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Noise Error Analysis in Fractal Dimension Estimation of Digital Images

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Abstract — In present paper the effect of noise and error occurring due to noise in fractal dimension of digital images has been analyzed. For this purpose, three digital images have been used which are added by Gaussian noise, salt and pepper noise and speckle noise. The fractal dimension of both noisy and non-noisy images has been estimated and corresponding error is reported in terms of RMSE. The study shows that noise affects the fractal dimension and there is an increase in fractal dimension due to noise. The average percentage error in fractal dimension has been estimated and reported as an offset for finding actual fractal dimension from noisy images.

Index Terms **—**Error analysis, fractal dimension, local fractal dimension, moving window, noisy images, noise models

I. Introduction

Fractal geometry has widely been used in digital image analysis for image classification, texture identification, object recognition, image segmentation, roughness measurement and various other applications. Study of texture is an important task using fractals [1], where texture is analysed and mapped on the basis of fractal features. Although measurement of texture is not an easy process, it could be studied using various parameters. In terms of fractals, the images might be modeled with fractals and then the fractal feature extracted could be used to explain the textural properties of the images. In this direction Pentland [1] modeled the images with the help of fractal geometry and showed that the textural properties follow the fractal behavior in digital images. Various image objects are studied on the basis of their size, shape, shade, tone, texture and other features. These features may be extracted statistically by studying the geometry of various image objects. The conventional geometry, i.e., Euclidean geometry deals with simple structures, e.g., lines, curves, circles, spheres, rectangles and other shapes while the objects experienced in practice are not simple. Since natural objects are so complex that ordinary geometry is not able to represent them accurately, a new kind of geometry was required to model them. Mandelbrot [2], discovered the fractal geometry and reported various applications of fractals in different fields of natural science.

In image analysis, fractal geometry became so famous that after its discovery it immediately took a big space. Texture analysis is another field where fractals became popular [3], [4], [5], [6], [7]. Since texture is estimated for a local neighborhood of pixels, the fractal features could easily be identified from it as fractal features are also estimated for the pixel neighborhood instead of individual pixel [5]. Other applications of fractals include face recognition, object detection and tracking in moving images and many more in digital image and video processing [8]. Further, beyond digital images, fractal based study is also in practice in remote sensing imagery and medical imaging systems [3]. In remote sensing, fractals are primarily used in analysis, classification, segmentation, feature extraction and many other applications. The main reason behind this popularity is the analogy of remotely sensed objects with the fractal objects [3]. Similarly, in medical imaging system, many image objects could easily be identified with the help of fractal geometry [5], [8].

Fractal geometry deals with specific objects called fractals. A fractal is defined to be an object having two properties, viz., self-similarity and fractal dimension [1], [2], [3]. Self-similarity means the object under consideration is exactly similar to original object when scaled down or scaled up to any level. The other property, i.e., fractal dimension is defined as a self-similar dimension denoted by D and given by

$$D = \frac{\log(N_r)}{\log(1/r)} \tag{1}$$

where N_r is the number of self-similar objects when the object is scaled down by ratio r, which may be any real number. This definition of fractal dimension is the simplest one, although various other definitions also exist. The self-similar dimension is very common, near to the self-similarity property and very easy to estimate for the practical point of view. These features justify the use of self-similarity dimension for estimation of fractal dimension of digital images.

In most cases, estimation of fractal features from digital images is application dependent and requires a keen attention [8], [9]. Fractal features include fractal dimension, self-similarity, scaling behavior, lacunarity and other features however fractal dimension is of primary interest. The fractal dimension of digital images is estimated by considering the image pixels with the pixel values oriented in 3D space. In most methods, the value of N_r is estimated as number of self-similar objects in which pixels are oriented in a similar pattern. This pattern is needed to be estimated efficiently and hence various methods evolved [3], [4], [8]. The methods seek for distribution of pixels in a predefined space and basically differ in the way how the pixels' distribution is measured. Here the notion of statistical self-similarity comes which says that digital images in which pixel values are digitized do not follow a strict self-similar behaviour, however they follow statistical self-similar property. Thus, the statistical properties of the images are studied for finding features of the digital images.

Since fractal dimension estimation methods deal with the distribution of pixels in local context, their statistical distribution matters for the study and noise plays a big role in this case. Noise alters the pixel values and thus it affects the fractal properties, i.e., fractal dimension value. Therefore, there is a strong requirement to explore the effect of noise in digital images while estimating their fractal dimension. Since fractal dimension can be estimated on a global basis or a local basis, the details are discussed in section III, corresponding effect of noise on both needs necessary investigation. The global or local fractal dimension is also application dependent and useful for various analyses, noise may play significant role in these specified applications. Noise affects the image at both global and local level and its effect may be interesting to test. In case of textural images, the effect of noise is local since the area of interest becomes local, however for non-textural images; the whole image becomes the area of interest. In such a case noise removal techniques are efficient which in fact work locally but affect globally. Once noise is removed, the images could be used for further processing. This strategy, i.e., noise removal could erase the fine details and hence the image texture [14]. Therefore noise removal cannot be applied as such when the image is subjected to textural analysis. Thus, the context global and local is significant at different places which as a consequence led to study the effect of noise at global and local level.

The paper has been organized in following sections: in Section II, noise models for digital images have been described briefly. In Section III, the fractal dimension estimation methods for digital images are explored among which triangular prism surface area method has been described in brief. Section IV covers the details of data set used in the study and Section V describes the methodology of present approach. Section VI describes the results and discussions and finally the study is concluded in section VII.

II. Noise Models For Digital Images

Noise may be defined as unwanted image components which are inherently present in images. Since noise is an inevitable part of images occurring due to various reasons, it cannot be ignored in image processing and analysis and therefore it is to be dealt efficiently [10]. One way to deal with the noise is its removal and the other to study its consequence without removing it. The general noise removal tools are smoothing, averaging and other image filtering options [14]; however the other approach, i.e., study of the objects in presence of noise also makes sense. The extent of noise affects the image visibility and thus various image features. Therefore, the study of noise and its quality are equally challenging for noisy image analysis. The noise may occur due to natural reasons, e.g., haze, visibility or due to sensor properties, e.g., camera lens or due to image transmission or processing or any other reason [11], [14]. Consequently there are various types of noise models defined [11], however in present study only 3 models have been used, viz. Gaussian noise, salt and pepper noise and speckle noise. These are very common kind of noise models and used frequently in digital image analysis.

A. Gaussian Noise

Gaussian noise is the additive noise which is the most frequently occurring noise in digital images. Gaussian noise is a part of almost all the signals since it occurs naturally and defined by the probability density function (pdf) [12], [13], [14]

$$p(z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{z-\mu}{2\sigma^2} \right) \tag{2}$$

for any random variable z with mean μ and standard deviation σ. In present study, a zero mean distribution, i.e., μ=0 has been considered.

B. Salt and Pepper Noise

Salt and pepper noise is a kind of tailed noise which refers to various processes resulting a degraded image [15], [16]. It appears as sprinkling black and white dots in the image and hence given the name. It is also called impulse noise and defined by the pdf [13], [14]

$$p(z) = \begin{cases} P_a & \text{if } z = a \\ P_b & \text{if } z = b \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where a and b are assumed to be minimum and maximum values of pixels, i.e., for gray level images a=0 and b=255 in general. When $b>a$, the gray level b appears as light dot and a appears as black dot in the image.

C. Speckle Noise

Speckle noise is a specific noise occurring in coherent light imaging which is signal dependent. It is a non-Gaussian kind of noise and consequently becomes one of the complex noise models [17], [18], [19]. Since coherent light imaging is used in laser and radar imaging, speckle noise is an inherent property of radar images and thus it has a specified area of interest in noise analysis. Contrary to the most common Gaussian noise, speckle noise is multiplicative in nature and given for any image I as $I_s=I+N\times I$ where I_s is speckled image N is uniform noise characterized by the pdf [13], [14], [20]

$$p(z)=\begin{cases}\dfrac{1}{b-a} & \text{if } a\le z\le b\\ 0 & \text{otherwise}\end{cases} \tag{4}$$

where a and b are real numbers and represent the bounds of data. The uniform distribution has been considered with a=0 and b=1 for present study.

III. Fractal Dimension Estimation Using Triangular Pristm Surface Area Method

By definition, fractal dimension is independent of scale and for a small portion of a fractal; the value of fractal dimension is same as of the original object. This fact is theoretically valid however, practically the value of fractal dimension changes at local level for statistically self-similar objects. This fact leads to estimate the fractal dimension in a local context together with the global one. Thus the fractal dimension can be estimated in a global or local way where the global fractal dimension represents the fractal dimension of the whole images and represents the overall distribution of image pixels [4]. This value is useful when the whole image is used for analysis and in this case fractal dimension represents the fractal signature of the image and mostly used when image size is small. The local fractal dimension, on the other hand, represents various image features at local level and consequently the fractal signatures of individual objects could be identified using local fractal dimension [21], [22]. In order to estimate the local fractal dimension, a moving window approach is usually followed in which the fractal dimension is estimated for this defined window. This scheme works exactly as a moving mask is used in spatial image filters. The size of window is an important issue to be dealt with, since this window represents the local neighborhood of pixels which as a consequence corresponds to the image features of various objects [4]. On the basis of window dependency, the local fractal dimension (D_w) can be estimated as

$$D_w=\frac{\log\left(N_{rw}\right)}{\log\left(\frac{1}{r}\right)} \tag{5}$$

where w represents the size of local window. The other notations are similar to those of equation (1).

In order to estimate the fractal dimension, triangular prism surface area method (TPSAM) has been used in present study. This method was proposed by Clarke [3], [4], [9], [21], [23], [24]. In this method, the image pixels are considered as height columns in 3D space where the pixel values represent the column heights. These columns are used to generate an imaginary prism in 3D with four pixels at four corners and their average value in the center. Thus, four triangular prisms are generated in 3D space and the total surface area of the four triangles at upper level is estimated. The base surface area is estimated for this particular configuration and the process is repeated for various base resolutions, starting from 2×2 pixels. The 2×2 base gives a square of size 1×1, where 1 represents the pixel distance. This base square with side as pixel distance is extended for the size 2×2, 4×4 and so on in multiple of 2. Thus, the base area varies as 1, 2, 4 and so on for which respective upper surface area is calculated. The corresponding upper surface area and base area is used to estimate the slope (s) of best fit line in a log-log plot and used to calculate fractal dimension D by the formula

$$D=2.0-s \tag{6}$$

Since total upper surface area decreases with an increase in base resolution, the value of s in above equation usually estimates to be negative and always be greater than −1. Thus the fractal dimension of surface is estimated between 2.0 and 3.0 [3], [4], [23]. The popularity of TPSAM can be seen through the fact that various researchers have used it for the study of fractal dimension from the day of its availability [3], [4], [9], [21], [24]. The method is a generalized one and can be used to estimate both global and local fractal dimension values.

There are certain issues in estimation of fractal dimension for digital images like uniqueness of the value, estimation methods, error in estimation, window size selection and few others [3], [4]. The issue of uniqueness of the fractal dimension value is a major one. The fractal dimension of two or more fractals may be same despite of their different construction and orientation [4]. TPSAM covers all the image pixels equally if considered from different image configurations, i.e., if the image is rotated by 90° or in its multiples, same value of fractal dimension is obtained [21]. The other issue is related to the methods available for fractal dimension estimation. Since various methods are now available, there is no unique or unified way which could estimate the fractal dimension of digital images accurately [3], [4], [8], [21]. All the methods have respective benefits and they can be used according to the problem requirements, like complexity and faster estimation. TPSAM is again suitable in these both factors and hence popular [3], [21]. Another issue is the outbound values of fractal dimension [4], [21]. Since the digital images are discretized images, they do not estimate the slope in log-log plot accurately in few cases leading an outbound value of D, which may be less than 2.0 or greater than 3.0. The reason for the error in estimation of the slope is discrete values of image pixels. However, this fact does not affect the study too much [3], [4], [21]. The issue of window selection while local fractal dimension is under consideration needs attention. The description of window size selection and its consequences could be found in [4]. The minimum suitable size of local window is 5×5 as described in [4] which has been followed in present study too.

IV. DATA SET USED

The data set comprises of three standard digital images namely Cameraman, Lena and Peppers which are displayed in Fig. 1 (a), (b) and (c). The standard images are grayscale images with the size 256×256 pixels. The gray values range from 0 to 255 in the images, thus the images are of 8-bit pixel depth. The single band grayscale images have been chosen intentionally since fractal dimension can be estimated for single band images only; however in multiband images, fractal dimension is needed to be estimated for each band individually.

The images are added with noise in appropriate extent to generate noisy images. The parameter for Gaussian noise is variance which is chosen as 0.001 for noise generation. The parameter for salt and pepper noise, i.e., noise density is considered as 0.01. Similarly, the parameter for speckle noise, i.e., variance is chosen to be 0.005. These values of parameters for each noise are not arbitrary and considered after a careful selection. For this purpose, various values of noise parameters are considered and applied on original images and corresponding observations are made so that the noise effect is explicitly visible in the images. The noisy images generated from first image using Gaussian, salt and pepper noise and speckle noise are shown in Fig. 2 (a), (b) and (c) respectively. Similarly, the noisy images corresponding to second and third image are shown in Fig. 3 and 4 (a)-(c) respectively.

(a)

(b)

(c)

Figure 1. (a) Cameraman image, (b) Lena, (c) Peppers

(a)

(b)

(c)

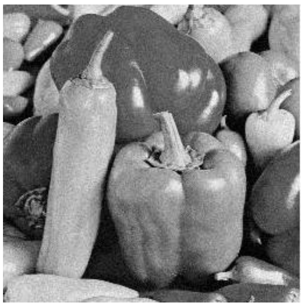
Figure 2. Noisy images of Cameraman with (a) Gaussian noise, (b) Salt and pepper noise and (c) Speckle noise

(a)

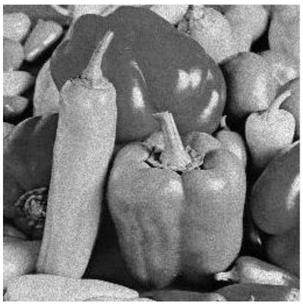
(b)

(c)

Figure 3. Noisy images of Lena with (a) Gaussian noise, (b) Salt and pepper noise and (c) Speckle noise




(a) (b)



(c)

Figure 4. Noisy images of Peppers with (a) Gaussian noise, (b) Salt and pepper noise and (c) Speckle noise

V. METHODOLOGY

The methodology of present work is straightforward. Since the input images are all grayscale images, they are used directly without any preprocessing for the estimation of fractal dimension. In first step, the value of global and local fractal dimension has been estimated for each image using TPSAM. The global fractal dimension representing the roughness of whole image is noted down for each image. This value is not much informative as far as the image roughness is concerned; however it is useful for error analysis and comparison purpose.

In second step, local fractal dimension for window size 5×5 has been estimated for each of the images. When a moving window approach is used for estimation of fractal dimension, new images could be generated with these local fractal dimension values. These images drawn with local fractal dimension values are called fractal images. The fractal images are also generated in this step.

In third step, using discussed noise models, noise is added to each image. Estimation of fractal dimension values is done in noisy image again in global and local way. The global values of fractal dimension for noisy images have been estimated and compared with those of non-noisy images and corresponding error is estimated. The local fractal dimension for noisy images for discussed window size is estimated and compared with that of non-noisy images. Corresponding error is also estimated for local fractal dimension.

In the last step error analysis is done. The error has been estimated in terms of root mean square error (RMSE) and percentage error. The RMSE, as the name indicates is the square root of mean squared error which is obtained by finding the squared error. Here the error represents the difference of actual value and estimated value. In case of present experiment, the error represents the difference of fractal dimension of noisy and non-noisy images which is obtained in previous step. The RMSE for global fractal dimension of noisy images has been estimated which is followed by the RMSE for local fractal dimension values. The percentage error for both global and local fractal dimension values has also been reported for each image.

All the steps are summarized in the flow diagram shown in Fig. 5. As the last step indicates, an offset value based on percentage error has been sought as a conclusion, which is of major concern of the study.

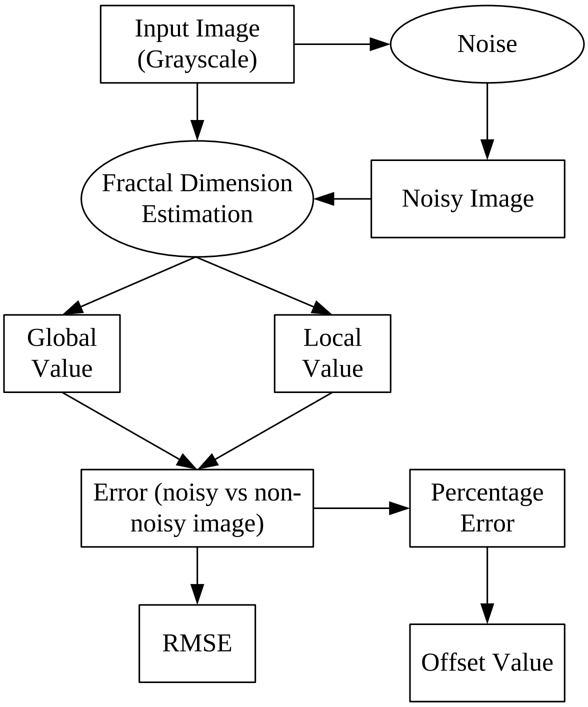
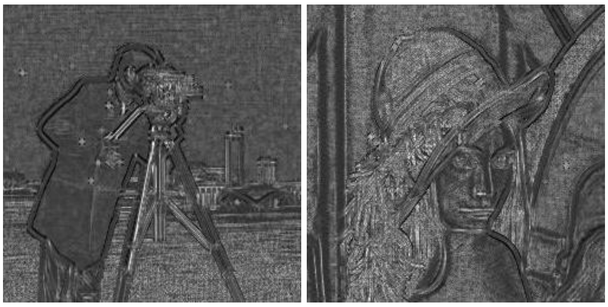


Figure 5. The Methodology of Proposed Approach

VI. RESULTS AND DISCUSSIONS

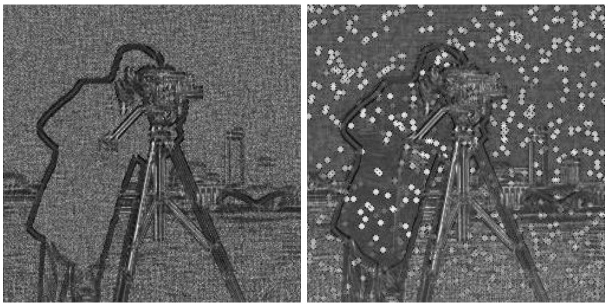
The fractal images generated by local fractal dimension values are generated for each of the images and displayed. The fractal images corresponding to the images of Fig. 1 (a)-(c) are shown in Fig. 6 (a)-(c). The fractal images of noisy images, i.e., Fig. 2, 3 and 4 (a)-(c) are shown in Fig. 7, 8 and 9 (a)-(c) respectively.



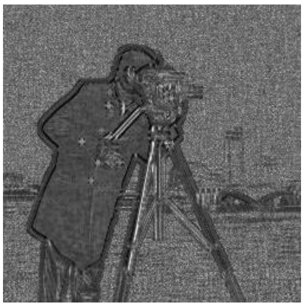
(a) (b)

(c)

Figure 6. Fractal images of Fig. 1 (a)-(c) generated with local window 5×5

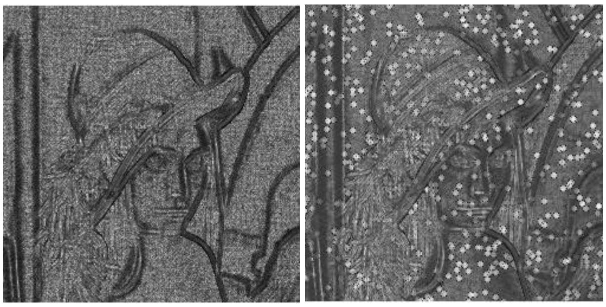


(a) (b)



(c)

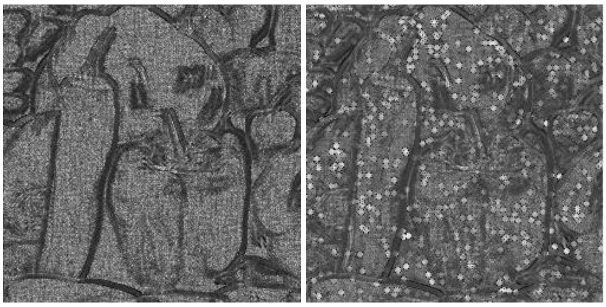
Figure 7. Fractal images of Fig. 2 (a)-(c) generated with local window 5×5



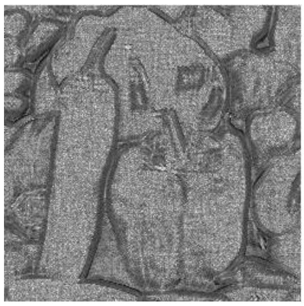
(a) (b)

(c)

Figure 8. Fractal images of Fig. 3 (a)-(c) generated with local window 5×5



(a) (b)



(c)

Figure 9. Fractal images of Fig. 4 (a)-(c) generated with local window 5×5

A visual inspection of the images also draws the attention. It can be seen in fractal images that the objects which are intermixed with neighbouring pixels are not visible in fractal images. Similarly, the edges of objects which have different pixel values as of their neighbouring pixels are highlighted in the fractal images. Although, these edges are not very sharp as obtained by conventional edge detectors, they are useful to separate various image objects from others. Another visual cue from the fractal images can be obtained for salt and pepper noise which is easy to identify in these images. Since salt and pepper noise creates the black and white dots in the images, these dots are clearly identifiable in fractal image.

The global fractal dimension for all the three images along with noisy images is listed in table 1. Corresponding percentage error and the RMSE has also been mentioned in the table. The average value of local fractal dimension using window size 5×5 for each image is listed in table 2. Since local fractal dimension is not a single value for each image, the average value has been used and reported. The effect of noise can be seen from the tables 1 and 2. Table 1 represents the values of global fractal dimension for all the images along with corresponding percentage error values. For each image with noisy image, the error has been represented together with the RMSE. A similar kind of data has been represented in table 2 where the values are calculated for local fractal dimension. It can be observed from table 1 and table 2 that the maximum value of RMSE is 0.03 for global value of fractal dimension whereas it is up to 0.14 for average value of local fractal dimension values. Since RMSE is not very high, for global fractal dimension, it can be said that the noise affects very slightly the images as far as the global fractal dimension is concerned. The RMSE for average local value is higher so the effect of noise on local level is significant.

TABLE 1. PERCENTAGE ERROR AND RMSE FOR GLOBAL VALUES OF FRACTAL DIMENSION

Image	Cameraman	Lena	Peppers
D (Global)	2.28	2.29	2.24
D (Gaussian)	2.31	2.32	2.27
% Error	1.26	0.97	1.60
D (Salt and Pepper)	2.29	2.30	2.26
% Error	0.78	0.02	0.97
D (Speckle)	2.30	2.32	2.28
% Error	0.98	1.14	1.78
RMSE	0.02	0.01	0.03

TABLE 2. PERCENTAGE ERROR AND RMSE FOR LOCAL VALUES OF AVERAGE FRACTAL DIMENSION FOR WINDOW SIZE 5×5

Image	Cameraman	Lena	Peppers
D (w=5×5)	2.24	2.25	2.26
D (Gaussian)	2.42	2.38	2.36
% Error	7.94	5.63	4.77
D (Salt and Pepper)	2.31	2.31	2.31
% Error	3.29	2.50	2.42
D (Speckle)	2.38	2.38	2.36
% Error	6.26	5.92	4.46
RMSE	0.14	0.06	0.12

The percentage error can be seen from table 1 and table 2. It is clear from the tabular data that in case of global value, the error is at most 1.78% which is for the image Peppers for speckle noise whereas it is 0.02 for Lena which is minimum. On an average, it is roughly 1% for all the images displayed in table 1. Thus the percentage error is very low for global fractal dimension. On the other hand, the percentage error is relatively high for average local fractal dimension. It is up to 7.94% for the image Cameraman for Gaussian noise. The minimum value of error for average local fractal dimension is 2.42% which is for the image Peppers for salt and pepper noise. Again, from table 2, the average value of error is 4.8%, i.e., about 5%, which is not so high.

These average values suggest that the error rate for global value of fractal dimension is about 1% whereas it is 5% for average local fractal dimension value. These values could be used as offset values for estimation of fractal dimension for noisy images. As a conclusion from table 1 and 2, it could be said that if fractal dimension is estimated for whole image, the noise effects the images by 1% on an average, i.e., if fractal dimension of noisy image is estimated, the actual value could be considered to be 1% lesser than the estimated. Similarly, it is about 5% less than that of estimated average local fractal dimension of noisy images.

VII. CONCLUSION

The present study dealt with the effect of noise in fractal dimension of digital images. The focus of the study was to analyse the error occurring due to the noise. It was observed that noise had a significant effect in fractal dimension of images. The noise changes the pixel values and hence the orientation of pixels when seen in a 3D space, i.e., the height of pixel columns gets altered. This causes a change in pixel distribution and hence the roughness of image surface. As a result the fractal dimension of the images changes due to this added noise. It is observed in the study that in general, the fractal dimension increased for noisy images as compared to non-noisy images.

Since noise caused error in actual fractal dimension, it required a careful analysis so that the extent of its consequence could be traced. Once the error rate is at hand, the noisy images could directly be used for estimation of fractal dimension without any preprocessing for noise removal. The error is tested on the basis of RMSE for both global and local fractal dimension of the images under experiment. The study showed that on an average 1% and 5% error occurred in global and local fractal dimension when noise was added to the images. Thus an offset of 1% and 5% respectively could be set to find the value of actual global and local fractal dimension. The initial results are encouraging and the study requires more attention and further investigations to generalize the results.

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