayesian Markov-switching of conditional volatility processes (Otherwise could be regarded as Bayesian MSGARCHs), which could lead to volatility forecasts and estimation of variations of risk measures in an unconditional volatility manner. This proposed Bayesian Markov-switching of conditional volatility processes differs from the classical existing solutions of GARCH-type variants because they incorporate prior information about the data in its chosen distributional form.

In light of the need to robustly contain the asymmetric impact on conditional volatility models, Skewed Generalized Error Distribution (SGED) random noise shall be adopted as error term for the mixture conditional volatility models of GARCH, EGARCH, GJR-GARCH, and TGARCH. In other words, regime-switching of the stated conditional volatility models via SGED random noise shall be expounded from a Bayesian paradigm and apply to jumps and volatility characterized ten (10) most used cryptocurrency coins and tokens. Cryptocurrencies shall be considered because of the recently reportedly swinging prices. The measure metric for the swinging prices of cryptocurrencies is volatility and this could be best reflected from the history of past swinging prices (prior knowledge). The adopted parameter estimation technique shall be via EM-algorithm powered from a Bayesian perspective.

This research is aimed at developing a novel regime-switching of conditional volatility processes of Generalized Autoregressive Conditional Heteroscedasticity (GARCH); Exponential GARCH (EGARCH); Glosten, Jagannathan, and Runkle GARCH (GJR-GARCH); and dependent variable-GARCH (TGARCH) from Bayesian paradigm based on shifting number of modes as stable location parameter instead of the mean. Primary research objectives are: (i.) To develop GARCH-type variants from Bayesian perspective using shifting number of modes in the marginal density of Skewed Generalized Error Distribution (SGED) to ascertain number of regimes; (ii.) SGED shall be adopted as the prior distributional random noise based on its ability to absolve and report skewedness, heteroscedasticity, and kurtosis effects. SGED shall also be adopted because of its effectiveness in modeling financial returns even when the sample size is small; (iii.) Juxtaposes model performances of all the considered GARCH-type variants with the aid of Bayesian model performance metric.

This paper is structured as follows: section 2 presents an in-depth formulation of the mixture conditional volatility models, as well as their in-depth parameter estimations from a Bayesian paradigm. Also, section 2 presents the needed algorithm for the Bayesian paradigm of the processes. Afterwards, section 3 presents the choosing of the appropriate GARCH-type orders for the considered conditional volatility models, as well as the needed algorithm. Section 4 presents the needed financial risk measures needed for appropriate interpretation. Section 5 presents deep interpretation of the shifting number of modes in the marginal density of SGED as number of mixture component processes in application to cryptocurrency coins and tokens. Also, section 5 presents an elaborate discussion of the applicative results to the conditional volatility models. In final, section 6 presents a conclusion of the Bayesian representation of the stated conditional volatility models.

2. Related Works

Manifestly, [11] collaborated with [12] to unfolded that multiple steps conditional distribution of estimates of futuristic observations from Mixture Autoregressive (MAR) process was tractable analytically. It was claimed that MAR process was a dynamicity of autoregressive processes of changing tenures or states of chronological successions of unobserved variables (otherwise known as latent variable) that could be used to select varying states or components. However, it was inferred that if a stationary process is mixed with non-stationary processes, the overall process becomes stationary.

Progressively, [13] introduced a point mixture process referred to as Location-Mixture Autoregressive (LMAR) process to capture multiple steps conditional densities. The parameter estimation technique adopted was the Expectation-Maximization (EM) algorithm and some multiple steps ahead predictions were carried-out. The LMAR process was reported to be superior in predictive performance of real-time computation when streamlined to location (mean-cluster) parameter estimation alone. Ref. [14] propounded a supposition and verifiable study of chance illation for AR processes with limited mixture of Scale Mixtures of Normal (SMN) innovation. The process involved AR processes with overall mean and mixture of mingling proportion innovation. The EM-algorithm technique was adopted and it was reported that the derivations needed for the variance-covariance matrix (Hessian matrix) could be developed simultaneously.

From Bayesian perspective, [15] described Markov-switching GARCH processes in terms of conditional variance dynamicity of time variant series. Simulation studies were carried-out, such that Maximum Likelihood (ML) and Markov Chain Monte Carlo were taken into account for estimations of involved parameters. Single-step and multi-step ahead predictions were considered for the completed conditional density in application to exchange rate and stock market return datasets. In extension, [16] extended the Gaussian Mixture Autoregressive (GMAR) to Fisher's z Mixture Autoregressive (ZMAR) process via the introduction of a four parameter Fisher's z random noise distribution and adopted Bayesian paradigm as a way of quantifying the uncertainty in the modified process. The process extension was based on mode as a stable location parameter of mixture of components of Fisher's z autoregressive processes with associated mixing weights. Markov Chain Monte Carlo (MCMC) algorithm was adopted as the parameter estimation technique such that, the process's evaluation was juxtaposed with GMAR and Student-*t* Mixture Autoregressive (TMAR) processes. Additionally, [17] propounded a flexible way to model MAR via a predictive distribution that depends on the history of the process, which accommodate asymmetry and multimodality with the use of Bayesian method. The merit of this process was that it incorporated the uncertainty in the estimated processes into the prediction, such that all the parameters were cover in the parameter space introduced. Unlike the known approaches, the MCMC approach adopted also introduced a re-labelling algorithm that deals with a posteriori label switching.

It is on this note that a robustly stable location parameter that is symmetric and skewed in nature, with the ability to contain heteroscedasticity and multimodality effects is needed to redefine time switching conditional variance processes. It is because of this need that mode as a location parameter will be adopted as an alternative stable location and shape parameters (with associated degrees of freedom for allowing more complex switching relationships, and as well to avoid over-fitting). Therefore, a Skewed-Generalized Error Distribution (SGED) with the ability to contain the pinpointed traits shall be adopted as prior random noise to drive, ameliorate, and redefine the switching mixture of conditional variance processes from a Bayesian perspective. Among the viable determinant of shifting number of regimes is the number of Bell-Curve shape densities associated to a financial data of interest, such that the mode is used as the central tendency, averages or representative values.

Notably, no effort about time regime-switching of GARCH-types have been considered to accommodate and capture distributional fluctuations, flat stretches, jumps, fluctuations, and full range shape changing predictive distributions from the dynamicity of mixture of conditional variances from a Bayesian perspective based on shifting number of modes in the marginal density of the prior distributional random noise considered. In light of this, novel mixture of conditional volatility models of GARCH-type variants of Generalized Autoregressive Conditional Heteroscedasticity (GARCH); Exponential GARCH (EGARCH); Glosten, Jagannathan, and Runkle GARCH (GJR-GARCH); and dependent variable-GARCH (TGARCH) shall be expounded from a Bayesian paradigm as mixture of components based on shifting number of modes in the marginal density of the Skewed Generalized Error Distribution (SGED) employed as the prior distributional random noise.

3. Methods

In this section, we present in-depth formulations of the mixture conditional volatility models. We also present the cognate reason for adopting SGED as the prior random noise. We structure-out the require likelihood functions need for the processes with the incorporation of latent variables. Choices of hyper-parameters that lead to setting-up of prior distributions with their corresponding posterior distributions shall be unveiled. Afterwards, Expectation-Maximization (EM) algorithm parameter estimation technique shall be used to estimate the involved parameters in the switching processes.

Mixture of conditional variance models of time series phenomena, say x_t of Generalized Autoregressive Conditional Heteroscedasticity-type models (GARCH-type models) propounded in a flexible way to model $\boldsymbol{x_t}$ that presents jumps, oscillation, heteroskedasticity, asymmetry and multimodality (See [17, 18]). However, whenever x_t is been conditioned on a regime variable, say $z_t = k$, with $\sigma_{k,t}$ available as a function of the past observations x_{t-1}, past variance $\sigma_{k,\,t-1}$ and additional regime-dependent vector of parameters, say Θ_k, then we have,

$$\sigma_{k,t} = \sigma\left(x_{t-1}, \sigma_{k,\,t-1}, \Theta_k\right). \tag{1}$$

z_t will be assumed to be sampled independently over time from a multinomial distribution with $k = \{1, \ldots, K\}$ and a vector of mixing weights $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_K)$, that is, $P[z_t = k] = \lambda_k$. The initial value of the variance recursions, that is, $\sigma_{k,1}(k = 1, \ldots, K)$ is the set of unconditional variances in each of the k-regime.

The single-regime GARCH-type models (that is, GARCH and its variants) identified are:

- The GARCH model as identified and propounded by [19] as:

$$\sigma_{k,t} \approx \alpha_{k,0} + \alpha_{k,1}x_{t-1}^2 + \eta_k\sigma_{k,t-1}. \tag{2}$$

For vectored parameter $\theta_k = (\alpha_{0,k}, \alpha_{1,\,k}, \eta_k)^T$ such that, $\alpha_{0,k} > 0$, $\alpha_{1,\,k} > 0$, and $\eta_k \geq 0$. For a non-explosive and Covariance-Stationarity Process (CSP) in each regime, $\alpha_{1,\,k} + \eta_k < 1$ for $k = \{1, \ldots, K\}$.

- The Exponential GARCH (EGARCH) model identified and propounded by [20] as:

$$\ln(\sigma_{k,t}) \approx \alpha_{k,0} + \alpha_{k,1}[|\delta_{k,t-1}| - E[|\delta_{k,t-1}|]] + \alpha_{k,2}\delta_{k,t-1} + \eta_k\ln(\sigma_{k,t-1}). \tag{3}$$

Where $E[|\delta_{k,t-1}|]$ is the distribution conditioned on k −regime vectored parameter $\theta_k = (\alpha_{0,k}, \alpha_{1,\,k}, \alpha_{2,\,k}, \eta_k)^T$. This GARCH-type of model takes into account leverage effect, such that past negative observations possessed larger influence on the conditional volatility than past positive observations of the same magnitude. For a non-explosive and Covariance-Stationarity Process (CSP) in each regime in this type of model, $\eta_k < 1$.

- The Glosten, Jagannathan, and Runkle GARCH-type model, usually refer to as GJR-GARCH model or GJR model as identified and proposed by [21] to capture asymmetry in the conditional volatility process as:

$$\sigma_{k,t} \approx \alpha_{k,0} + \left(\alpha_{k,1} + \alpha_{k,\,2}I\{x_{t-1} < 0\}\right)x_{t-1}^2 + \eta_k\sigma_{k,\,t-1}. \tag{4}$$

For vectored parameter $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \eta_k)^T$ such that $I\{\cdot\}$ is the indicator function of value one if it holds, and zero otherwise. For positivity conditions on $\alpha_{0,\,k} > 0$, $\alpha_{1,\,k} > 0$, $\alpha_{2,\,k} \geq 0$, and $\eta_k > 0$. $\alpha_{2,\,k}$ adjusts the degree of asymmetry in the conditional volatility response to past shock in k-regime. For a non-explosive and Covariance-Stationarity Process (CSP) in each regime, $\alpha_{1,k} + \alpha_{2,k}E[\delta_{k,t}^2 I\{\delta_{k,t} < 0\}] + \eta_k < 1$ for $= \{1, \ldots, K\}$.

- The TGARCH model as identified and proposed by [22] to be the situation where the conditional volatility is conceived as the dependent variable instead of the conditional variance. The model is given as:

$$\sigma_{k,t}^{1/2} \approx \alpha_{k,0} + (\alpha_{k,1}I\{x_{t-1} \geq 0\} - \alpha_{k,\,2}I\{x_{t-1} < 0\})x_{t-1} + \eta_k\sigma_{k,t-1}^{1/2}. \tag{5}$$

For vectored parameter $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \eta_k)^T$ and required positivity of $\alpha_{0,k} > 0$, $\alpha_{1,k} > 0$, $\alpha_{2,k} > 0$, and $\eta_k > 0$. For non-explosive and Covariance-Stationarity Process (CSP) in each regime,

$$\alpha_{1,k}^2 + \eta_k^2 - 2\eta_k(\alpha_{1,k} + \alpha_{2,k})E[\delta_{t,k}I\{\delta_{t,k} < 0\}] - (\alpha_{1,k}^2 - \alpha_{2,k}^2)E[\delta_{k,t}^2 I\{\delta_{k,t} < 0\}] < 1 \text{ for } k = \{1, \ldots, K\}.$$

Mixture conditional volatility models aimed at detecting outliers, break points, and changes in the behavior of a time series flexibility in a way to model data points that presents regime-switching, oscillations, heteroskedasticity, asymmetry and multimodality. Mathematically, mixture of GARCH, EGARCH, GJR-GARCH and TGARCH processes are systems of equations as follows:

$$x_t(GARCH) = \begin{cases} \sigma_{1,t} = \alpha_{1,0} + \alpha_{1,1}x_{t-1}^2 + \eta_1\sigma_{1,t-1} + \varepsilon_{t,1} & \ni \quad \lambda_1 \\ \sigma_{2,t} = \alpha_{2,0} + \alpha_{2,1}x_{t-1}^2 + \eta_2\sigma_{2,t-1} + \varepsilon_{t,2} & \ni \quad \lambda_2 \\ \vdots \qquad\qquad \vdots & \qquad \vdots \\ \sigma_{k-1,t} = \alpha_{k-1,0} + \alpha_{k-1,1}x_{t-1}^2 + \eta_{k-2}\sigma_{k-2,t-1} + \varepsilon_{t,k-2} & \ni \quad \lambda_{k-1} \\ \sigma_{k,t} = \alpha_{k,0} + \alpha_{k,1}x_{t-1}^2 + \eta_k\sigma_{k,t-1} + \varepsilon_{t,k} & \ni \quad \lambda_k \end{cases} \tag{6}$$

$$x_t(EGARCH) = \begin{cases} \ln(\sigma_{1,t}) \approx \alpha_{1,0} + \alpha_{1,1}[|\delta_{1,t-1}| - E[|\delta_{1,t-1}|]] + \alpha_{1,2}\delta_{1,t-1} + \eta_1 \ln(\sigma_{1,t-1}) + \varepsilon_{t,1} & \ni \quad \lambda_1 \\ \ln(\sigma_{2,t}) \approx \alpha_{2,0} + \alpha_{2,1}[|\delta_{2,t-1}| - E[|\delta_{2,t-1}|]] + \alpha_{2,2}\delta_{2,t-1} + \eta_2 \ln(\sigma_{2,t-1}) + \varepsilon_{t,2} & \ni \quad \lambda_2 \\ \vdots \quad \vdots \quad \vdots & \vdots \\ \ln(\sigma_{k-1,t}) \approx \alpha_{k-1,0} + \alpha_{k-1,1}[|\delta_{k-1,t-1}| - E[|\delta_{k-1,t-1}|]] + \alpha_{k-1,2}\delta_{k-1,t-1} + \eta_{k-1} \ln(\sigma_{k-1,t-1})\ \varepsilon_{t,k-1} & \ni \quad \lambda_{k-1} \\ \ln(\sigma_{k,t}) \approx \alpha_{k,0} + \alpha_{k,1}[|\delta_{k,t-1}| - E[|\delta_{k,t-1}|]] + \alpha_{k,2}\delta_{k,t-1} + \eta_k \ln(\sigma_{k,t-1}) + \varepsilon_{t,k} & \ni \quad \lambda_k \end{cases} \tag{7}$$

$$x_t(GJR) = \begin{cases} \sigma_{1,t} \approx \alpha_{1,0} + (\alpha_{1,1} + \alpha_{1,\,2} I\{x_{t-1} < 0\}) x_{t-1}^2 + \eta_1 \sigma_{1,\,t-1} + \varepsilon_{t,1} & \ni \quad \lambda_1 \\ \sigma_{2,t} \approx \alpha_{2,0} + (\alpha_{2,1} + \alpha_{2,\,2} I\{x_{t-1} < 0\}) x_{t-1}^2 + \eta_2 \sigma_{2,\,t-1} + \varepsilon_{t,2} & \ni \quad \lambda_2 \\ \vdots \quad \vdots & \vdots \\ \sigma_{k-1,t} \approx \alpha_{k-1,0} + (\alpha_{k-1,1} + \alpha_{k-1,\,2} I\{x_{t-1} < 0\}) x_{t-1}^2 + \eta_{k-1} \sigma_{k-1,\,t-1} + \varepsilon_{t,k-1} & \ni \quad \lambda_{k-1} \\ \sigma_{k,t} \approx \alpha_{k,0} + (\alpha_{k,1} + \alpha_{k,\,2} I\{x_{t-1} < 0\}) x_{t-1}^2 + \eta_k \sigma_{k,\,t-1} + \varepsilon_{t,k} & \ni \quad \lambda_k \end{cases} \tag{8}$$

$$x_t(TGARCH) = \begin{cases} \sigma_{1,t} \approx \alpha_{1,0} + (\alpha_{1,1} + \alpha_{1,\,2} I\{x_{t-1} < 0\}) x_{t-1}^2 + \eta_1 \sigma_{1,\,t-1} + \varepsilon_{t,1} & \ni \quad \lambda_1 \\ \sigma_{2,t} \approx \alpha_{2,0} + (\alpha_{2,1} + \alpha_{2,\,2} I\{x_{t-1} < 0\}) x_{t-1}^2 + \eta_2 \sigma_{2,\,t-1} + \varepsilon_{t,2} & \ni \quad \lambda_2 \\ \vdots \quad \vdots & \vdots \\ \sigma_{k-1,t} \approx \alpha_{k-1,0} + (\alpha_{k-1,1} + \alpha_{k-1,\,2} I\{x_{t-1} < 0\}) x_{t-1}^2 + \eta_{k-1} \sigma_{k-1,\,t-1} + \varepsilon_{t,k-1} & \ni \quad \lambda_{k-1} \\ \sigma_{k,t} \approx \alpha_{k,0} + (\alpha_{k,1} + \alpha_{k,\,2} I\{x_{t-1} < 0\}) x_{t-1}^2 + \eta_k \sigma_{k,\,t-1} + \varepsilon_{t,k} & \ni \quad \lambda_k \end{cases} \tag{9}$$

Respectively, where $\varepsilon_{t1}, \varepsilon_{t2}, \dots, \varepsilon_{tk}$ are the white noise whose z_t evolves around the unobserved first-order Ergodic homogeneous Markov chain as k by k transition probability matrix of say P as,

$$P = \begin{bmatrix} \lambda_{1,1} & \lambda_{1,2} & \cdots & \lambda_{1,K} \\ \lambda_{2,1} & \lambda_{2,2} & \cdots & \lambda_{2,K} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{K,1} & \lambda_{K,2} & \cdots & \lambda_{K,K} \end{bmatrix} \quad \forall\{i,j\} = \{1, \dots, K\}. \tag{10}$$

Such that $\lambda_{i,j} = P[z_t = j | z_{t-1} = 1]$ is the mixing weight of transiting from state $z_{t-1} = i$ to $z_t = j$ $\forall\ 0 < \lambda_{ij} < 1$, $\ni$ $\sum_{j=1}^{K} \lambda_{ij} = 1$ and $\forall\{i,j\} = \{1, \dots, K\}$. This implies that the chance of a given state to occur is constant over time and does not depend on the previous regime.

For time series $x_1, \dots x_n$, the likelihood functions of the mixture conditional variance models are the product of the conditional densities of GARCH, EGARCH, GJR-GARCH and TGARCH as:

$$L_{(GARCH)}(\theta_k | x_t) = \prod_{t=1}^{n} \sum_{k=1}^{K} \frac{\lambda_k}{\sigma_k} g_k\left(\frac{\sigma_{k,t}}{\sigma_k}\right) = \prod_{t=1}^{n} \sum_{k=1}^{K} \frac{\lambda_k}{\sigma_k} g_k\left(\frac{\alpha_{k,0} + \alpha_{k,1} x_{t-1}^2 + \eta_k \sigma_{k,t-1}}{\sigma_k}\right). \tag{11}$$

$$L_{(EGARCH)}(\theta_k | x_t) = \prod_{t=1}^{n} \sum_{k=1}^{K} \frac{\lambda_k}{\sigma_k} g_k\left(\frac{\ln(\sigma_{k,t})}{\sigma_k}\right) =$$
$$\prod_{t=1}^{n} \sum_{k=1}^{K} \frac{\lambda_k}{\sigma_k} g_k\left(\frac{\alpha_{k,0} + \alpha_{k,1}[|\delta_{k,t-1}| - E[|\delta_{k,t-1}|]] + \alpha_{k,2}\delta_{k,t-1} + \eta_k \ln(\sigma_{k,t-1})}{\sigma_k}\right). \tag{12}$$

$$L_{(GJR)}(\theta_k | x_t) = \prod_{t=1}^{n} \sum_{k=1}^{K} \frac{\lambda_k}{\sigma_k} g_k\left(\frac{\sigma_{k,t}}{\sigma_k}\right) = \prod_{t=1}^{n} \sum_{k=1}^{K} \frac{\lambda_k}{\sigma_k} g_k\left(\frac{\alpha_{k,0} + (\alpha_{k,1} + \alpha_{k,\,2} I\{x_{t-1} < 0\}) x_{t-1}^2 + \eta_k \sigma_{k,\,t-1}}{\sigma_k}\right). \tag{13}$$

$$L_{(TGARCH)}(\theta_k | x_t) = \prod_{t=1}^{n} \sum_{k=1}^{K} \frac{\lambda_k}{\sigma_k} g_k\left(\frac{\sigma_{k,t}^{1/2}}{\sigma_k}\right)$$
$$= \prod_{t=1}^{n} \sum_{k=1}^{K} \frac{\lambda_k}{\sigma_k} g_k\left(\frac{\alpha_{k,0} + (\alpha_{k,1} I\{x_{t-1} \geq 0\} - \alpha_{k,\,2} I\{x_{t-1} < 0\}) x_{t-1} + \eta_k \sigma_{k,t-1}^{1/2}}{\sigma_k}\right). \tag{14}$$

Where x denotes the full data $x_t = (x_1, \dots, x_n)$. σ_k is the scale parameter, such that $\tau_k = 1/_{\sigma_k^2}$ is the associated precision parameter per each regime, such that, $k = \{1, \dots, K\}$.

3.1. Reason for the Choice of SGED Random Noise

Notably, the Student-*t* distribution is embedded in Generalized Error Distribution (GED), also known as the Exponential Power Distribution (EPD), but the Skewed-Generalized Error Distribution (SGED) is notably a special case of the skewed Student-*t* distribution which includes the student-*t* distribution as another special case (See [23]). In other words, Skewed-Student-*t* distribution (SSTD) is a special and limiting case of SGED. SGED is more efficient than SSTD for modeling financial returns. In fact, it has been ascertained that SGED outperformed SSTD in terms of goodness-of-fit, MSE, MAE, and whenever the sample size is small (See [24]).

The student-*t* distribution turns Laplace, Normal, Uniform, and GED distributions when its degree of freedom v is one (1), two (2), infinite (∞) and $v > 0$ respectively. The SGED shall be of interest because of its robustness and efficiency in containing heteroskedasticity, asymmetry effects, and jumps that contributed to shocks (multimodality) in financial return.

The Probability Density Function (PDF) of SGED is given by,

$$g_{x_t} = \frac{\xi^{1-\frac{1}{\xi}}}{2\varphi} \Gamma(\tfrac{1}{\xi})^{-1} \exp(-\frac{1}{\xi} \frac{|\delta|^{\xi}}{(1+\text{sgn}(\delta)\psi)^{\xi}\varphi^{\xi}}) \,. \tag{15}$$

Where, $\delta = x_t - \mu^*$, such that, μ^* is the mode of the time series x_t. φ is the scaling parameter in relation to the standard deviation of x_t. ψ is the skewedness parameter, such that, it ranges from $-1 \le \psi \le 1$ with the aim to control the rate of descent of the density around the mode (μ^*), ξ is the kurtosis coefficient, sgn is the sign function that assumes -1 and +1 when $\mu^* < 0$ and $\mu^* > 0$ respectively. $\Gamma(\cdot)$ is the gamma function, such that, $\Gamma(a) = \int_0^\infty y^{a-1} e^{-y} dy$ for a variable y.

3.2. The Likelihood Functions and the Latent Variable Formulation of the Conditional Volatile Models

Assuming that $x_1, \dots, x_n$ are the switching time-variant observations, then the tractable likelihood functions of equation (11) to equation (14) could then be written as,

$$L_{(GARCH)}(\theta_k, \xi_k, \varphi_k, \delta_k, \mu_k^*, \psi_k | x_t) =$$
$$\prod_{t=1}^{n} \sum_{k=1}^{K} \frac{\lambda_k}{\sigma_k} \left[\frac{\xi_k^{1-\frac{1}{\xi_k}}}{2\varphi_k} \Gamma\left(\frac{1}{\xi_k}\right)^{-1} \exp\left(-\frac{\sigma_k}{\xi_k} \frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|^{\xi_k}}{(1+\text{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k\varphi_k)^{\xi_k}} \right) \right]^{z_t} . \tag{16}$$

Where $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \eta_k)^T$ and μ_k^* is the mode as the stable location parameter per each regime.

$$L_{(EGARCH)}(\theta_k, \xi_k, \varphi_k, \delta_k, \mu_k^*, \psi_k | x_t) = \prod_{t=1}^{n} \sum_{k=1}^{K} \frac{\lambda_k}{\sigma_k} [\frac{\xi_k^{1-\frac{1}{\xi_k}}}{2\varphi_k} \Gamma(\frac{1}{\xi_k})^{-1} \times$$
$$\exp(-\frac{\sigma_k}{\xi_k} \frac{|\alpha_{k,0}+\alpha_{k,1}[|\delta_{k,t-1}|-E[|\delta_{k,t-1}|]]+\alpha_{k,2}\delta_{k,t-1}+\eta_k\ln(\sigma_{k,t-1})+\delta_k+\mu_k^*|^{\xi_k}}{(1+\text{sgn}(\alpha_{k,0}+\alpha_{k,1}[|\delta_{k,t-1}|-E[|\delta_{k,t-1}|]]+\alpha_{k,2}\delta_{k,t-1}+\eta_k\ln(\sigma_{k,t-1})+\delta_k+\mu_k^*)\psi_k\varphi_k)^{\xi_k}})]^{z_t}. \tag{17}$$

Where $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \eta_k)^T$ and μ_k^*

$$L_{(GJR)}(\theta_k, \xi_k, \varphi_k, \delta_k, \mu_k^*, \psi_k | x_t) =$$
$$\prod_{t=1}^{n} \sum_{k=1}^{K} \frac{\lambda_k}{\sigma_k} [\frac{\xi_k^{1-\frac{1}{\xi_k}}}{2\varphi_k} \Gamma(\frac{1}{\xi})^{-1} \exp(-\frac{\sigma_k}{\xi_k} \frac{|\alpha_{k,0}+(\alpha_{k,1}+\alpha_{k,2}I\{x_{t-1}<0\})x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|^{\xi_k}}{(1+\text{sgn}(\alpha_{k,0}+(\alpha_{k,1}+\alpha_{k,2}I\{x_{t-1}<0\})x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k\varphi_k)^{\xi_k}})]^{z_t} \tag{18}$$

Where $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \eta_k)^T$ and μ_k^*

$$L_{(TGARCH)}(\theta_k, \xi_k, \varphi_k, \delta_k, \mu_k^*, \psi_k | x_t) = \prod_{t=1}^{n} \sum_{k=1}^{K} \frac{\lambda_k}{\sigma_k} [\frac{\xi_k^{1-\frac{1}{\xi_k}}}{2\varphi_k} \Gamma(\frac{1}{\xi_k})^{-1} \times$$
$$\exp(-\frac{\sigma_k}{\xi_k} \frac{|\alpha_{k,0}+(\alpha_{k,1}I\{x_{t-1}\ge 0\}-\alpha_{k,2}I\{x_{t-1}<0\})x_{t-1}+\eta_k\sigma_{k,t-1}^{1/2}+\delta_k+\mu_k^*|^{\xi_k}}{(1+\text{sgn}(\alpha_{k,0}+(\alpha_{k,1}I\{x_{t-1}\ge 0\}-\alpha_{k,2}I\{x_{t-1}<0\})x_{t-1}+\eta_k\sigma_{k,t-1}^{1/2}+\delta_k+\mu_k^*)\psi_k\varphi_k)^{\xi_k}})]^{z_t}. \tag{19}$$

Where $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \eta_k)^T$ and μ_k^*

This makes much easier to handle when $z_t = 1$, at each time t, because the supplemented likelihoods of equation (16) to (19) will be a product each for the equations. z_t and ξ are not usually available, so both shall be referred to as unobserved variables in the preceding sections.

3.3. Prior Set-up and Choice of Hyper-parameters for the Conditional Volatile Models

Bayesian paradigm deals with the unavailability of $z_t's$. Setting-up appropriate prior distributions for the latent variables, mixing weights, and other involved coefficients of the conditional volatile models in the absence of vital and relevant prior information might be tasking. We shall rely on the known attributes of the parameters to be estimated. Since each observation has the equal likely chance to be generated sequential to make-up mixture components, it implies that, $\lambda_1 = \cdots = \lambda_K = \frac{1}{K}$. It connotes a discrete uniform distribution of $z_t's$ that happens to be a general case of the multinomial distribution

$$\lambda_1, \lambda_2 \cdots, \lambda_k \sim Dir(\omega_1, \omega_2, \dots, \omega_k).$$

The prior distribution of each ξ is conditioned on each of z_t on the mixture component generated by observations by x_t. For a generic $z_{t,k} = 1$, the prior distribution on ξ_t is

$$\xi_t | z_t \sim Gamma(\tfrac{a_k}{2}, \tfrac{b_k - 2}{2}) \quad k = \{1, \dots, K\}. \tag{20}$$

The prior distribution on the mode components (μ_k^*) follows a normal distribution with common hyper-parameters μ (the mean) and the specified precision coefficient τ_k such that,

$$\mu_k^* \sim \aleph(\mu_k, \tau_k^{-1}) \quad \text{for} \quad k = \{1, \dots, K\}. \tag{21}$$

For the precision τ_k,

$$\tau_k \sim Gamma(\tfrac{a_k}{2}, \tfrac{\omega}{2}) \ \ k = \{1, \dots, K\}. \tag{22}$$

For a fixed ω such that,

$$\omega \sim Gamma(\tfrac{a_k}{2}, \tfrac{b_k - 2}{2}) \ \ k = \{1, \dots, K\}. \tag{23}$$

We build priors of other coefficients from independent diffuse priors are as follows:

Assuming $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \eta_k)^T$, then, $\hbar = (\xi_k, \varphi_k, \delta_k, \mu_k^*, \psi_k)^T$, $\Theta_{GARCH} = \{\theta_k, \hbar, P\}$.

Assuming $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \eta_k)^T$, then, $\hbar = (\xi_k, \varphi_k, \delta_k, \mu_k^*, \psi_k)^T$, $\Theta_{EGARCH} = \{\theta_k, \hbar, P\}$.

Assuming $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \eta_k)^T$, then, $\hbar = (\xi_k, \varphi_k, \delta_k, \mu_k^*, \psi_k)^T$, $\Theta_{GJR} = \{\theta_k, \hbar, P\}$.

Assuming $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \eta_k)^T$ then, $\hbar = (\xi_k, \varphi_k, \delta_k, \mu_k^*, \psi_k)^T$, $\Theta_{TGARCH} = \{\theta_k, \hbar, P\}$.

For GARCH:

$$g_{GARCH}(\Theta) = g(\theta_1, \hbar_1) \cdots g(\theta_K, \hbar_K) g(P) \qquad \text{for} \qquad k = \{1, \dots, K\}. \tag{24}$$

$$g_{GARCH}(\theta_k, \hbar_k) \propto g(\theta_k) g(\hbar_k) I\{(\theta_k, \hbar_k) \in CSP_k\} \quad \text{for} \qquad k = \{1, \dots, K\}. \tag{25}$$

$$g_{GARCH}(\theta_k) \propto g_n(\theta_k; \mu_{\theta_k}^*, diag(\sigma_{\theta_k}^*)) I\{\theta_k \in (\Re_{k+})\} \quad \text{for} \qquad k = \{1, \dots, K\}. \tag{26}$$

$$g_{GARCH}(\hbar_k) \propto g_n(\hbar_k;\ \mu_{\theta_k}^*, diag(\sigma_{\theta_k}^2)) I\{\hbar_{k,1} > 0, \hbar_{k,2} > 2\} \ \text{for} \qquad k = \{1, \dots, K\}. \tag{27}$$

Same manner of diffuse priors for EGARCH, GJR, and TGARCH shall be employed. $g(P) \propto \prod_{i=1}^{K} (\prod_{j=1}^{K} \lambda_{ij}) I\{0 < \lambda_{ii} < 1\}$ for all of GARCH, EGARCH, GJR, and TGARCH. CSP_k is the Covariance-Stationarity Process per each regime of the stated processes. $\Re_{k+}$ is the positivity condition placed on each of the regime.

3.4. Choice of Hyper-parameters for the Conditional Volatile Models

Specifications are needed for each of the stated hyper-parameters of $a,\ b,\ \xi,\ \omega$ and τ. It is to be noted that ω is also a hyper-parameter and shall be treated as a random variable by full specification once a and b has been correctly chosen. According to [25], if $\Re_x = \max(x) - \min(x)$ is the length of variation of the dataset, then the hyper-parameters are:

$$a = \frac{2}{10}; \omega = 2; b = \frac{100a}{\omega\Re_x^2} = \frac{10}{\Re_x^2}; \xi = \min(x) + \frac{\Re_x}{\omega}; \tau = \Re_x^{-1}.$$

Such that the peak of the Gamma distribution at $a_k > 1$ shall be used to drive the posterior distribution towards the peak. The mode of the Gamma distribution at $\mu_k^* = \frac{a_k - 1}{b_k}$ will be used to choose the peak of the SGED around the point of maximum likelihood, such that,

$$\tilde{v}_k^{EM} = \frac{a_k - 1}{b_k}. \tag{28}$$

$\tilde{v}_k^{EM}$ denotes the estimate of degrees of freedom using the EM-algorithm approach. If interested in approximately the same prior variance with degrees of freedom for all components, given the truncated nature of the prior, then we shall have,

$$s^2 = \frac{a_k}{b_k}. \tag{29}$$

3.5. Posterior Distribution of the Involved Coefficients of the Conditional Volatile Models

For GARCH:

Assuming $\phi(\cdot)$ denotes the PDF of the SGED distribution. As specified, if $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \eta_k)^T$, then, $\Theta = (\alpha_{0,k}, \alpha_{1,k}, \eta_k, \xi_k, \varphi_k, \delta_k, \mu_k^*, \psi_k)^T$ and if Θ_{-x} is the vectored parameter with the exclusion of x. The following notations shall be introduced as:

$$\varepsilon_{k,t} = \alpha_{k,0} + \alpha_{k,1}x_{t-1}^2 + \eta_k\sigma_{k,t-1};\ t = 1, \dots, n\ \ k = 1, \dots, K; n_k = \sum_{t=1}^{n} z_{tk};$$

$$\bar{\varepsilon}_k = \frac{1}{n_k}\sum_{t:z_{kt}} \varepsilon_{kt}; b_k = 1 - \sum_{i=1}^{K} \alpha_{ki}; c_k = \sum_{t:\ z_{kt}=1} \xi_t(\varepsilon_{kt} - \bar{\varepsilon}_k); d = \sum_{t:z_{kt}=1} \xi_t.$$

The posterior probability of the latent variable of observations x_t being generated from the k-component is nothing but,

$$P(z_{kt} = 1|x_t, \xi_t, \Theta) = \frac{\frac{\lambda_k}{\sigma_k} g_k\left(\frac{\varepsilon_{kt}}{\sigma_k/\sqrt{\xi_t}}\right)}{\sum_{k=1}^{K} \frac{\lambda_k}{\sigma_k} g_k\left(\frac{\varepsilon_{kt}}{\sigma_k/\sqrt{\xi_t}}\right)}. \tag{30}$$

Such that z is drawn from the multinomial distribution with it corresponding probability.

Posterior distribution of ξ_t:

For GARCH:

For $\xi_t|z_t = k \sim Gamma(\frac{a_k}{2}, \frac{b_k}{2})\quad k = \{1, \dots, K\}$.

$$P(\xi_t|x_t, z_t, \Theta) \propto \left[\frac{\xi_k^{1-\frac{1}{\xi_k}}}{2\varphi_k}\Gamma\left(\frac{1}{\xi_k}\right)^{-1}\exp\left(-\frac{\tau_k}{\xi_k}\frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|^{\xi_k}}{\left(1+\operatorname{sgn}\left(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*\right)\psi_k\varphi_k\right)^{\xi_k}}\right)\right]^{z_t}. \tag{31}$$

$$= \left(\frac{\xi_k^{1-\frac{1}{\xi_k}}}{2\varphi_k}\Gamma(\frac{1}{\xi_k})^{-1}\right)^{z_t}\exp\left(-\frac{\tau_k}{\xi_k}\frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|^{\xi_k}}{(1+\operatorname{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k\varphi_k)^{\xi_k}}\right)^{z_t}.$$

$$= \frac{\xi_k^{z_t - \frac{z_t}{\xi_t}}}{2\varphi_k^{z_t}} \frac{z_t}{\Gamma(\frac{1}{\xi_k})} \exp - \left(\frac{\tau_k^{z_t}}{\xi_k^{z_t}}\right) \exp\left(\frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|^{z_t\xi_k}}{\left(1+\text{sgn}\left(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*\right)\psi_k\varphi_k\right)^{z_t\xi_k}}\right).$$

$$\propto \frac{\xi_k^{z_t - \frac{z_t}{\xi_t}}}{\Gamma(\frac{1}{\xi_k})} \exp(\xi_k^{z_t}) \exp\left(\frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|^{z_t\xi_k}}{(1+\text{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k\varphi_k)^{z_t\xi_k}}\right) . \tag{32}$$

$$= \frac{\xi_k^{z_t(1-\frac{1}{\xi_t})}}{\Gamma(\frac{1}{\xi_k})} \exp\left(\xi_k^{z_t} + \frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|}{(1+\text{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k\varphi_k)}\right) . \tag{33}$$

Recall that,

$\Gamma(a) = \int_0^\infty y^{a-1}e^{-y}dy$, so $\Gamma(\frac{1}{a}) = \int_0^\infty y^{1-\frac{1}{a}}e^{-y}dy$, $\Gamma(\frac{1}{2}) = \sqrt{\pi} = \pi^{1/2}$; $\Gamma(\frac{3}{2}) = \frac{\sqrt{\pi}}{2} = \frac{\pi^{1/2}}{2}$. So, $\Gamma\left(\frac{1}{\xi_t}\right) = \pi^{1/\xi_t}$.

$$= \frac{1}{\pi^{1/\xi_t}} \exp\left(1 - \frac{1}{\xi_t} + \frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|}{(1+\text{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k\varphi_k)}\right).$$

$$= \pi^{-1/\xi_t}\sqrt{7}\exp\left(-\frac{1}{\xi_t} + \frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|}{(1+\text{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k\varphi_k)}\right) . \tag{34}$$

Hence,

$$\xi_t|x_t, z_t, \Theta \sim Gamma\left(\pi\sqrt{7}, \frac{|\varepsilon_t|}{(1+\text{sgn}(\varepsilon)\psi_k\varphi_k)}\right). \tag{35}$$

Where,

$$\varepsilon_t = \alpha_{k,0} + \alpha_{k,1}x_{t-1}^2 + \eta_k\sigma_{k,t-1} + \delta_k + \mu_k^* . \tag{36}$$

For EGARCH:

$$\xi_t|x_t, z_t, \Theta \sim Gamma\left(\pi\sqrt{7}, \frac{|\varepsilon_t|}{(1+\text{sgn}(\varepsilon)\psi_k\varphi_k)}\right). \tag{37}$$

Where,

$$\varepsilon_t = \alpha_{k,0} + \alpha_{k,1}\left[\left|\delta_{k,t-1}\right| - E\left[\left|\delta_{k,t-1}\right|\right]\right] + \alpha_{k,2}\delta_{k,t-1} + \eta_k\ln\left(\sigma_{k,t-1}\right) + \delta_k + \mu_k^*. \tag{38}$$

For GJR:

$$\xi_t|x_t, z_t, \Theta \sim Gamma\left(\pi\sqrt{7}, \frac{|\varepsilon_t|}{(1+\text{sgn}(\varepsilon)\psi_k\varphi_k)}\right) . \tag{39}$$

$$\varepsilon_t = \alpha_{k,0} + (\alpha_{k,1} + \alpha_{k,\,2}I\{x_{t-1} < 0\})x_{t-1}^2 + \eta_k\sigma_{k,\,t-1} + \delta_k + \mu_k^* . \tag{40}$$

For TGARCH:

$$\xi_t|x_t, z_t, \Theta \sim Gamma\left(\pi\sqrt{7}, \frac{|\varepsilon_t|}{(1+\text{sgn}(\varepsilon)\psi_k\varphi_k)}\right). \tag{41}$$

$$\varepsilon_t = \alpha_{k,0} + (\alpha_{k,1}I\{x_{t-1} \geq 0\} - \alpha_{k,\,2}I\{x_{t-1} < 0\})x_{t-1} + \eta_k\sigma_{k,t-1}^{1/2} + \delta_k + \mu_k^*. \tag{42}$$

3.6. Update of Posterior Probability of the Latent Variable

$$P(z_{kt} = 1|x_t, \xi_t, \Theta) = \frac{\frac{\lambda_k}{\sigma_k} g_k\left(\frac{\varepsilon_{kt}}{\sigma_k/\sqrt{\xi_t}}\right)}{\sum_{k=1}^{K} \frac{\lambda_k}{\sigma_k} g_k\left(\frac{\varepsilon_{kt}}{\sigma_k/\sqrt{\xi_t}}\right)}. \tag{43}$$

Such that ε_t could be any of equation (36), (38), (40), and (42) for GARCH, EGARCH, GJR, and TGARCH respectively.

Posterior distribution of λ:

The prior distribution of the mixing weight (λ) is $p(\lambda) \sim D(1, \dots, 1)$. The posterior distribution is:

$$p(\lambda|z, x) \propto \prod_{k=1}^{K} \lambda_k^{n_k}. \tag{44}$$

Therefore, $\lambda|z, x \sim D(n_1 + 1, \dots, n_K + 1)$.

Posterior distribution of μ_k^*:

For GARCH:

The posterior distribution of the mode (μ_k^*) as the **stable** location parameter could be derived as,

$$\mu_k^* \sim \aleph(\mu_k,\ \tau_k^{-1}).$$

$$P(\mu_k^*|y, \xi, z, \Theta_{-\mu_k^*}) \propto \left[\frac{\xi_k^{1-\frac{1}{\xi_k}}}{2\varphi_k}\Gamma\left(\frac{1}{\xi_k}\right)^{-1}\exp\left(-\frac{\tau_k}{\xi_k}\frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|^{\xi_k}}{(1+\mathrm{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)z_t\xi_k)^{\xi_k}}\right)\right]^{z_t}. \tag{45}$$

$$\propto \exp\left(-\frac{\tau_k}{\xi_k}\frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|^{\xi_k}}{(1+\mathrm{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k\varphi_k)^{\xi_k}}\right)^{z_t}.$$

$$= \exp\left(-\frac{\tau_k}{\xi_k}\right)^{z_t}\exp\left(\frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|^{\xi_k}}{(1+\mathrm{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k\varphi_k)^{\xi_k}}\right)^{z_t}.$$

$$\propto \exp\left(\frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|^{\xi_k}}{(1+\mathrm{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k\varphi_k)^{\xi_k}}\right)^{z_t}.$$

$$= \exp\left(\frac{|(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k)+\mu_k^*|^{z_t\xi_k}}{(1+\mathrm{sgn}((\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k)+\mu_k^*)\psi_k\varphi_k)^{z_t\xi_k}}\right).$$

$$= \exp\left(\frac{|(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k)+\mu_k^*|^{z_t\xi_k}}{(1+\mathrm{sgn}((\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k)+\mu_k^*)\psi_k\varphi_k)^{z_t\xi_k}}\right).$$

$$= \exp\left(\frac{||\mu_k^*|-|-((\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k))||^{z_t\xi_k}}{(1+\mathrm{sgn}(||\mu_k^*|-|-((\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k))||\psi_k\varphi_k)^{z_t\xi_k}}\right).$$

$$= \exp\left(\frac{\left||\mu_k^*|-|-(\varepsilon_t)|\right|^{z_t\xi_k}}{(1+\mathrm{sgn}(||\mu_k^*|-|-(\varepsilon_t)||\psi_k\varphi_k)^{z_t\xi_k}}\right).$$

$$= \exp\left(\frac{||\mu_k^*|-|(\varepsilon_t)||^{z_t\xi_k}}{(1+\mathrm{sgn}(||\mu_k^*|-|(\varepsilon_t)||\psi_k\varphi_k)^{z_t\xi_k}}\right) = \exp\left(\frac{||\mu_k^*|-|(\varepsilon_t)||^{z_t\xi_k}}{(1+\mathrm{sgn}(||\mu_k^*|-|(\varepsilon_t)||\psi_k\varphi_k)^{z_t\xi_k}}\right). \tag{46}$$

Where,

$$\varepsilon_t = (\alpha_{k,0} + \alpha_{k,1}x_{t-1}^2 + \eta_k\sigma_{k,t-1} + \delta_k).$$

Therefore, we conclude that

$$\mu_k^* \sim \aleph\left(1,\ \frac{\varepsilon_t}{(1+\mathrm{sgn}|\varepsilon_t|)\psi_k\varphi_k}\right). \tag{47}$$

$$\mu_k^* \sim \aleph(1, \varepsilon_t\, \vartheta_k^{-1}) \quad \text{where, } \vartheta = (1 + \mathrm{sgn}|\varepsilon_t|)\psi_k\varphi_k$$

Similarly for EGARCH, GJR, and TGARCH, such that their error terms ε_t and ϑ are:

$$\varepsilon_t = \alpha_{k,0} + \alpha_{k,1}[|\delta_{k,t-1}| - E[|\delta_{k,t-1}|]] + \alpha_{k,2}\delta_{k,t-1} + \eta_k \ln(\sigma_{k,t-1}) + \delta_k, \text{ where } \vartheta = (1+\text{sgn}|\varepsilon_t|)\psi_k\varphi_k;$$

$$\varepsilon_t = \alpha_{k,0} + (\alpha_{k,1} + \alpha_{k,\,2}I\{x_{t-1} < 0\})x_{t-1}^2 + \eta_k\sigma_{k,\,t-1} + \delta_k; \text{ where } \vartheta = (1+\text{sgn}|\varepsilon_t|)\psi_k\varphi_k; \text{ and}$$

$$\varepsilon_t = \alpha_{k,0} + (\alpha_{k,1}I\{x_{t-1} \geq 0\} - \alpha_{k,\,2}I\{x_{t-1} < 0\})x_{t-1} + \eta_k\sigma_{k,t-1}^{1/2} + \delta_k \text{ where } \vartheta = (1+\text{sgn}|\varepsilon_t|)\psi_k\varphi_k \text{ respectively.}$$

Posterior distribution of φ_k:

For GARCH:

$$P(\varphi_k|y,\xi,z,\Theta_{-\mu_k^*}) \propto \left[\frac{\xi_k^{1-\frac{1}{\xi_k}}}{2\varphi_k}\Gamma\left(\frac{1}{\xi_k}\right)^{-1}\exp\left(-\frac{\tau_k}{\xi_k}\frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|^{\xi_k}}{(1+\text{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k)^{\xi_k}\varphi_k^{\xi_k}}\right)\right]^{z_t}. \tag{48}$$

$$\propto \left[\frac{1}{(\varphi_k)^{z_t}}\exp\left(-\frac{\tau_k}{\xi_k}\frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|^{\xi_k}}{(1+\text{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k)^{\xi_k}\varphi_k^{\xi_k}}\right)\right]^{z_t}.$$

$$\propto \left[\frac{1}{(\varphi_k)^{z_t}}\exp\left(\frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|^{z_t\xi_k}}{(1+\text{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k)^{z_t\xi_k}}\right)\frac{1}{\left(\varphi_k^{\xi_k}\right)^{z_t}}\right].$$

$$= (\varphi_k)^{-z_t}\exp\left(\frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|^{z_t\xi_k}}{(1+\text{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k)^{z_t\xi_k}}(\varphi_k)^{-\xi_k z_t}\right).$$

$$= (\varphi_k)^{-z_t}\exp\left(\left(\frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|}{(1+\text{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k)}\right)^{z_t\xi_k}(\varphi_k)^{-\xi_k z_t}\right).$$

$$= (\varphi_k)^{-z_t}\exp\left(\left(\frac{(1+\text{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k)}{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|}\right)^{-z_t\xi_k}(\varphi_k)^{-\xi_k z_t}\right). \tag{49}$$

Hence, $\varphi_k|y,\xi,z,\Theta_{-\mu_k^*} \sim Gamma(-z_t, \frac{1+(\text{sgn}(\varepsilon_t))\psi_k}{|\varepsilon_t|})$, where $\varepsilon_t = \alpha_{k,0} + \alpha_{k,1}x_{t-1}^2 + \eta_k\sigma_{k,t-1} + \delta_k + \mu_k^*$.

For EGARCH:

$$\varphi_k|y,\xi,z,\Theta_{-\mu_k^*} \sim Gamma(-z_t, \frac{1+(\text{sgn}(\varepsilon_t))\psi_k}{|\varepsilon_t|}, \text{where} \varepsilon_t = \alpha_{k,0} + \alpha_{k,1}[|\delta_{k,t-1}| - E[|\delta_{k,t-1}|]] + \alpha_{k,2}\delta_{k,t-1} + \eta_k\ln(\sigma_{k,t-1}) + \delta_k + \mu_k^*.$$

For GJR

$$\varphi_k|y,\xi,z,\Theta_{-\mu_k^*} \sim Gamma(-z_t, \frac{1+(\text{sgn}(\varepsilon_t))\psi_k}{|\varepsilon_t|}), \text{ where } \varepsilon_t = \alpha_{k,0} + \left(\alpha_{k,1} + \alpha_{k,\,2}I\{x_{t-1} < 0\}\right)x_{t-1}^2 + \eta_k\sigma_{k,\,t-1} + \delta_k + \mu_k^*.$$

For TGARCH:

$$\varphi_k|y,\xi,z,\Theta_{-\mu_k^*} \sim Gamma(-z_t, \frac{1+(\text{sgn}(\varepsilon_t))\psi_k}{|\varepsilon_t|}), \text{ where, } \varepsilon_t = \alpha_{k,0} + (\alpha_{k,1}I\{x_{t-1} \geq 0\} - \alpha_{k,\,2}I\{x_{t-1} < 0\})x_{t-1} + \eta_k\sigma_{k,t-1}^{1/2} + \delta_k + \mu_k^*.$$

Posterior distribution of ψ_k:

$$P(\psi_k|y,\xi,z,\Theta_{-\mu_k^*}) \propto [\exp(-\frac{\tau_k}{\xi_k}\frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|^{\xi_k}}{(1+\text{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k)^{\xi_k}\varphi_k^{\xi_k}})]^{z_t}. \tag{50}$$

$$\propto \exp(\frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|^{\xi_k}}{(1+\text{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\psi_k)^{\xi_k}\varphi_k^{\xi_k}})^{z_t}.$$

$$\propto \exp\left(\frac{|\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*|^{\xi_k z_t}}{(1+\text{sgn}(\alpha_{k,0}+\alpha_{k,1}x_{t-1}^2+\eta_k\sigma_{k,t-1}+\delta_k+\mu_k^*)\varphi_k)^{z_t\xi_k}\psi_k^{z_t\xi_k}}\right).$$

Therefore, we conclude that

$$\psi_k \sim \aleph\left(1,\ \frac{|\varepsilon_t|}{1+\left(\operatorname{sgn}(\varepsilon_t)\right)\varphi_k}\right). \tag{51}$$

Such that ψ_k in EGARCH, GJR, and TGARCH could also be estimated in the same manner.

Posterior distributions of $\alpha_{k,i}$ such that $i = 1,2,\ldots,t-1$:

Random walk metropolis of $\alpha_{k,i}(k = 1,\ldots,K)$ shall be make use of. If the current chain of $\alpha_{k,i}$ possessed simulating candidate of $\alpha^*_{k,i}$ from a proposed Multivariate Normal Distribution of $(\alpha_{k,i}, \gamma_k I_{t-1})$, where γ_k is the turning parameter and I_{t-1} is the $(t-1) \times (t-1)$ identity matrix.

The $\alpha^*_{k,i}$ is then accepted with probability,

$$\mho_{GARCH}\left(\alpha_{k,i}, \alpha^*_{k,i}\right) = \min\left(1, \frac{\exp\left(-\frac{\tau_k}{\xi_k}\frac{\left|\alpha^*_{k,0}+\alpha^*_{k,1}x^2_{t-1}+\eta_k\sigma_{k,t-1}+\delta_k+\mu^*_k\right|^{\xi_k}}{\left(1+\operatorname{sgn}\left(\alpha^*_{k,0}+\alpha^*_{k,1}x^2_{t-1}+\eta_k\sigma_{k,t-1}+\delta_k+\mu^*_k\right)\psi_k\varphi_k\right)^{\xi_k}}\right)^{z_t}}{\exp\left(-\frac{\tau_k}{\xi_k}\frac{\left|\alpha_{k,0}+\alpha_{k,1}x^2_{t-1}+\eta_k\sigma_{k,t-1}+\delta_k+\mu^*_k\right|^{\xi_k}}{\left(1+\operatorname{sgn}\left(\alpha_{k,0}+\alpha_{k,1}x^2_{t-1}+\eta_k\sigma_{k,t-1}+\delta_k+\mu^*_k\right)\psi_k\varphi_k\right)^{\xi_k}}\right)^{z_t}}\right). \tag{52}$$

$i = 0,1$ & $k = 1,\ldots,K$.

$$\mho_{EGARCH}\left(\alpha_{k,i}, \alpha^*_{k,i}\right) = \min\left(1, \frac{\exp\left(-\frac{\tau_k}{\xi_k}\frac{\left|\alpha^*_{k,0}+\alpha^*_{k,1}\left[\left|\delta_{k,t-1}\right|-E\left[\left|\delta_{k,t-1}\right|\right]\right]+\alpha^*_{k,2}\delta_{k,t-1}+\eta_k\ln\left(\sigma_{k,t-1}\right)+\delta_k+\mu^*_k\right|^{\xi_k}}{\left(1+\operatorname{sgn}\left(\alpha^*_{k,0}+\alpha^*_{k,1}\left[\left|\delta_{k,t-1}\right|-E\left[\left|\delta_{k,t-1}\right|\right]\right]+\alpha^*_{k,2}\delta_{k,t-1}+\eta_k\ln\left(\sigma_{k,t-1}\right)+\delta_k+\mu^*_k\right)\psi_k\varphi_k\right)^{\xi_k}}\right)^{z_t}}{\exp\left(-\frac{\tau_k}{\xi_k}\frac{\left|\alpha_{k,0}+\alpha_{k,1}\left[\left|\delta_{k,t-1}\right|-E\left[\left|\delta_{k,t-1}\right|\right]\right]+\alpha_{k,2}\delta_{k,t-1}+\eta_k\ln\left(\sigma_{k,t-1}\right)+\delta_k+\mu^*_k\right|^{\xi_k}}{\left(1+\operatorname{sgn}\left(\alpha_{k,0}+\alpha_{k,1}\left[\left|\delta_{k,t-1}\right|-E\left[\left|\delta_{k,t-1}\right|\right]\right]+\alpha_{k,2}\delta_{k,t-1}+\eta_k\ln\left(\sigma_{k,t-1}\right)+\delta_k+\mu^*_k\right)\psi_k\varphi_k\right)^{\xi_k}}\right)^{z_t}}\right). \tag{53}$$

$i = 0,1$ & $k = 1,\ldots,K$.

$$\mho_{GJR}(\alpha_{k,i}, \alpha^*_{k,i}) = \min(1, \frac{\exp(-\frac{\tau_k}{\xi_k}\frac{|\alpha^*_{k,0}+(\alpha^*_{k,1}+\alpha^*_{k,\,2}I\{x_{t-1}<0\})x^2_{t-1}+\eta_k\sigma_{k,\,t-1}+\delta_k+\mu^*_k|^{\xi_k}}{(1+\operatorname{sgn}(\alpha^*_{k,0}+(\alpha^*_{k,1}+\alpha^*_{k,\,2}I\{x_{t-1}<0\})x^2_{t-1}+\eta_k\sigma_{k,\,t-1}+\delta_k+\mu^*_k)\psi_k\varphi_k)^{\xi_k}})^{z_t}}{\exp(-\frac{\tau_k}{\xi_k}\frac{|\alpha_{k,0}+(\alpha_{k,1}+\alpha_{k,\,2}I\{x_{t-1}<0\})x^2_{t-1}+\eta_k\sigma_{k,\,t-1}+\delta_k+\mu^*_k|^{\xi_k}}{(1+\operatorname{sgn}(\alpha_{k,0}+(\alpha_{k,1}+\alpha_{k,\,2}I\{x_{t-1}<0\})x^2_{t-1}+\eta_k\sigma_{k,\,t-1}+\delta_k+\mu^*_k)\psi_k\varphi_k)^{\xi_k}})^{z_t}}). \tag{54}$$

$i = 0,1$ & $k = 1,\ldots,K$.

$$\mho_{TGARCH}(\alpha_{k,i}, \alpha^*_{k,i}) = \min(1, \frac{\exp(-\frac{\tau_k}{\xi_k}\frac{|\alpha^*_{k,0}+(\alpha^*_{k,1}I\{x_{t-1}\geq 0\}-\alpha^*_{k,\,2}I\{x_{t-1}<0\})x_{t-1}+\eta_k\sigma^{1/2}_{k,t-1}+\delta_k+\mu^*_k|^{\xi_k}}{(1+\operatorname{sgn}(\alpha^*_{k,0}+(\alpha^*_{k,1}I\{x_{t-1}\geq 0\}-\alpha^*_{k,\,2}I\{x_{t-1}<0\})x_{t-1}+\eta_k\sigma^{1/2}_{k,t-1}+\delta_k+\mu^*_k)\psi_k\varphi_k)^{\xi_k}})^{z_t}}{\exp(-\frac{\tau_k}{\xi_k}\frac{|\alpha_{k,0}+(\alpha_{k,1}I\{x_{t-1}\geq 0\}-\alpha_{k,\,2}I\{x_{t-1}<0\})x_{t-1}+\eta_k\sigma^{1/2}_{k,t-1}+\delta_k+\mu^*_k|^{\xi_k}}{(1+\operatorname{sgn}(\alpha_{k,0}+(\alpha_{k,1}I\{x_{t-1}\geq 0\}-\alpha_{k,\,2}I\{x_{t-1}<0\})x_{t-1}+\eta_k\sigma^{1/2}_{k,t-1}+\delta_k+\mu^*_k)\psi_k\varphi_k)^{\xi_k}})^{z_t}}). \tag{55}$$

$i = 0,1$ & $k = 1,\ldots,K$.

In a similar vein, the posterior distributions of η_k and δ_k could be estimated. It is to be noted that whenever any of $\mho_{GARCH}(\alpha_{k,i}, \alpha^*_{k,i})$, $\mho_{EGARCH}(\alpha_{k,i}, \alpha^*_{k,i})$, $\mho_{GJR}(\alpha_{k,i}, \alpha^*_{k,i})$, or $\mho_{TGARCH}(\alpha_{k,i}, \alpha^*_{k,i})$ is equal to zero the candidate value shall be automatically rejected.

4. Choosing the Appropriate Volatile GARCH-Type Orders

At each iteration, one of the K-mixture components of k shall be chosen at random. If α_k is the current order of any of the GARCH-type component. Furthermore, if α_{max} is the largest possible coefficient of any of the GARCH-type order chosen arbitrarily. The idea here is to increase the order of the GARCH-type to $\alpha^*_k = \alpha_k + 1$ with probability $b(\alpha_k)$ or decreasing probability of $d(\alpha_k)$ with order $\alpha^*_k = \alpha_k - 1$. $b(\cdot)$ and $d(\cdot)$ shall assume values from $[0, 1]$ as a 1−1 mapping function satisfying $b(\alpha_{\max}) = 0$ and $d(\alpha_k) = 1 - b(\alpha_k)$ between the current and candidate process, since the only difference between the two is the addition or subtraction of the largest order of any of the GARCH-type coefficient. Therefore, its Jacobian will always be one (1).

For $\alpha^*_k = \alpha_k - 1$, the acceptance probability is the product of the likelihood and the proposed ratio is,

$$\varsigma(\alpha_k, \alpha_k^*) = \min\left\{1, \frac{g_{x_t}(x|\alpha_{k,i}^*)}{g_{x_t}(x|\alpha_{k,i})} \times \frac{b(\alpha_{k,i}^*)}{d(\alpha_{k,i})} \times \phi\left(\frac{\alpha_{k,i}^* - \alpha_{k,i}}{1/\sqrt{\gamma_k}}\right)\right\}. \tag{56}$$

For $\alpha_k^* = \alpha_k + 1$, the acceptance probability is the product of the likelihood and the proposed ratio is,

$$\varsigma(\alpha_k, \alpha_k^*) = \min\left\{1, \frac{g_{x_t}(x|\alpha_{k,i}^*)}{g_{x_t}(x|\alpha_{k,i})} \times \frac{b(\alpha_{k,i}^*)}{d(\alpha_{k,i})} \times 3\right\}. \tag{57}$$

Where the magnitude three (3) is the inverse density of $\alpha_{k,i}$ usually under uniform interval of $U(-1.5, 1.5)$ of the proposed distribution of any of the GARCH-type model. Where $g_{x_t}(x|\alpha_{k,i})$ and $g_{x_t}(x|\alpha_{k,i}^*)$ are the Multivariate Normal Distributions (MVNs) of the current chain of the $\alpha_{k,i}$ possessed by it simulating candidate of $\alpha_{k,i}^*$ respectively. Therefore, If the candidate model is not stable, that is, if $\varsigma(\alpha_k, \alpha_k^*) = 0$, the current model is retained.

4.1. Choosing the Number of Mixture Components

The appropriate and correct specification of the associated number of mixture of GARCH-type model components associated to series of interest could be worked-out via the number of curvy SGED densities associated to the regime-switching series. It can also be selected via marginal likelihood identity technique, that is, the marginal likelihood function that is conditioned on the number of mixture components used from the output extracted from the Gibbs and Metropolis-Hastings sampling as,

$$g(x_t|g) = \sum_\lambda \int g(x_t|\theta_k, \lambda, K)\, p(\theta_k, \lambda|K) d\theta_k. \tag{58}$$

From Bayes theorem of

$$p(K|x_t) \propto g(x_t|K)p(K).$$

Such that $p(K)$ is the prior distribution on K, and $g(x_t|K)$ is the marginal likelihood function.
Such that θ_k can either be $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \eta_k)^T$ or $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \eta_k)^T$.
For any value of θ_k^* and $\alpha_{i,k}^*$; and number of components K and observed data x_t, the marginal likelihood identity can be decomposed into marginal likelihood into parts that could be estimated via,

$$g(x_t|K) = \frac{g(x_t|\theta^*, \alpha_{i,k}^*, K)p(\theta^*, \alpha_{i,k}^*|K)}{p(\theta^*, \alpha_{i,k}^*|x_t, K)}. \tag{59}$$

$$= \frac{g(x_t|\theta^*, \alpha_{i,k}^*, K)p(\theta^*|\alpha_{i,k}^*, K)p(\alpha_{i,k}^*|K)}{p(\alpha_{i,k}^*|K^*, x_t, K)p(\alpha_{i,k}^*|x_t, K)}. \tag{60}$$

The only quantity not readily available in equation (60) is $p(\theta^*|\alpha_{i,k}^*, x_t, K)$. However, $p(\theta^*|\alpha_{i,k}^*, x_t, K)$ can be estimated via reduced MCMC simulations for fixed $\alpha_{i,k}^*$ via,

$$\widehat{p}\left(\theta^*\middle|\alpha_{i,k}^*, x_t, K\right) = \hat{p}\left(\theta^*\middle|x_t, \alpha_{i,k}^*, K\right). \tag{61}$$

$$\hat{p}\left(\mu_k^{**}\middle|\theta^*, x_t, \alpha_{i,k}^*, K\right). \tag{62}$$

$$\widehat{p}(\tau^*|\mu_k^{**}, \theta^*, x_t, \alpha_{ik}^*, K). \tag{63}$$

$$\hat{p}(\lambda^*|\tau^*, \mu_k^{**}, \theta^*, x_t, \alpha_{i,k}^*, K)\ . \tag{64}$$

Such that equation (61) to (64) shall be plugged into equation (60) together with the other known quantities, to obtain the marginal likelihood for the model with fixed number of K-components.

4.2. Algorithm

- For $k \leftarrow 2, \ldots, K$ do
- MCMC and determine $\lambda_1^*, \ldots, \lambda_k^*$
- Calculate $g(x_t|k)$
- Select $k^* = \max g(x_t|k),\ k = 2, \ldots, K$
- Estimate $g(\Phi|x_t k^*, \lambda^*)$

5. Financial Risk Measures

Financial risk measure in financial econometric and financial time series estimates the chance associated with an estimated loss. Formally, financial risk measures are usually defined around a class ρ of random variables in the support (0, 1) which satisfy the,

- Normalized property: $\rho(0) = 0$: that is no risk if no assets are held.
- Translative property: It says that for any real constant say υ. Then $\rho(X + \upsilon) = (X) + \upsilon$
- Monotone property: $X \leq Y \Rightarrow \rho(X) \geq \rho(Y)$. It says that if portfolio Y possessed a profitable portfolio than X, then the risk associated with portfolio Y will be lower than that associated with portfolio X.
- Positive homogeneity: If υ is the real constant. Then $\rho(cX) = c\rho(X)$
- Sub-additively: $\rho(X + Y) = \rho(X) + \rho(Y)$

Two of the most notable risk measures for assessing risk associated with a financial investment, financial returns, and assets are Value-at-risk (VaR) and Expected Shortfall (ES)

Value-at-Risk (VaR): Value-at-Risk (VaR) estimates how much an investment, asset or financial returns might lose, with a given chance ρ in a normal market condition. Formally, Var $\rho(x) = \theta$ means that an investment x is likely to result in a Loss of at least θ with probability ρ in a set time window. The VaR at level ρ is:

$$VaR_{\rho}(X) = -\inf\{x \in \Re, F_X(x) = 1 - \rho\} = -F_X^{-1}(1 - \rho) \tag{65}$$

Tells that VaR θ associated with X at level ρ is minus the $1 - \rho$ quantile of the distribution of X.

Expected Shortfall: Expected shortfall is somehow related to Value-at-Risk (VaR). ES estimates the expected loss on an investment, financial asset and financial returns, assuming that a loss larger than VaR has occurred. ES is sometimes preferred to VaR since it is more sensitive about the tails of the distribution considered, and most importantly because is a coherent risk measure. ES is associated with X at level ρ could be obtained by solving the integral:

$$ES_{\rho}(X) = \frac{1}{\rho}\int_0^{\rho} VaR_{\alpha}(X)\partial\alpha. \tag{66}$$

Numerical integration methods, such as the Monte Carlo are sometimes preferred in solving equation (66).

6. Applicative Results

Cryptocurrencies are intended for payments, transmitting value (akin to digital money) across a decentralized network of users. Cryptocurrencies are in form of blockchain-based tokens and coins meant not only to serve as different purposes from that of money, but also to serve as a means for trade by barter and asset detainment. Coins are cryptocurrencies (digital assets that can be bought or sold) that use their own independent blockchains (e:g Bitcoin), while tokens are a non-native asset that uses another blockchain's infrastructure, eg: Ethereum, Chainlink. Ten (10) noticeable and mostly used coins and tokens: Bitcoin (BTC), Bitcoin Cash (BTH), Avalanche (AVAX), Binance coin (BNB), Cardano (ADA), Chainlink (Link), Cosmos (ATOM), Filecoin (FLC), Litecoin (LTC), and Ethereum (ETH) shall be subjected to the four (4) considered conditional volatility models of GARCH, EGARCH, GJR-GARCH and TGARCH from a Bayesian. All the considered coins and tokens, there data points considered were from date of inception to 23/08/2022. All considered tokens and coins entails opening, lower, higher, closing and volume daily transactions after the conversion to USD.

Mode as a central tendency of each of the stated coins and tokens shall firstly be calculated; afterwards the density curve of each coin and tokens shall be carved-out with the use of mode as the representative value (central tendency). Progressively, each coin and tokens shall be subjected to the proposed mixture of conditional volatility of GARCH-type variants.

Tab. 1 shows the model information criteria of all the ten (10) cryptocurrency studied coins and tokens from Bayesian perspective. Deviance Information Criterion (DIC) is the widely used Bayesian model comparison index to select and estimate candidacy model as an hierarchical and complex modeling generalization of the Alkaike Information Criterion (AIC). As per Bitcoin (BTC) and Bitcoin Cash (BTH), their higher daily conversion of coins to USD in application to four conditional volatile processes from the same date of inception yielded smallest DICs of 81023.8548 and 22200.1018 apiece via GARCH volatile process with an associated seven (7) and three (3) regime-switching phenomena respectively. For Avalanche (AVAX), Binance Coin (BNB) and Cardano (ADA), it was that low-AVAX, low-BNB, and low-ADA that yielded miniature DICs of 5166.1022, 3651.1935 and 19636.5431 via a two regime-switching (for the first two) and four regime-switching of TGARCH (for low-AVAX (daily lower Avalanche price) and low-BNB (daily lower Binance coin price)) and GJR-GARCH (for low-ADA) processes respectively.

Table 1. Model selection of the bayesian conditional volatile processes of the ten (10) studied cryptocurrency coins and tokens

Crypto currency Coins	**Deviance Information Criterion (DIC)**				
	Open	**High**	**Low**	**Close**	**Volume**
Bitcoin (BTC)	**7 reg.**	**7 reg.**	**7 reg.**	**7 reg.**	**7 reg.**
GARCH	81023.8548***	81023.8548***	99939.6259	94906.4499	168262.4767
EGARCH	81023.8548***	99939.6259	94906.4499	168262.4767	44052.93
GJR-GARCH	81023.8548***	99939.6259	94906.4499	168262.4767	44052.93
TGARCH	81023.8548***	99939.6259	94906.4499	168262.4767	44052.93
Bitcoin Cash (BTH)	**3 Reg.**	**3 Reg.**	**3 Reg.**	**3 Reg.**	**3 Reg.**
GARCH	60076.4815	22200.1018***	3787988.5944	111804.924	74093.2153
EGARCH	22200.1018***	3787988.5944	111804.924	74093.2153	5377302.8193
GJR-GARCH	22200.1018***	3787988.5944	111804.924	74093.2153	5377302.8193
TGARCH	22200.1018***	3787988.5944	111804.924	74093.2153	5377302.8193
Avalanche (AVAX)	**2 Reg.**	**2 Reg.**	**2 Reg.**	**2 Reg.**	**2 Reg.**
GARCH	5853.1305	5880.0663	5703.848	5856.1292	5300.1093
EGARCH	5310.2171	5349.1191	5166.13	5329.7422	23379.9068
GJR-GARCH	5855.2103	5885.6064	5751.7122	5853.3641	23379.9068
TGARCH	5297.466	5885.6064	5166.1022***	5300.1093	23379.9068
Binance Coin (BNB)	**2 Reg.**	**2 Reg.**	**2 Reg.**	**2 Reg.**	**1 Reg.**
GARCH	4242.8131	4338.3898	4113.6838	4220.646	3782.5012
EGARCH	3770.3604	3807.8926	169762.9487	3782.0604	27760.7869
GJR-GARCH	4220.1435	4271.29	4118.0703	4513.4443	27760.7869
TGARCH	3766.6401	3816.9291	**3651.1935*****	3782.5012	27760.7869
Cardano (ADA)	**4 Reg.**	**4 Reg.**	**4 Reg.**	**4 Reg.**	**4 Reg.**
GARCH	608888.7585	271745.7083	483131.1895	608739.126	20031827.7667
EGARCH	265540.1075	1744270.7187	19636.5431	65079.4764	440523.93
GJR-GARCH	1994474.1964	3979019.1767	19636.5431***	29213.8051	440523.93
TGARCH	1074567.7183	1835800.6819	335328.2918	20031827.7667	440523.93
Chainlink (Link)	**3 Reg.**	**3 Reg.**	**3 Reg.**	**3 Reg.**	**3 Reg.**
GARCH	2084334.5301	1263721.6336	407827.2823	731420.4216	770467.3461
EGARCH	13397.783	6933879.3002	33555.3705	3711930.5333	412885.886
GJR-GARCH	152208.8144	253332.4952	33555.3705	5409.544***	412885.886
TGARCH	90013.0515	232802.5659	942312.6867	770467.3461	412885.886
Cosmos (ATOM)	**7 Reg.**	**7 Reg.**	**7 Reg.**	**7 Reg.**	**7 Reg.**
GARCH	14896.1759	14789.0279	5325.1574	43342.2563	17285.3721
EGARCH	29561.2413	5909.1884	6255.195	7892.044	440523.93
GJR-GARCH	10289.4441	6393.4267	10899.5562	11913.4015	440523.93
TGARCH	4582.5153***	14169.552	31010.0892	17285.3721	440523.93
Ethereum (ETH)	**7 Reg.**	**7 Reg.**	**7 Reg.**	**7 Reg.**	**7 Reg.**
GARCH	37755.1001***	37755.1001	65513.9151	53772.3835	52547.3036
EGARCH	37755.1001***	65513.9151	53772.3835	52547.3036	440523.93
GJR-GARCH	37755.1001***	65513.9151	53772.3835	52547.3036	440523.93
TGARCH	37755.1001***	65513.9151	53772.3835	52547.3036	440523.93
Filecoin (FIL)	**8 Reg.**	**8 Reg.**	**8 Reg.**	**8 Reg.**	**8 Reg.**
GARCH	48113.6574	56447.8819	23874.4466	25943.363	31845.1242
EGARCH	143044.6148	26783.1787	54431.0997	25396.3585	440523.93
GJR-GARCH	56447.8819	16575.8276	143044.6148	31845.1242	440523.93
TGARCH	56447.8819	16575.8276	19000.1302***	31845.1242	440523.93
Litecoin (LTC)	**5 Reg.**	**5 Reg.**	**5 Reg.**	**5 Reg.**	**5 Reg.**
GARCH	114809.7119	114809.7119	3922445.9635	8522349.5635	440523.93
EGARCH	114809.7119	3922445.9635	32499.6357	8522349.5635	440523.93
GJR-GARCH	114809.7119	3922445.9635	32495.6357***	8522349.5635	440523.93
TGARCH	114809.7119	3922445.9635	32495.6357***	8522349.5635	440523.93

Keywords: Reg. =Regime   Source: Authors' Computation (2025)

This literally implies that lower daily conversion of AVAX, BNB, ADA to USD yielded miniature DICs via TGARCH and GJR-GARCH volatile processes. As regards Chainlink (LINK) and Cosmos (ATOM); low-LINK and high-ATOM yielded smallest DICs of 5409.544 and 4582.5153 for GJR-GARCH and TGARCH processes via three (3) and seven (7) regime-switching of their series **respectively**. For Ethereum (ETH), all the four (4) volatile processes yielded a DIC of 37755.1001 apiece for open-ETH daily transactions to USD, while Filecoin (FIL) and Litecoin (LTC) yielded DICs of 19000.1302 (for TGARCH **process**) and 32495.6357 (for GJR-GARCH and TGARCH **processes**) for low-FLC and low-LTC respectively. This implies that GJR-GARCH and TGARCH processes yielded the same DIC for low-LTC. Overall, Binance Coin (BNB) yielded the best smallest optimal DIC via TGARCH process. Additionally, TGARCH and GJR-GARCH are the leading candidates' conditional volatile models among the four conditional volatile models subjected to a Bayesian paradigm.

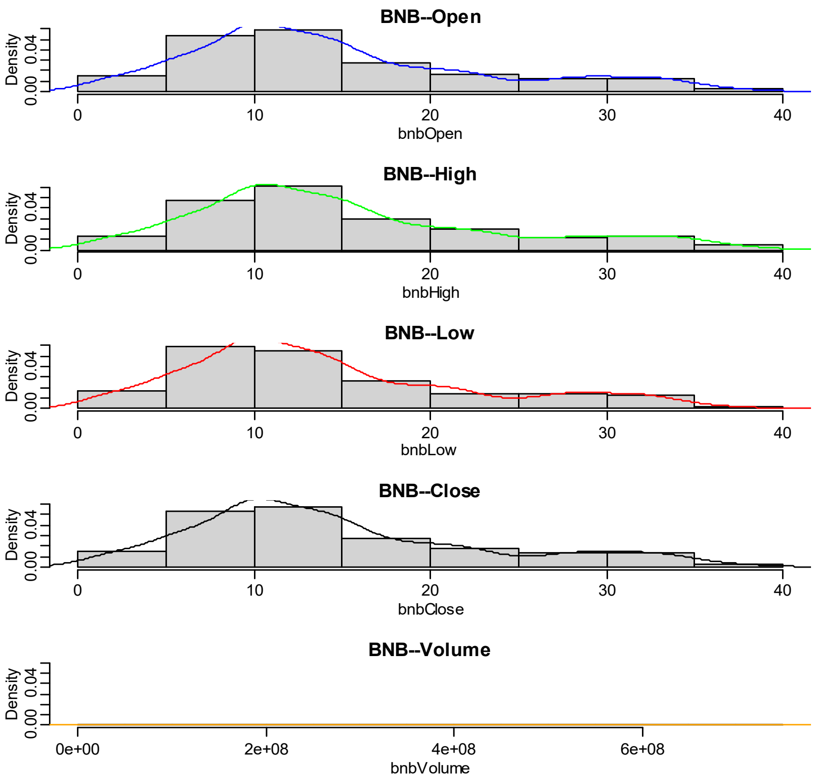

Fig.1. Histogram plots of the ten (10) cryptocurrency coins and tokens' indexes of open, high, low, close, and volume daily transacted in usd from inception

Source: Authors' Computation (2025)

From Fig. 1 above, the histogram of BNB's open, high, low, and close yielded a two-switching density curve from the mode to suggest a two-regime switching prototype jump as a confirmation to Tab. 1 assertion. BNB-volume produced a flat curvy density for the whole period of study that resulted into a unified regime. Mode was considered as the stable location parameter instead of the density mean curve, because the conditional volatile processes were conceived as mixture of components based on shifting number of modes in the marginal density of SGED. A more stable location parameter (via a symmetric and skewed contained mode location distribution) was adopted to redefine the switching time-variant of GARCH-type processes as also suggested by [26]. Inferentially, this does not only present a conditional modal distribution given the immediate past value from the "most likely mode value", but also present the merit of captivating the conditional modal distribution of the observations given the immediate past value that could either be perceived as an asymmetric or asymmetric distributed variates.

Table 2. Coefficients of the posterior mean of the volatile mtgarch (2:2, 2) process of the low-bnb coins with the most miniature and optimal dic

Coefficients	Mean	SD	SE	TSSE	RNE
$\alpha_{1,0}$	0.0178	0.0196	0.0006	0.0044	0.0195
$\alpha_{1,1}$	0.1397	0.0496	0.0016	0.0098	0.0257
$\alpha_{1,2}$	0.4997	0.2125	0.0067	0.0386	0.0302
η_1	0.1461	0.0371	0.0012	0.0074	0.0254
μ_1^*	0.7001	0.0001	0.0000	0.0000	0.0168
ξ_1	0.1989	0.0155	0.0005	0.0013	0.1481
φ_1	0.9995	0.0004	0.0000	0.0001	0.0395
$\alpha_{2,0}$	0.3517	0.5022	0.0159	0.0644	0.0609
$\alpha_{2,1}$	0.0678	0.1970	0.0062	0.0642	0.0094
$\alpha_{2,2}$	0.5163	0.1995	0.0063	0.0288	0.0481
η_2	0.1904	0.1417	0.0045	0.0561	0.0064
μ_2^*	0.7105	0.0068	0.0002	0.0009	0.0552
ξ_2	0.3424	0.0411	0.0013	0.0024	0.3019
φ_2	0.0003	0.0002	0.0000	0.0001	0.0095

Posterior mean transition matrix: DIC: 3651.1935;

$$\begin{pmatrix} 2\times 2 & t+1|\text{k}=1 & t+1|\text{k}=2 \\ t|\text{k}=1 & 0.9995 & 0.0005 \\ t|\text{k}=2 & 0.0003 & 0.9997 \end{pmatrix}$$

UncVol. 1 = 16.9666; UncVol. 2= 585.6190

Posterior mean stable probabilities: Reg.1 VaR= -13.4091 Reg. 1 ES= -62.3324

State 1 (λ_1) State 2 (λ_2) Reg. 2 VaR= -19.5175 Reg. 2 ES= -51.9592

0.3329 0.6671

Keys: Mean= Posterior mean; SD=Standard Deviation; SE=Naive Standard Error of the mean;
TSSE= Time Series Standard Error based on an estimate of the Spectral Density at zero;
RNE= Relative Numerical Efficiency= (SE/TSSE)^[2]; Unc. Vol.=Unconditional Volatility.

MTGARCH (2:2, 2) =

$$\frac{0.3329}{0.8033}\left[\frac{1}{7.343}\exp\left(-\frac{0.8033}{0.1989}\frac{\left|0.0178+\left(0.1397I\{x_{t-1}\geq 0\}-0.4997\{x_{t-1}<0\}x_{t-1}+0.1461+\sqrt{0.8033_{t-1}}+x_t\right)\right|^{0.1989}}{\left(1+\operatorname{sgn}\left(0.0178+\left(0.1397I\{x_{t-1}\geq 0\}-0.4997\{x_{t-1}<0\}x_{t-1}+0.1461+\sqrt{0.8033_{t-1}}+x_t\right)\right)0.999\right)^{0.1989}}\right)\right].+$$

$$\frac{0.6671}{1.1262}\left[\frac{1}{3.624}\exp\left(-\frac{0.6671}{1.1262}\frac{|0.3517+(0.0678I\{x_{t-1}\geq 0\}-0.5163\{x_{t-1}<0\}x_{t-1}+0.1904+\sqrt{1.1262_{t-1}}+x_t)|^{0.3424}}{(1+\operatorname{sgn}(0.3517+(0.0678I\{x_{t-1}\geq 0\}-0.5163\{x_{t-1}<0\}x_{t-1}+0.1904+\sqrt{1.1262_{t-1}}+x_t))0.9997)^{0.3424}}\right)\right].$$

Since $\delta_k = x_t - \mu_k^*$

Discussion of Results

From Tab. 2 above shows the coefficients of the posterior mean of the volatile MTGARCH (2:2, 2) (Mixture of TGARCH) process for low-BNB. Also, from Tab. 2 above, a two regime-switching was confirmed for the Bayesian conditional variance (volatilized) process of Binance Coin (BNB) with the most reduced DIC of 3651.1935 with four (4) and two (2) parameters each to the variance distributed process via TGARCH process. The process can be termed as a two-mixture TGARCH process of 2 by 2, that is, MTGARCH (2: 2, 2). Interestingly, it is to be noted that the MTGARCH (2: 2, 2) Markov chain evolves persistently over time with stable probabilities of two states centered around 33% and 66% respectively. Deductively, when the posterior mean stable probabilities of the regimes are close to one (1), the filtered volatility in each regime sharply increases. Inferentially, since the probability of the posterior mean stability in regime two (2) is 66%, it implies that the filtered volatility in regime two (2) process rapidly increases as twice as that of regime one (1). The volatility process is heterogeneous across the two regimes of the TGARCH process associated to low-BNB. The two regimes produced different unconditional volatiles of 16.9666 and 585.61901 respectively (that is, the volatility persistence in the two regimes differs) as past negative returns: $\alpha_{2,0} = 0.3517$; $\alpha_{2,1} = 0.0678$; $\alpha_{2,2} = 0.5163$. $\alpha_{1,0} + \alpha_{1,1} + \frac{1}{2}\alpha_{1,2} + \eta_1 \approx$ 0.5535 and $\alpha_{2,0} + \alpha_{2,1} + \frac{1}{2}\alpha_{2,2} + \eta_2 \approx$ 0.8681 confirms the non-explosive, positivity constraint and Covariance-Stationarity Process (CSP) in the first and second regime respectively. The first regime was attributed with low unconditional volatility of 16.96664 (while the second regime was traded with high unconditional volatility of 585.6190); the first regime was characterized by strong volatility reaction (while the second was attributed to weak volatility reaction) to past negative returns; the first regime was attributed with low persistence (while the second was attributed with high persistence) volatility process. In a generalized term, the first regime could be perceived to be flooded by market participants as interplay of "tranquil market conditions" because of these mentioned attributes, while second regime was flooded by market participants as interplay of "tumultuous market conditions" or otherwise known as the "stormy market conditions". In other words, Binance Coin (BNB) was a mixture of tranquil market and stormy market conditions. The conditional volatility for the two regimes is 7.343 and 3.624 respectively. The five-step monthly risk measure prediction ahead for VaR and ES measures at 1% and 5% risk levels computed for regime one is -13.4091 and -62.3324; while the measures for regime two are -19.5175 and -51.9592 respectively. Advantageously, Bayesian paradigm via MCMC procedures explored the joint posterior distribution of the mixture variance process models' coefficients, thus hence, avoiding convergence to local maxima commonly encountered by the Maximum Likelihood (ML) method.

Significantly, the Bayesian approach offers some additional advantages: (i.) the exact SGED distribution in the nonlinear mixture functions of the GARCH-type variants of the model coefficients was obtained at low cost simulation from the joint posterior distribution, such the parameter uncertainty was easily integrated in the forecasts through the predictive distribution; (ii.) the EM parameter estimation from the Bayesian viewpoint of random-walk Metropolis-Hastings algorithm exhibited coerced acceptance rate with excellent performance in the context of identifying mixture models.

7. Conclusions

In this article, four novel mixture conditional volatile processes of GARCH, EGARCH, GJR-GARCH and TGARCH were expounded from Bayesian paradigm with Skewed Generalized Error Distribution (SGED) as the random noise. The adopted parameter estimation technique was the Expectation-Maximization (EM)-algorithm. Mode was considered as the stable location parameter instead of the mean, because it could robustly adapt to symmetric, skewedness, heteroscedasticity and multimodality effects needed to redefine switching conditional variance processes conceived as mixture components based on shifting number of modes in the marginal density of the Skewed Generalized Error Distribution (SGED) as the prior distributional random noise.

In application to ten (10) most used cryptocurrency coins and tokens via their daily open, high, low, close and volume conversions to USD from the same inception date to 23/08/2022. Binance Coin (BNB) via its lower daily unit conversion, that is, low-BNB yielded the most reduced DIC of 3651.1935 via a two-regime process of TGARCH, that is, MTGARCH (2: 2, 2) with stable probabilities of 33% and 66% respectively. The first regime was attributed with low unconditional volatility of 16.96664, while the second regime was traded with high unconditional volatility of 585.6190. The conditional volatilities associated to the two regimes were 7.343 and 3.624 respectively. In summary, the first regime was perceived to have possessed tranquil market conditions, while the second regime was regarded to have possessed stormy market conditions.

Other Recommendations

In recommendation, the proposed mixture conditional volatility models via Bayesian paradigm could be extended to Kalman filter model as a mixture of the process driven by a Bayesian method.

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