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In-depth Study of Quantum Hadamard Gate Edge Detection: Complexity Analysis, Experiments, and Future Directions

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Abstract: Quantum computing is a rapidly developing field with faster computational capabilities than classical computing. The popularity of quantum computing has reached the field of image processing, particularly with a breakthrough method known as Quantum Hadamard Edge Detection. This approach represents a significant advancement in edge detection techniques using quantum computing. Quantum Hadamard Edge Detection is a method that can detect image edges more quickly than classical methods with exponential acceleration. This paper explains the Quantum Hadamard Edge Detection method in detail, including how it is implemented, a time complexity explanation, some experiments, and future research directions. Our experiments utilize a quantum computer simulator and employ four measurement metrics: Structural Similarity Index, Figure of Merit, Entropy, and a Proposed Metric with radius-based features, to detect simple binary images, MNIST images, and the Berkeley Segmentation datasets. We recognize the potential of quantum computing and believe that image processing with quantum representation will make processing more efficient and significantly valuable in the future.

Index Terms: Image Processing, Quantum Computation, Quantum Image Processing, Quantum Edge Detection, Quantum Hadamard Edge Detection

1. Introduction

1.1. Background

Digital image processing keeps growing from year to year. These developments are matched by technological advances in various fields. In the present era, technology can process large amounts of data, which is then referred to as the era of big data. There have been many researches that use Big Data as the main focus or reference in their research [1, 2, 3]. In order to keep up with the development of Big Data, we need computers that can run large computations optimally. Accordingly, quantum computing has been a major concern in the field of computer science and academia [4, 5, 6, 7, 8]. The advantages promised by quantum computing are speed and processing capabilities that far surpass classical computing. The potential of quantum computing can solve difficult and complex problems such as large number factorization [9], molecular simulation [10], and faster random data searching [11]. Therefore, research on quantum computing is still growing until now.

Quantum computing has entered various fields, including the field of image processing which is specifically called Quantum Image Processing (QImP). Research on quantum in the field of QImP started from image representation [12, 13, 14, 15], image processing [16, 17, 18, 19], until their application as feature extraction in Quantum Machine Learning methods [20, 21, 22]. Edge detection is one of the image processing that is widely discussed, various well-known methods such as Canny, Sobel, and Prewitt edge detection are the foundation of edge detection methods in quantum representations such as Quantum Sobel [23] and Quantum Hadamard Edge Detection (QHED) [15]. We reviewed more about the work of [15]. The research discusses QImP and its application to edge detection. The QImP is described in detail, including quantum image representation, transformation, and experimental results. The researchers also explained quantum wavelet transform, image filtering, and image similarity. Specifically, they propose a new algorithm with the quantum concept known as QHED. QHED method is an edge detection that claims its computation is much faster than classical computations with an exponential speedup. In \cite{yao2017quantum}, it was concluded that the time complexity of the QHED method is $O(1)$, and the classical edge detection method is $O(2^n)$. Based on these promising observations, we discuss more in-depth about the QHED method from the method explanation, time complexity analysis, experiments, and future directions.

1.2. Related Work

In this section, we explain the related researches to the Quantum Edge Detection method. Yuan, et al proposed a quantum edge detection method in 2019 [24]. The detection method has three steps: image smoothing, gradient determination, and edge tracking. The gradient determination uses the direction of $0°, 90°, 45°,$ dan $135°$, and then the maximum gradient is selected using the Quantum Comprator. There are also classical methods that are transformed into quantum representations in order to obtain more optimal results, such as Quantum SUSAN [25]. The classical SUSAN method is limited to horizontal and vertical directions, therefore it's optimized by proposing a double-chain quantum genetic algorithm. Another example is the Marr-Hildreth method and the Robinson operator which are performed in quantum representation [26, 27]. Furthermore, besides classical methods that are transformed to quantum representations, there are methods that directly use quantum characteristics, such as QHED, which will be discussed in more depth in this paper. The QHED method was first proposed by Xi-Wei Yao, et al in 2017 [15]. In summary, the QHED method is an edge detection method that utilizes Hadamard gates as its fundamentals. A detailed explanation of this method is discussed in the next section. The development of QHED was proposed in [28]. QHED method finds the gradient at each integer index, e.g. $c_0 - c_1, c_1 - c_2, \dots, c_k - c_{k+1}$, meanwhile in [28], they proposed the gradient calculation based on the difference of an even index, i.e. $c_0 - c_2, c_1 - c_3, c_2 - c_4, \dots, c_k - c_{k+2}$. According to this idea, the QHED method can still be modified as further research. The next discussion is a more in-depth explanation of QHED.

2. Methods

Quantum Hadamard Edge Detection (QHED) is an image edge detection method that utilizes quantum characteristics. QHED works like edge detection in general, which is to find the gradient of the image pixels. The gradient is searched using Hadamard gate $\boldsymbol{H}$ such that there is a difference between image pixels. We briefly review the definition of a Hadamard gate, which is a gate that transforms a qubit $|0\rangle$ into $(|0\rangle + |1\rangle)/\sqrt{2}$ and $|1\rangle$ into $(|0\rangle - |1\rangle)/\sqrt{2}$. Therefore, the Hadamard gate is actually a matrix as follows:

$$\boldsymbol{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{1}$$

Furthermore, we divide into three parts, Quantum Probability Image Encoding as input image of QHED method, a simple QHED method, and QHED development by utilizing auxiliary qubit.

2.1. Quantum Probability Image Encoding

Quantum Probability Image Encoding (QPIE) is one of the quantum image representations that are also proposed by paper [15]. QPIE works by encoding image pixels with probability amplitudes. In general, the number of qubits needed to convert N pixels is $n = \lceil \log_2 N \rceil$. Suppose there is a binary image $\boldsymbol{B}$ with size $N_1 \times N_2$, then the quantum representation of the image is $|I\rangle$,

$$|I\rangle = \sum_{i=0}^{2^n-1} c_i |i\rangle, \tag{2}$$

where c_i is the normalized pixel intensity such that the square sum of all probability amplitudes is one. If we transform the image $\boldsymbol{B}$ into a vector of

$$\boldsymbol{B}' = \left(\boldsymbol{B}'_0, \boldsymbol{B}'_1, \dots, \boldsymbol{B}'_{\{N1\times N_2\}}\right) = \left(\boldsymbol{B}(0,0), \dots, \boldsymbol{B}(0, N_2), \boldsymbol{B}(1,0), \dots, \boldsymbol{B}(N_1, N_2)\right),$$

then the value of c_i is

$$c_i = \frac{\boldsymbol{B}_i'}{\sqrt{\sum(\boldsymbol{B}_i')^2}}. \tag{3}$$

2.2. Simple Quantum Hadamard Edge Detection

Based on (2), we get the quantum image $|I\rangle = (c_0, c_1, c_2, \dots, c_{2^n-1})^T$ and want to retrieve the gradient of image pixels i.e. $c_0 - c_1, c_1 - c_2, \dots, c_k - c_{k+1}$. In quantum computing, values such as $c_k - c_{k+1}$ cannot be obtained directly. Instead, it must be calculated through the design of a specific sequence of quantum gates that allow the extraction of such information indirectly through quantum operations. Simplistically, we can find the pixel gradient of image $|I\rangle$ by transforming Hadamard gate $\boldsymbol{H}$ into

$$\mathbf{I}^{\otimes(n-1)} \otimes \boldsymbol{H} = \begin{pmatrix} 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 1 & -1 \end{pmatrix}, \tag{4}$$

where $\mathbf{I}$ is the identity matrix with size 2×2, such that we get

$$\left(\mathbf{I}^{\otimes(n-1)} \otimes \boldsymbol{H}\right)|I\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} c_0 + c_1 \\ c_0 - c_1 \\ c_2 + c_3 \\ c_2 - c_3 \\ \vdots \\ c_{2^n-1} + c_{2^n-1} \\ c_{2^n-1} - c_{2^n-1} \end{pmatrix}. \tag{5}$$

In (5), the gradients of image pixels $c_k - c_{k+1}$ with k even are obtained. Edge detection with these gradients is not optimal, because there are gradients that are not calculated, especially when k is odd. Therefore, QHED was further developed with a modification by utilizing an auxiliary qubit.

2.3. Quantum Hadamard Edge Detection with Auxiliary Qubit

As explained in the previous subsection, QHED needs improvement such that $c_k - c_{k+1}$ is obtained for both even and odd k. Hence, $|I\rangle$ must first be modified to $|I'\rangle$ as follows:

$$|I'\rangle = |I\rangle \otimes (\boldsymbol{H}|0\rangle) = \frac{1}{\sqrt{2}}(c_0, c_0, c_1, c_1, \dots, c_{2^n-1}, c_{2^n-1})^T, \tag{6}$$

with $\boldsymbol{H}|0\rangle$ means Hadamard gate $\boldsymbol{H}$ applied to auxiliary qubit $|0\rangle$. Quantum amplitude permutation is required in order to make $|I'\rangle$ change into $(c_0, c_1, c_1, c_2, \dots, c_{2^n-1}, c_0)^T/\sqrt{2}$, that process uses a gate amplitude permutation $\boldsymbol{D}$ of size $2^{n+1} \times 2^{n+1}$ with matrix as follows:

$$\boldsymbol{D} = \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}, \tag{7}$$

Applying the same procedure as in (5), we get

$$(\mathbf{I}^{\otimes n} \otimes \boldsymbol{H})(\boldsymbol{D}|I'\rangle) = \frac{1}{2} \begin{pmatrix} c_0 + c_1 \\ c_0 - c_1 \\ c_1 + c_2 \\ c_1 - c_2 \\ \vdots \\ c_{2^n-1} + c_0 \\ c_{2^n-1} - c_0 \end{pmatrix}. \tag{8}$$

At this point, $c_k - c_{k+1}$ with k even or odd has been obtained. However, we highlight that the work described is the process of horizontal gradient determination, hence it is necessary to perform a vertical gradient determination to get the overall edge detection. To detect vertical gradients, the method involves transposing the image before converting it to a quantum representation. This transposition process converts the vertical axis into a horizontal axis, allowing the same QHED process to be used without additional modifications to the quantum circuit. After edge detection is performed on the transposed image, the result is then transposed back to the original orientation to be consistent with the initial coordinate system. In this way, both horizontal and vertical gradients can be extracted separately and then combined to obtain two-dimensional edge detection results. The quantum circuit of the QHED method with auxiliary qubit is shown in Fig. 1.

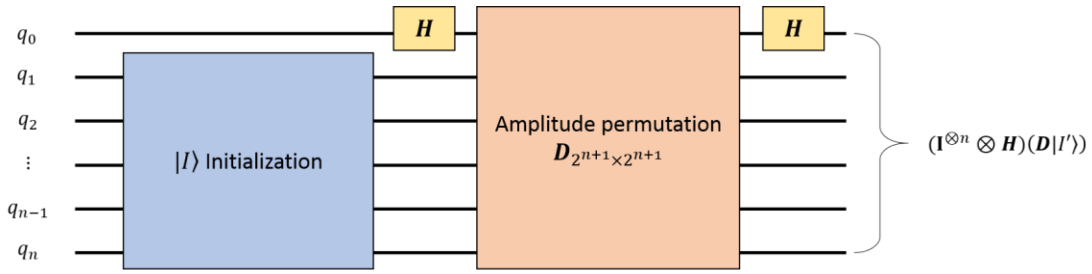


Fig. 1. Quantum circuit of QHED method with auxiliary qubit. The input image is termed as $|I\rangle$, an initial qubit is denoted by q_i with a value of $|0\rangle$, amplitude permutation is expressed by matrix $\boldsymbol{D}$ with dimension $2^{n+1} \times 2^{n+1}$, and Hadamard gate is represented by $\boldsymbol{H}$. Quantum circuit simulated using state vector backend to extract pixel-based edge indication. As the simulation does not involve real quantum measurement, uncertainty from probabilistic measurement is not considered at this stage.

3. Experimental Preparation

3.1. Test Images

We used various images to evaluate the QHED's performance as an edge detection method. First, we employed a simple black and white image that we created as an initial visual inspection as shown in Fig. 2, and then we also used an image sourced from the MNIST dataset [29], a digit dataset which is still widely used as data processing [30], as an additional visual inspection conclusion. We use the Berkeley Segmentation Dataset (BSDS500) [31] as the main test image to analyze the QHED performance. The BSDS500 dataset contains more than 100 images accompanied by ground truth images, which makes it suitable for use as an edge detection object. However, since the QHED method is limited to simple inputs, we used 100 test images with segmentation images as input and the corresponding ground truth images as a comparison.

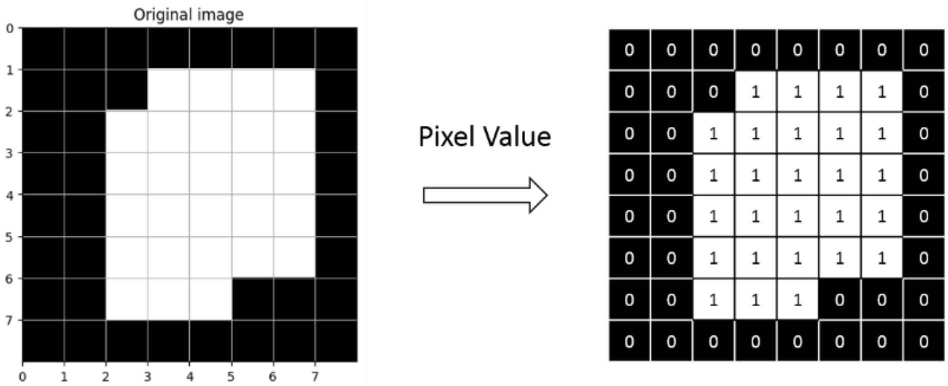


Fig. 2. Simple Black and White Binary Image Example.

3.2. Metrics

The survey paper [32] states that three categories of assessment methods can be used to measure edge detection quality, i.e., subjective evaluation based on human subjectivity results, quality assessment of reference images complete with ground truth information, and image quality analysis without reference methods such as time complexity. We use these three categories to measure QHED's edge detection performance. The first category is a preliminary inference step to determine the quality of edge detection by visual inspection. The images we used are our simple image and the MNIST dataset. The second category was used as measured results to benchmark the QHED method on the Berkeley segmentation image. The last category serves as a final and comprehensive comparison of the algorithm's quality, as shown by the time complexity analysis. We refer to the edge detection quality assessment in [32], there are more than 10 metrics available to use. However, we consider that some metrics such as Peak Signal to Noise Ratio (PSNR), Intersection Over Union (IoU), Receiver Operating Characteristic (ROC), and Confusion Matrix-based measures are not suitable for quality assessment. The reason is the quality of these metrics must be precise at each location and if there are edge detections that differ in pixel position, it will make the value worse. We conclude that there are four metrics that are suitable for edge detection, i.e. Structural Similarity Index (SSIM), Figure of Merit (FOM), Entropy, and the metric we propose in this paper which is inspired by Buffer Analysis Method. A brief explanation is as follows:

a) Structural Similarity Index

Structural Similarity Index (SSIM) is a quality assessment method by comparing the ground truth image I_{gt} and the edge detection result I_{ed} based on the luminance value, contrast, and structure. The SSIM value simply follows the following formula:

$$\mathrm{SSIM}(I_{gt}, I_{ed}) = \frac{\left(2\mu_{I_{gt}}\mu_{I_{ed}}+C_1\right)\left(2\sigma_{I_{gt}I_{ed}}+C_2\right)}{\left(\mu_{I_{gt}}^2+\mu_{I_{ed}}^2+C_1\right)\left(\sigma_{I_{gt}}^2+\sigma_{I_{ed}}^2+C_2\right)}, \tag{9}$$

where $\mu_{(\cdot)}$, $\sigma_{(\cdot)}^2$, $\sigma_{(\cdot)(\cdot)}$, C_i denoting the mean, variance, covariance, and stabilizer variables.

b) Figure of Merit

Figure of Merit (FOM) is a well-known method that is widely used to evaluate the edge detection method's performance. FOM works by comparing I_{gt} and I_{ed} based on scale α to find out d, the distance between the two images. FOM is described as:

$$\mathrm{FOM}(I_{gt}, I_{ed}) = \frac{1}{\max\{E(I_{gt}),E(I_{ed})\}}\sum_i \frac{1}{1+\alpha\cdot d_i^2}, \tag{10}$$

where $E(\cdot)$ denotes the function to calculate the number of detected edges.

c) Entropy

We use Shannon Entropy to measure the diversity and complexity of elements in a certain distribution. The larger $SE(I_{ed})$ value, the more information is obtained. Information from entropy can be obtained with the following formula:

$$\mathrm{SE}(I_{ed}) = -\sum_{i\in\mathcal{I}(I_{ed})} p_i \log_2(p_i), \tag{11}$$

where $\mathcal{I}(I_{ed})$ is the set of intensities at I_{ed} and p_i is the pixel repetition rate with intensity i.

d) Proposed Metric

We propose a simple and more efficient assessment metric for the quality of edge detection algorithms. This metric assesses the edges co-located with the ground truth pixels and the edges surrounding the ground truth. Based on the surrounding radius concept, we reference the Buffer Analysis Method to measure the actual edge [33]. We proposed the idea that if an edge is detected around radius r from the ground truth edge, the detected edge represents the same edge. We prefer this assessment, as it can handle the difference in edge location without being precise in pixel location. We call this metric True Positive Rate Based on Radius (TPRBR), presented in Algorithm 1. If $r = 1$, then TPRBR is equal to TPR, often called Recall in Confusion Matrix-based measurements.

Algorithm 1 True Positive Rate Based on Radius

Input: ground truth I_{gt}, edge detection I_{ed}, and radius r
Output: TPRBR value
1: $edgeCount \leftarrow 0$, $totalEdges \leftarrow E(I_{gt})$
2: **for** each edge e in I_{gt} **do**
3:     **if** any edge exists in the r-neighborhood of e-location in I_{ed} **then**
4:         $edgeCount \leftarrow edgeCount + 1$
5:     **end if**
6: **end for**
7: TPRBR $\leftarrow edgeCount/totalEdges$

4. Results

4.1. Experimental Results

Quantum computers are currently not as widely available as classical computers. One easy way to implement quantum computing theory is to use quantum computer simulator. We try to detect the edges of a simple black and white image as an early test, the image size is 8×8 as shown in Fig. 2. The edge of the image is detected with QHED algorithm such that horizontal edge and vertical edge detection is obtained. Combination of two results led to the edge detection as shown in Fig. 3. Total qubits required are $\log_2(8^2) + 1 = 7$, which has been calculated with one auxiliary qubit. Our visual inspection of the QHED detection results shows that the results are pretty good and in line with expectations.

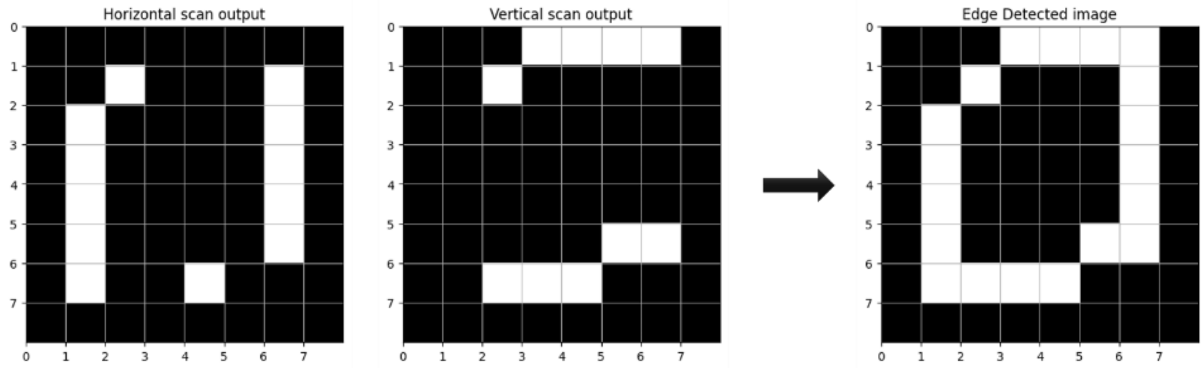


Fig. 3. Edge detection results using QHED for a simple black and white image, from left to right are the results of horizontal, vertical, and combined edge detection.

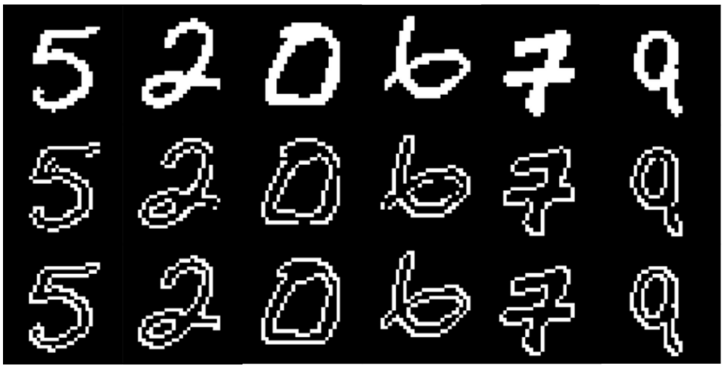


Fig. 4. Edge detection results using QHED for a binary conversion image of size 32×32. According to the image from top to bottom (original image, Canny edge detection, QHED method), there is a slight difference in the obtained results, the QHED method is better able to detect image edges at certain pixels. If we compare the computational speed, then QHED has better computation and is supported by its complexity analysis.

Subsequently, experiments are conducted on a grayscale MNIST image that is converted to binary images. We resize all images to 32×32, so the total qubits required are 11 (10 qubits and one auxiliary qubit). We also performed edge detection using the Canny method for additional comparison. The results of the experiments are shown in Fig. 4. The QHED edge detection in Fig. 4 is relatively good results when compared to the Canny method. The difference is that some boundary pixels are detected in the QHED method, giving the QHED method a better visual inspection. We also conducted a trial for grayscale images, the results obtained were not as good as detection in binary images, instead several detection results were unreadable. We can still detect the result if we can choose the optimal threshold, considering normalized pixel values in (3) will significantly impact the result in (8) for grayscale images. Moreover, because we use probabilistic-based quantum computing, the decision of the threshold to transform into a binary image must be precise. Therefore, we emphasize the binary image as the experimental input. We also conducted experiments for large images, but we explain further in the discussion section. Our early conclusion subjectively suggests that the QHED method is already an excellent one for edge detection of binary images by visual inspection, and the results can even be better than well-known detection methods such as Canny.

The main experiment in this research is assessed by four measurement metrics: SSIM, FOM, entropy, and TPRBR. The Berkeley Segmentation image used as input has the smallest size of 321×481, while the quantum computer simulator needs an image size of $2^m \times 2^m, m \in \mathbb{Z}^+$ with an m-limit according to the device's capabilities. Hence, we resize the input and ground truth images to 64×64 (Note that resizing the ground truth image might remove the edges; thus, applying the dilation morphology operation is necessary before resizing the image. Obviously, not all pixels are well covered, but the dilation process severely reduces edge loss). The segmentation image in BSDS500 that we used has a maximum intensity of fewer than 64, which makes it more complex than the binary MNIST image but still suitable as an input to the QHED method. However, the segmentation image is not bordered by zero pixels as in the simple black-and-white or MNIST image, so we put zero pixels around the border of our edge-detected image with the QHED method to reduce detection errors. The average measurement metrics of the 100 segmented test images are shown in Table 1. Based on the experimental results according to Table 1, the QHED method outperforms other edge detection methods with almost all metrics. The QHED result difference is 4.27% compared to the Canny method at zero-radius TPBR. Overall, the TPBR value of zero-radius in all methods is inadequate because when the radius is zero, it is equal to recall evaluation, which requires pixel location precision to achieve high results. This result is matched by strong similarity and a small FOM value. Further analysis is shown in Fig. 5 - Fig. 7.

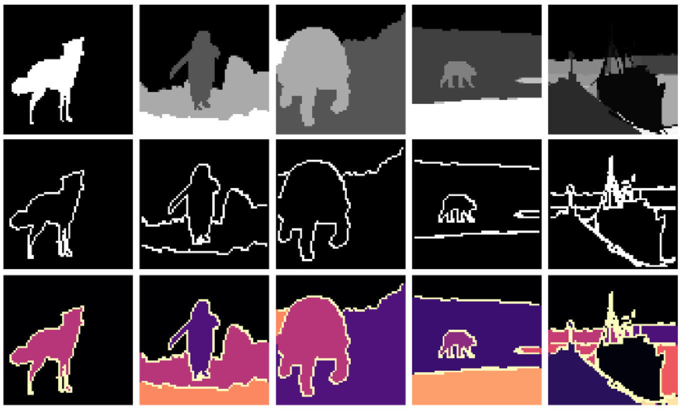


Fig. 5. Sample edge detection result of the test image variation. According to the image from top to bottom: Resized segmentation image, QHED edge detection result, and stacked edges in the segmentation image (white edges).

Table 1. Comparison of edge detection results with four measurement metrics

Method	**TPRBR** ($r = 0$)	**TPRBR** ($r = 1$)	**TPRBR** ($r = 2$)	**SSIM**	**Entropy**	**FOM**
Prewitt	0.288316	0.454095	0.557095	0.997041	0.259899	0.007476
Sobel	0.468328	0.622174	0.702448	0.997352	0.335403	0.007078
Canny	**0.547749**	0.984893	0.996366	0.997138	0.377983	0.006817
QHED	0.524329	**0.992089**	**0.997443**	**0.998194**	**0.449145**	**0.005498**

Method	**TPRBR** ($r = 0$)	**TPRBR** ($r = 1$)	**TPRBR** ($r = 2$)	**SSIM**	**Entropy**	**FOM**
Prewitt	0.288316	0.454095	0.557095	0.997041	0.259899	0.007476
Sobel	0.468328	0.622174	0.702448	0.997352	0.335403	0.007078
Canny	**0.547749**	0.984893	0.996366	0.997138	0.377983	0.006817
QHED	0.524329	**0.992089**	**0.997443**	**0.998194**	**0.449145**	**0.005498**

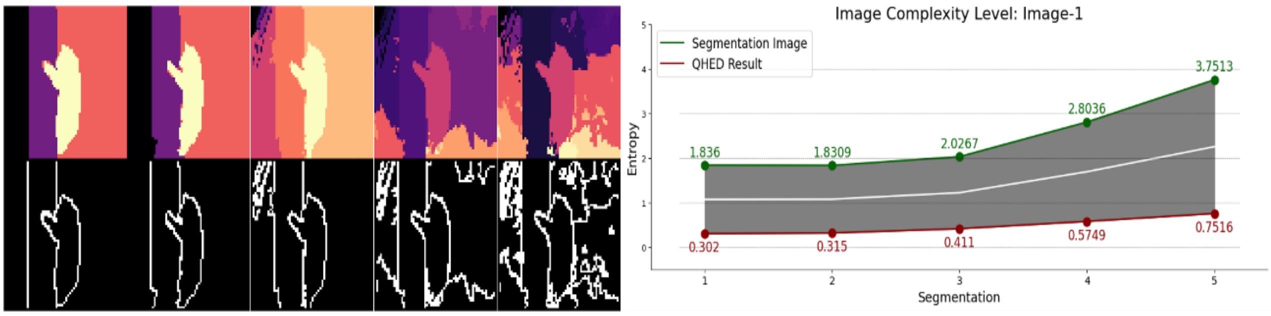


Fig. 6. Sample edge detection results of segmentation variations and corresponding entropy.

In Fig. 5, there are five examples of segmentation images as input and the edge detection results. In the presented image, we can see that the edges of the image are detected very well. Each object, such as the wolf, penguin, and bear, is clearly and precisely rendered. Each object's detailed contours and shapes can be seen clearly, indicating the high quality of edge detection. In this image, the edges are very accurate and match the object's original shape. For example, in the wolf image, the body's and fur edges are well detected. Similarly, the edges of the distinguishing contours are sharp in the image of a penguin and environs. Therefore, the QHED edge detection results by visual inspection have covered the image segmentation perfectly, confirming that a good detection should be followed by a suitable quality assessment. The entropy value in Fig. 6 is semi-linearly related to the image being processed. A high-level image can be identified by its larger entropy value, indicating higher complexity in the image. In addition, the resulting image edge is also proportional to the entropy of the input image; the greater the entropy of the image, the greater the entropy of the edge. In Fig. 7, we can see the comparison between the maximum and minimum values of TPRBR. A difference between the maximum and minimum values indicates uncertainty or variability. However, the results obtained are relatively good for each test image; the overall average TPBR is 0.992089, which suggests that the edges produced are as expected. Based on the impressive visual inspection results in Fig. 5 and the expected performance metric values in Table 1, we conclude the QHED method is a great edge detection method and relevant to classical computer detection results.

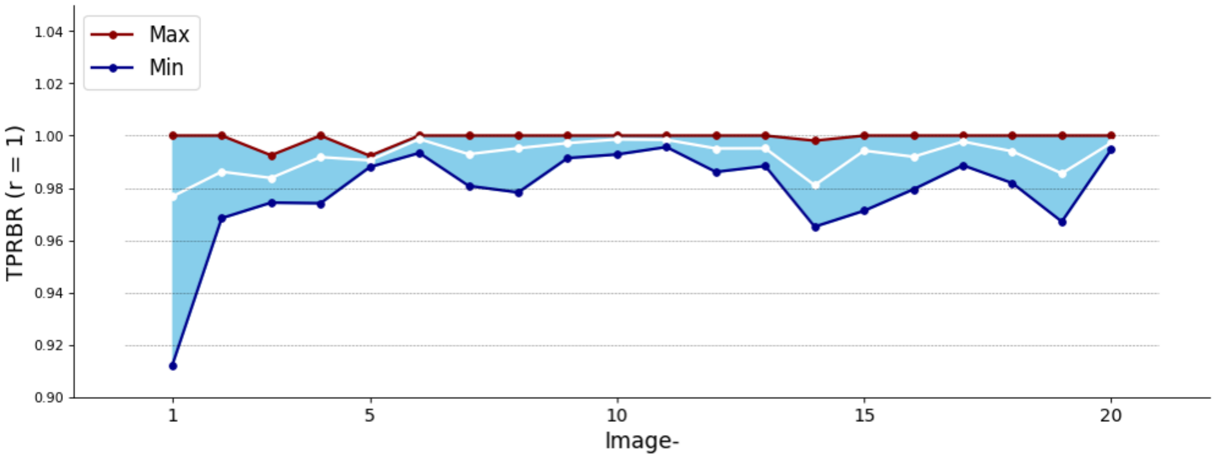


Fig. 7. TPRBR value for each image with 20 images and five segmentation variations in each image.

4.2. Time Complexity Analysis

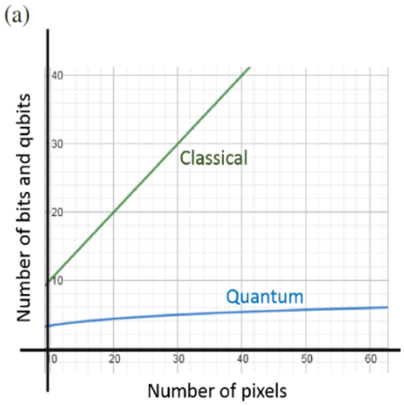
According to every introduction in the quantum paper, one of the highlights of quantum computing is its computational speed. However, we cannot directly compare quantum computing speed with classical due to device limitations. Therefore, all researches always emphasize time complexity to determine the performance of quantum algorithms that have been created and compared with classical algorithms. We refer to [15], the complexity of QHED is $O(1)$ independent of the image size and better than classical algorithms which have an average complexity of $O(2^n)$. For an image with dimensions $N = N_1 \times N_2$, a total of $n = \lceil \log_2 N \rceil$ qubits are required. Hence, the complexity of the classical edge detection algorithm is $O(N) = O(2^n)$. The QHED algorithm has a complexity of $O(1)$ since it only requires one Hadamard gate qubit in its implementation. If a auxiliary qubit are used, then the addition of further gates will also count in complexity $O(1)$. We aim to emphasize again why this result can be obtained. First, we need to realize that the complexity calculation can be based on the number of basic operations used. For example, if the basic operation used in classical computing is addition, then for an algorithm that has M addition operations, the complexity is $O(M)$. In [13, 14, 23], it was explained that the complexity calculation in the quantum algorithm is based on the number of simple gates used. Hence, the basic operation used is the matrix multiplication operation associated with gate utilization. However, compared to Grover's algorithm [11], to find data from N sequence data, the oracle multiplication operation and Grover diffusion operator are performed $\pi\sqrt{\mathrm{N}}/4$ times. Therefore, the complexity of Grover's algorithm is $O(\sqrt{\mathrm{N}})$.

The basic operation used is the matrix multiplication operation, the matrix multiplication operation in quantum is assumed to have significant computation with the addition operation in classical computing. To be clear, if we reconsider the existing classical algorithms, then the average complexity calculation uses the multiplication operation as the basic operation. This can be done because in the modern era, there are already devices that can perform multiplication operations as fast as addition operations on past devices. Therefore, if a quantum computer is a computer that can perform matrix multiplication operations quickly and is considered equivalent to basic operations on classics, then we can compare computational speed based on the complexity of existing algorithms.

We must consider specific processes in the complexity analysis of the QHED algorithm, particularly the complexity of image representation and amplitude permutation. The QPIE image representation has the worst complexity $O(n^2)$ [34] this can happen because it requires n-qubits with each qubit computed n times, refer to (3). Moreover, the amplitude permutation requires complexity $O(\mathrm{poly}(n))$ [35]. However, we suppose that if QHED has a complexity of $O(1)$ because it uses one Hadamard gate qubit, then it can also be interpreted that the matrix multiplication operation in (5) is a basic operation that has a significant speed. Therefore, the amplitude permutation $\boldsymbol{D}|I'\rangle$ should have complexity $O(1)$. In summary, if we neglect the complexity before QHED method and strictly focus on the edge detection process, the complexity of QHED is $O(1)$ and has an exponential speedup when compared to the complexity of the classical algorithm as $O(2^n)$. We summarize the time complexity explanation with a comparison chart in Fig. 8.

5. Discussion

We recognize that the development of quantum computing must be followed by the device-owned. There are several problems that will arise when we use a simulator for quantum computing, such as inability to process very large images, relatively longer computation time, and finding a suitable simulator. Currently, our devices at home are able to process large image data, so it will be a problem if the quantum concept is expected to solve problems very quickly but cannot be realized due to device limitations. Let's take an example of how to deal with a large image, suppose 256×256. The total qubits used are 17. Therefore, if we use a simulator to implement QHED method, we need an amplitude permutation matrix $\boldsymbol{D}$ with size 131072×131072. The size of the matrix $\boldsymbol{D}$ is huge and impossible to compute using the simulator. Hence, a little trick is needed to detect the edges of large images in order to run QHED in the simulator. We can simply separate the 256×256 image into a collection of smaller images, then perform edge detection for each image, and recombine them back into a single image. However, this method also needs to be clarified on how it is implemented, for example, if we divide the image in a simple way as shown in Fig. 9, then the edge detection result is not optimal. This can happen because the sub-image boundaries are treated as edges when using (8). There are several suggestions that could be improved, such as non-universal cropping or adding zero padding for each sub-image. Another big problem with QHED method is its image representation rather than QPIE representation. We recognize that the conversion process from 2D to 1D image representation in the form of quantum state with QPIE may produce artifacts at the image boundaries. This is a common challenge in quantum image computing and is one of the focuses of future development.



(b)

	QPIE	Amplitude Permutation	Edge Detection
QHED	$O(n^2)$	$O(\text{poly}(n^2))$	$O(1)$
Classical	-	-	$O(2^n)$

Fig. 8. (a) The resource comparison of the number of bits or qubits to the number of pixels N. (b) The time cost shown in the time complexity form with $n = \lceil \log_2 N \rceil$.

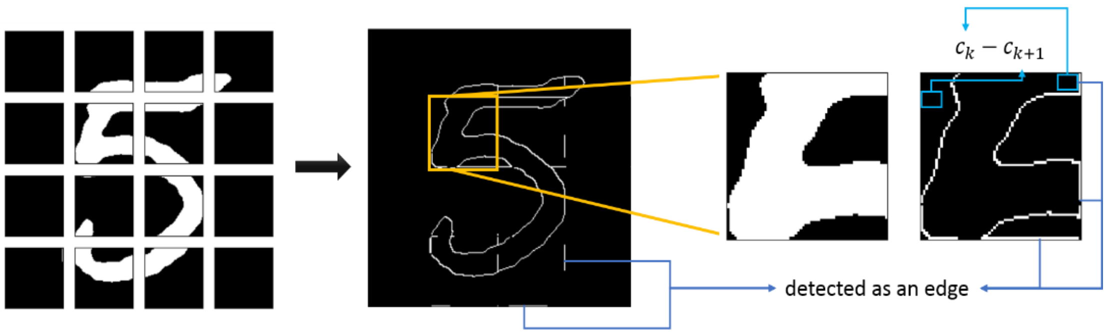


Fig. 9. Detection of a 256×256 image by splitting the image into several sub-images and concatenating the results. There are some detection errors due to the gradient between sub-image boundaries not being zero, hence a line is detected at some sub-pixel locations.

We recognize other existing image representations such as Flexible Representation for Quantum Images (FRQI) [13], and Novel Enhanced Quantum Representation (NEQR) [14]. The FRQI representation is an image representation such that the image $|I\rangle$ can be written as

$$|I\rangle = \frac{1}{2^n} \sum_{i=0}^{2^{2n}-1} (\cos(\theta_1)\,|0\rangle + \sin(\theta_i)\,|1\rangle) \otimes |i\rangle\,, \tag{12}$$

and the NEQR representation can be constructed as

$$|I\rangle = \frac{1}{2^n} \sum_{i=0}^{2^{2n}-1} \sum_{j=0}^{2^{2n}-1} |I(i,j)\rangle |ij\rangle\,, \tag{13}$$

The main issue with FRQI is its accuracy, this is because the intensity in this representation is based on an angular value approximation. Therefore, the QHED method on FRQI representation has a greater possibility of error. Meanwhile, the NEQR method requires more qubits. According to the discussion in the previous problem, the QHED method requires more qubits to detect edges of large images. Hence, the computational power will be a problem in the NEQR representation. Similar challenges could also exist with other quantum representations, requiring specific research on edge detection in each quantum representation.

We recognize one main problem with the QHED method. The problem is the conversion process of the quantum representation in (2) or (6). Converting an image of size $2^m \times 2^m$ to 1×2^{2m} for $m \in \mathbb{Z}^+$ makes the image boundaries detected as edges. For example, in Fig. 9, the sub-image boundaries are detected as edges and affect the final result. Therefore, we need to preprocess the input image or modify the QHED method where the quantum representation does not make the QHED method miss detect the image boundary. We suggest adding padding or converting the boundary into the same pixels. We suggest optimizing the $\boldsymbol{D}$ amplitude permutation gate in (7) by utilizing other quantum gates. Further discussion is on the implementation of the QHED method. QHED's primary key is utilizing the Hadamard gate in (1). The gates can be applied to quantum computers to make the computational process much faster. However, according to the quantum image representation, the matrix multiplication procedure, and the gate utilizations, the edge detection in (8) can supposedly be implemented on a classical computer with a similar matrix size. We haven't tested this idea because a classical computer shouldn't be able to run it for bigger image sizes. If quantum computers are widely used in the future, the current implementation of QHED will be much more efficient as a future edge detection method. One crucial aspect to consider in a real implementation is the development of a hybrid interface between classical and quantum systems. A hybrid approach allows certain processing stages, such as data preprocessing, segmentation, or result interpretation, to remain classical. At the same time, the central part of edge detection is maintained in the quantum domain. This approach can improve processing efficiency while keeping the potential benefits of quantum computing.

6. Conclusion

This paper comprehensively studies the Quantum Hadamard Edge Detection (QHED) method, including method description, time complexity, and experiments. The QHED method utilizes the Quantum Probability Image Encoding representation, which has a complexity square of the total qubit. The QHED method can detect image edges with constant time complexity without limiting the image size. In that case, the QHED edge detection method has a high-speed computation with exponential speedup compared to classical computing, as proven by time complexity. The QHED method is a superior approach for edge detection, as confirmed by time complexity, visual inspection, and performance metrics on experimental results. However, this method is still limited to binary images to achieve optimal performance. Hence, in future research, we plan to develop this method for more general applications, including color images. We also aim to modify the QHED method by referencing classical detection methods, such as Canny, Prewitt, and Sobel, to explore encoding techniques that better preserve spatial locality and reduce the error in transforming the quantum representation. Therefore, it is possible to obtain a classical image edge method that is more efficient when applied in quantum computing.

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References

[1] Mihai Andronie, George Lăzăroiu, Mariana Iatagan, Iulian Hurloiu, Roxana Ştefănescu, Adrian Dijmărescu, and Irina Dijmărescu. “Big data management algorithms, deep learning-based object detection technologies, and geospatial simulation and sensor fusion tools in the internet of robotic things”. *ISPRS International Journal of Geo- Information*, 12(2):35, 2023.

[2] Mihai Andronie, George Lăzăroiu, Oana Ludmila Karabolevski, Roxana Ştefănescu, Iulian Hurloiu, Adrian Dijmărescu, and Irina Dijmărescu. “Remote big data management tools, sensing and computing technologies, and visual perception and environment mapping algorithms in the internet of robotic things”. *Electronics*, 12(1):22, 2022.

[3] Sjoukje A Osinga, Dilli Paudel, Spiros A Mouzakitis, and Ioannis N Athanasiadis. “Big data in agriculture: Between opportunity and solution”. *Agricultural Systems*, 195:103298, 2022.

[4] Gui-Long Jiang,Wen-Qiang Liu, and Hai-RuiWei. “Optimal synthesis of general multi-qutrit quantum computation”. *arXiv preprint arXiv*:2310.11996, 2023.

[5] Xiaozhou Pan, Pengtao Song, and Yvonne Y Gao. “Continuous-variable quantum computation in circuit qed”. *Chinese Physics Letters*, 2023.

[6] Warit Asavanant, Akito Kawasaki, Ryuhoh Ide, Takumi Suzuki, Hector Brunel, Baramee Charoensombutamon, Atsushi Sakaguchi, Kosuke Fukui, Takahiro Kashiwazaki, Asuka Inoue, et al. “Continuous-variable quantum computation with optical quantum entanglement and quantum teleportation in time domain”. In *Quantum Communications and Quantum Imaging XXI*, volume 12692, page 1269202. SPIE, 2023.

[7] Raquel Pérez-Antón, Alberto Corbi, José Ignacio López-Sánchez, and Daniel Burgos. “Reliability of ibm’s public quantum computers”. 2023.
[8] Raquel Pérez-Antón, José Ignacio López-Sánchez, and Alberto Corbi. “The game theory in quantum computers: A review”. 2023.
[9] Salvatore M Giampaolo, Gerardo Adesso, and Fabrizio Illuminati. “Theory of ground state factorization in quantum cooperative systems”. *Physical review letters*, 100(19):197201, 2008.
[10] Adam M Kaufman and Kang-Kuen Ni. “Quantum science with optical tweezer arrays of ultracold atoms and molecules”. *Nature Physics*, 17(12):1324–1333, 2021.
[11] Lov K Grover. “A fast quantum mechanical algorithm for database search”. In *Proceedings of the twenty-eighth annual ACM symposium on Theory of computing*, pages 212–219, 1996.
[12] Salvador E Venegas-Andraca and Sougato Bose. “Storing, processing, and retrieving an image using quantum mechanics”. In *Quantum information and computation*, volume 5105, pages 137–147. SPIE, 2003.
[13] Phuc Q Le, Fangyan Dong, and Kaoru Hirota. “A flexible representation of quantum images for polynomial preparation, image compression, and processing operations”. *Quantum Information Processing*, 10:63–84, 2011.
[14] Yu-Guang Yang, Ju Tian, He Lei, Yi-Hua Zhou, and Wei-Min Shi. “Novel quantum image encryption using onedimensional quantum cellular automata”. *Information Sciences*, 345:257–270, 2016.
[15] Xi-Wei Yao, Hengyan Wang, Zeyang Liao, Ming-Cheng Chen, Jian Pan, Jun Li, Kechao Zhang, Xingcheng Lin, Zhehui Wang, Zhihuang Luo, et al. “Quantum image processing and its application to edge detection: theory and experiment”. *Physical Review X*, 7(3):031041, 2017.
[16] Meina Yu, Mohsin Hassan Saeed, Shuaifeng Zhang, Huiyun Wei, Yanzi Gao, Cheng Zou, Lanying Zhang, and Huai Yang. “Luminescence enhancement, encapsulation, and patterning of quantum dots toward display applications”. *Advanced Functional Materials*, 32(13):2109472, 2022.
[17] Randy Kuang and Maria Perepechaenko. “Quantum encryption with quantum permutation pad in ibmq systems”. *EPJ Quantum Technology*, 9(1):26, 2022.
[18] Lu Wang, Zhiliang Deng, and Wenjie Liu. “An improved two-threshold quantum segmentation algorithm for neqr image”. *Quantum Information Processing*, 21(8):302, 2022.
[19] Sreetama Das, Jingfu Zhang, Stefano Martina, Dieter Suter, and Filippo Caruso. “Quantum pattern recognition on real quantum processing units”. *Quantum Machine Intelligence*, 5(1):16, 2023.
[20] Patrick Rebentrost, Masoud Mohseni, and Seth Lloyd. “Quantum support vector machine for big data classification”. *Physical review letters*, 113(13):130503, 2014.
[21] Iris Cong, Soonwon Choi, and Mikhail D Lukin. “Quantum convolutional neural networks”. *Nature Physics*, 15(12):1273–1278, 2019.
[22] Afrad Basheer, A Afham, and Sandeep K Goyal. “Quantum k-nearest neighbors algorithm”. *arXiv preprint arXiv*:2003.09187, 2020.
[23] Yi Zhang, Kai Lu, and YingHui Gao. “Qsobel: a novel quantum image edge extraction algorithm”. *Science China Information Sciences*, 58:1–13, 2015.
[24] Suzhen Yuan, Salvador E Venegas-Andraca, Yuchan Wang, Yuan Luo, and Xuefeng Mao. “Quantum image edge detection algorithm”. *International Journal of Theoretical Physics*, 58:2823–2833, 2019.
[25] Chenyi Wu, Fei Huang, Jingyi Dai, and Nanrun Zhou. “Quantum susan edge detection based on double chains quantum genetic algorithm”. *Physica A: Statistical Mechanics and its Applications*, 605:128017, 2022.
[26] Sanjay Chakraborty, Soharab Hossain Shaikh, Amlan Chakrabarti, and Ranjan Ghosh. “Quantum image edge extraction based on classical robinson operator”. *Multimedia Tools and Applications*, 81(23):33459–33481, 2022.
[27] Panchi Li, Tong Shi, Aiping Lu, and BingWang. “Quantum implementation of classical marr–hildreth edge detection”. *Quantum Information Processing*, 19:1–26, 2020.
[28] Giacomo Cavalieri and Dario Maio. “A quantum edge detection algorithm”. *arXiv preprint arXiv*:2012.11036, 2020.
[29] Li Deng. “The mnist database of handwritten digit images for machine learning research”. *IEEE Signal Processing Magazine*, 29(6):141–142, 2012.
[30] Ridho Nur Rohman Wijaya, Budi Setiyono, Mahmud Yunus, and Dwi Ratna Sulistyaningrum. “Operator-n layer construction for optimizing capsule network methods in image classification problems”. *Journal of Information Hiding and Multimedia Signal Processing*, 14(3):90–101, 2023.
[31] D. Martin, C. Fowlkes, D. Tal, and J. Malik. “A database of human segmented natural images and its application to evaluating segmentation algorithms and measuring ecological statistics”. In *Proc. 8th Int’l Conf. Computer Vision*, volume 2, pages 416–423, July 2001.
[32] Nazish Tariq, Rostam Affendi Hamzah, Theam Foo Ng, Shir Li Wang, and Haidi Ibrahim. “Quality assessment methods to evaluate the performance of edge detection algorithms for digital image: A systematic literature review”. *IEEE Access*, 9:87763–87776, 2021.
[33] Ziwen Zhang, Yijun Liu, Tie Liu, Yang Li, and Wujian Ye. “Edge detection algorithm of a symmetric difference kernel sar image based on the gan network model”. *Symmetry*, 11(4):557, 2019.
[34] Xiao-Ming Zhang, Man-Hong Yung, and Xiao Yuan. “Low-depth quantum state preparation”. *Physical Review Research*, 3(4):043200, 2021.
[35] Amir Fijany and Colin P Williams. “Quantum wavelet transforms: Fast algorithms and complete circuits”. In *Quantum Computing and Quantum Communications: First NASA International Conference, QCQC’98 Palm Springs, California, USA February 17–20, 1998 Selected Papers*, pages 10–33. Springer, 1999.

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