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Application of Tensor Networks Analysis to Optimize Traffic Management in a Critical Information and Telecommunications Network

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Abstract: The article investigates the task of optimising traffic management in critical information and telecommunication networks in order to ensure a guaranteed quality of user service, particularly in emergencies. A method of tensor analysis of networks is proposed, using a formalised description of the system in the form of tensors of message lengths, delays and bandwidth of channels. The network is modelled as a simplified complex, and routing is implemented through a tensor equation of connection between network parameters in different coordinate systems. Experimental calculations using examples with dynamically variable topology have shown:

- Reduction of average multipath message delivery latency by 9–40% depending on traffic intensity,
- Probability of packet delivery at or above 0.999 under high loads (200-300 messages/s),
- Zero jitter due to the even distribution of delays between paths,
- The ability to adaptively fragment messages in nodes to reduce latency,
- Increasing the efficiency of resource use compared to single-track models.

The use of a tensor apparatus provides stable and scalable routing in an unstable network topology. The method allows you to take into account the heterogeneity of traffic, adapt to the loss of nodes or channels, and maintain guarantees of quality of service in real time. The proposed approach is of practical importance for information and telecommunication systems used in emergencies, in particular for coordinating the actions of emergency rescue services, emergency medicine, civil protection, military units, control of drones and robotic means in the face of infrastructure loss. Potential stakeholders include state and municipal security services, operators of critical networks (energy, transport, healthcare), developers of automated control systems, and manufacturers of secure communication equipment. The proposed method can be integrated into decentralised networks with limited resources and variable topology, where traditional routing approaches do not guarantee sufficient quality of service.

Index Terms: Telecommunication Network; Information Exchange; Multipath Routing; Traffic Management; Tensor Analysis of Networks; Network-Centric Principle (Single Information Field)

1. Introduction

We live in an era of historical changes, significant challenges and turbulence. The world is changing. Nature is changing, too. The number of challenges that humanity faces is constantly increasing. The number of emergencies of both natural and artificial origin is growing. Integration and coordination of efforts are what can significantly increase the effectiveness of overcoming emergency consequences.

Emergency rescue, special services and other law enforcement agencies (units of the National Guard, police, emergency rescue service, medical, military, etc.) are involved in emergency response in the event of an emergency. In such critical conditions, rapid and effective management of all involved units comes to the fore. Optimization of management processes allows for the effective use of available resources and, as a result, reduces time and achieves significant savings in the workforce and in the use of resources for solving tasks in any field of activity.

Today, the world's experience in developing a law enforcement management system consists of integrating control, communication, computing, intelligence, surveillance, navigation and comprehensive support systems. Such a system must be highly secure and resistant to cyber threats, quickly adapt to changing circumstances, and provide the necessary level of quality of user service [1-7]. The principles of the network-centric concept of law enforcement management of leading countries in the world best meet these requirements.

Since 2014, all law enforcement agencies in Ukraine have been undergoing a process of re-equipping with modern digital means of communication and telecommunications [8]. Currently, various law enforcement agencies are equipped with multiple types of communication means that use data exchange protocols in networks of different manufacturers. This fact significantly affects the interaction of various units involved in the elimination of the consequences of the emergency. At the same time, the number and volume of information is constantly increasing, the number of management tasks is increasing, and the time for their solution by management bodies is continuously decreasing.

An essential task of telecommunication networks (TCN) of various functional purposes is to ensure a given level of quality of service (QoS). One of the keys means of ensuring QoS and managing network resources is routing protocols [5-6, 9-10]. Solving the problem of multipath routing in TCM of critical purpose is complicated by the need to take into account multiprotocol and hybrid nature, which is associated with the peculiarities of servicing the use of different user traffic, the use of other protocols in a network formed by various types of communication means [7, 11]. Heterogeneity is manifested not only in different directions of traffic transmission but also in the need to provide differentiated communication services since the intensity of served traffic, their priority and the requested guarantees of communication quality can differ significantly.

Thus, the issue of finding new approaches and methods for optimizing traffic management in heterogeneous information and telecommunication networks of critical purpose is relevant. The purpose of the article is to optimize traffic management in a vital information and telecommunications network to reduce information transmission time while ensuring guaranteed quality of user service through the use of tensor network analysis.

The purpose of this study is to develop a mathematically grounded approach to optimising traffic management in critical ITN using the tensor analysis apparatus. The main difference between the proposed approaches is the use of a multidimensional description of the system in the form of tensor equations, which allows simultaneous modelling of the topology, delays, and bandwidth of channels.

The proposed model provides a solution to the problem of multipath routing with zero jitter, automatic load balancing and the ability to adapt in case of loss of network structural elements. The simulation results show a reduction in average latency of up to 40% depending on the intensity of traffic.

The limitations of the model relate to the computational complexity when scaling, the need for complete information about the network structure, and the lack of consideration of external influences (interference). Despite this, the tensor approach has significant potential for use in automated control systems in emergencies.

Main objectives of the study

1. Optimisation of traffic management in critical information and telecommunication networks (CITNs) for:
    - Minimising the delay in the delivery of messages;
    - Ensuring guaranteed quality of service (QoS);
    - Increasing resistance to changes in the network topology (destruction of nodes, channels, etc.).
2. Application of the Tensor Apparatus for CITN Simulation:
    - Taking into account the multidimensional nature of the network;
    - Using equations in the form of tensor transformations;
    - To build an adaptive multipath routing system.
3. Simulation of emergency scenarios:
    - With a dynamic change in the network structure;
    - Taking into account limited and damaged communication channels.

Main limitations of the proposed technology

1. Computational complexity. The tensor model requires matrix and tensor operations (inversions, products), which are resource-intensive for vast networks.
2. The need for complete knowledge of topology. The system assumes the availability of global information about the network - this is possible only in centralised or well-coordinated systems.
3. Initial static. Although the method allows dynamics (changes in time), the network initialisation process itself is static and requires a detailed preliminary description (matrices C, A, M, etc.).
4. There is no complete simulation of delays in communication channels with losses or interference (for example, interference from electronic warfare).

The scientific novelty of the work lies in the first proposed generalised approach to optimising traffic management in critical information and telecommunication networks (ITN) based on tensor analysis, which:

- combines network topology, channel characteristics and delays into a single tensor model;
- allows you to carry out adaptive multipath routing with automatic load balancing;
- provides synchronisation of message delivery and elimination of jitter by equalising average delays in different routes;
- Takes into account dynamic changes in topology without the need to rebuild the model structure, only by converting tensors between coordinate systems.

The main contribution of the article:

1. A formal tensor model of ITN has been developed, which describes the system in the form of tensors of message length, delays and bandwidths in different coordinates.
2. It is proposed to use a mathematical apparatus for transitioning between coordinate systems (branches, contours, pairs of nodes) to optimize traffic management in a critical information and telecommunications network, which ensures stable routing in the event of structural changes in the network.
3. It is proposed to use the tensor equation of the connection between network parameters ($S = MT$) in a critical information and telecommunications network to compare routes with different characteristics, which allows to avoid jitter.
4. The effectiveness of the approach is demonstrated on the example of a variable network in three emergency scenarios, with a decrease in the average latency by 9–40% and the provision of QoS in case of node destruction.

Structure of the article. The article consists of five main sections. Chapter 2 provides an overview of existing approaches to optimising traffic management in critical information and telecommunication networks and justifies the need to use tensor analysis. Chapter 3 describes the methodology of tensor analysis for routing in such networks, including details of modelling and tensor operations. Chapter 4 is devoted to experimental results, where the effectiveness and benefits of the proposed approach are compared with traditional methods. Chapter 5 discusses the main limitations and possibilities for further research, as well as conclusions on the applicability of the technique in real-world critical networks.

2. Related Works

Modern routing approaches in critical ITNs are mainly based on heuristic or combinational algorithms that do not allow for simultaneous consideration of QoS, load balancing, and real-time topology change. In addition, most solutions are not able to ensure the coordinated functioning of the system with increasing network complexity or fragmentation. We live in an era of profound changes and global instability, accompanied by an increase in the number of emergencies of both natural and artificial origin. In such conditions, the effectiveness of the coordination of rescue and special services directly depends on the ability of information and telecommunication networks (ITN) to provide guaranteed quality of service (QoS) even in the case of loss of individual nodes or channels. To this end, it is necessary to have flexible, adaptive, and damage-resistant traffic management solutions.

Table 1. What differs from existing approaches

Criterion	Existing models	Proposed approach
Model type	Combinatorial, heuristic, graph	Tensor, analytical, generalised
QoS/Traffic	Either QoS or balancing (separately)	QoS + traffic + topology at the same time
Scalability	Local models that are not systemically consistent	Global model covering the entire network
Reaction to changes	Recalculation from zero when changing the structure	Adaptive transformation without rebuilding
Jitter / Sync	High jitter through different paths	Jiter ≈ 0 due to tensor alignment
Emergency Networks	Poorly taken into account	Directly simulated (destruction, losses)

A review of existing technologies and mechanisms for managing network resources has shown that most solutions in this area are based mainly on heuristic schemes or the simplest combinatorial and flow models of the critical information and telecommunications network (CITN), which for the most part do not meet the requirements of a system nature. The use of such models in the formalization of system routing tasks channel and buffer resource allocation often

entails making inadequate decisions on the use of network capacities, which significantly reduces performance and quality of service indicators [2, 5-6]. As the Analysis has shown, a number of approaches have been used in the modelling of such systems and the solution of basic network tasks, in which, most often, the functions of structural and functional synthesis are solved independently, in the best case, determining the initial data for each other, accepted as assumptions and constraints. For example, models and methods that are focused on providing QoS do not allow for a complete description of the load-balancing processes in the CITN [12, 13]. Conversely, in models that adequately describe the load balancing problems solution, when trying to ensure that QoS is taken into account, computational complexity increases significantly [12, 13]. Accordingly, these models do not allow for a comprehensive consideration of the most critical factors that have a significant impact on the qualitative solution of information exchange management problems between the elements of the CITN [6, 9, 10].

In this regard, one of the possible ways to study the network architectures of the CITN is by using tensor models and analysis methods. In this aspect, the research of G. Kron [14], based on the use of invariant quantities - tensors, is of particular relevance. The possibility of a joint study of the structure of a complex system and the processes occurring in it is the main advantage of the tensor research methodology. Based on the tensor model of the system, it is possible to study the system properties of the CITN regardless of the possible coordinate systems of its consideration [15, 16]. Suppose it is necessary to analyse the system from different points of view (aspects of consideration). In that case, it is possible to obtain the required interpretation of the tensor model by projecting the tensors that form this model into one or another coordinate system. Thus, the tensor approach to system modelling of complex systems most fully meets modern requirements [17-18]. It allows solving problems related to taking into account interrelated and simultaneously contradictory requirements that arise (are put forward) during the management of information exchange in the CITN.

In general, tensor analysis, tensor methods of research and modelling of complex systems are quite widely used by scientists in various fields. They allow us to consider multidimensional structures and complex systems in the form of matrices and vectors of high order. The composition/decomposition of complex systems is carried out using tensor expansions, which allows us to approximate high-dimensional problems. As an example, a number of the following studies can be cited.

In order to increase the reliability of the decomposition to identify the features of individual components of a complex system, Tucker tensor decomposition is used in studies [19, 20]. This approach is based on principal component analysis matrices, but does not take into account changes in the state of the complex system and its qualitative characteristics over time.

The collection and processing of multidimensional data sets [21], their interpretation, prediction [22] and the recovery of sparse components from noisy and incomplete observation data [23] are based on the generalized application of matrix and tensor decomposition methods and tensor discriminant analysis (TDA), which allows establishing the exact boundaries of the HD-TDA classifier. However, the issues of classifying multidimensional data sets do not solve the problem of optimizing traffic management in information and telecommunication networks of critical purpose.

Information and telecommunication networks of critical purpose are characterized by a number of discrete multidimensional indicators that describe the state of the system, its topology, the level of information load, traffic heterogeneity, etc. In this aspect, the issues of processing multidimensional data [24] and [25] deserve special attention. Here, the tensor multi-species clustering method for image segmentation (TMCNIS) is based on the Schatten-p tensor norm, and the Jensen–Shannon divergence is used to obtain nonlinear data structures, which allows for a better approximation of the target rank of the tensor and fully captures the spatial structure in the tensor. However, these studies do not allow describing the state of critical information and communication systems taking into account the change in their state and characteristics over time.

The article [26] is devoted to the use of the weighted low-rank tensor as a basis for unsupervised multi-species feature selection (WLTL). The method includes an adaptive strategy for automatically assigning data representation weights, optimizing the feature selection process, by integrating multi-species spectral clustering and the weighted low-rank tensor to generate high-quality pseudo-labels for feature selection. However, it does not consider the issues of optimizing the transmission of information flows in telecommunication systems.

Data ranking tasks for further improving the reliability [27] and efficiency [28] of training neural networks (GNN) by representing their structure in tensor form allow facilitating decision-making based on the characteristics of the time series of criteria, where the data is structured in three dimensions [29]. However, these works do not consider the issue of increasing the efficiency of the functioning of information and communication systems of critical purpose as an element of providing guaranteed quality services in such systems. The use of tensor decomposition of higher-order singular values [30] allows, by analysing spatiotemporal data on the transport flow and the correlation between the initial data about it, to fairly effectively carry out short-term forecasting of the behaviour of the transport flow to improve the productivity of its management. The article [31] also considers solving the problem of optimizing the movement of animals between different points of the farm using tensor analysis, but the structure of the initial network remains constant. Critical information and communication networks with their bandwidths are similar to transport networks, but require, unlike [30] and [31], long-term load forecasting taking into account changes in the network structure over time, including under the influence of external factors.

The learning and estimation protocol of a channel without interference for estimating the signal of a specific channel with reflection in such complex systems as MIMO [32], as well as in conditions of intentional interference [33] allow finding solutions for the joint estimation of communication channels and transmitted messages [34], but the issues of information flow transmission in critical information and communication systems under conditions of a possible sharp change in network topology are not considered.

Tensor research methods also find their place in signal processing [35], where an algorithm for adaptive decomposition of such a multidimensional structure as a multisensory signal is proposed. Deep singular tensor decomposition is quite effective for processing multisensory information fusion and preserving the relationships between attributes of multisensory signals. However, this work does not investigate the issues of guaranteed message delivery quality during the transmission of information flows in rapidly changing conditions.

The search for new principles and approaches to the study of critical information structures as in [36] makes it possible to study the indicators of the efficiency of functioning of information and communication systems of critical purpose, but leaves without attention the issue of guaranteed quality of functioning of such systems in conditions of possible sharp changes in the structure and characteristics of network channels.

Modelling and research of network structures take place in [37], [38] and [39]. Thus, the study [37] is devoted to increasing the efficiency of anomaly detection in large networks. A multimodal incremental tensor method of decomposition (MMITD) has been developed, which allows processing network traffic that grows dynamically in several directions (modes). In the article [38], a new multilayer temporal network model is proposed, which considers the interaction between different layers in one timestamp and the interaction between different timestamps, which allows ranking nodes, layers and timestamps with greater efficiency. And in [39], a deep tensor projection network (DTPN) model is considered, which is built by replacing a layer in DCNN with a tensor projection layer (TPL). The constructed model can reduce the dimensionality of higher-order tensor data while ensuring that the structure of tensor data is taken into account and preventing the loss of information about the features of this data. However, these works do not consider the issues of transmitting information flows in telecommunication systems in conditions of a possible sharp change in the topology of a network that uses heterogeneous channels. This, in turn, does not allow simultaneously taking into account the dynamic change in the structure of complex systems and their quantitative characteristics. These works do not investigate the issue of guaranteed quality of message delivery during the transmission of information flows in rapidly changing conditions, and do not take into account the issue of the influence of intentional interference on information transmission channels. The issues of possible optimization of traffic management in critical information and telecommunication networks remain unsolved.

3. Methods and Methodology

In critical conditions, including those emerging *in situ* of scheduled processing, *i.e*. at any time, in situations of rapid topological changes caused by numerous factors (displacement, failure, destruction, or suppression of network elements), different equations should account for the variety of arising circumstances such as geographical dislocation conditions, the number, composition, and characteristics of network elements, connecting lines (wire, radio relay, fibre optic, space, etc.), environmental conditions and external influences, etc.

The state of the system, its topology, the level of information load, and the heterogeneity of traffic over time can be described by the corresponding functional equations. In other words, in a general modelling case, the system should be characterized either by a set of tensors that represent the state of the system at each particular moment or as a geometric object of mixed dimensionality (multi-tensor).

Our study is based on a fragment of the mobile component of the communication system, shown in Fig. 1. The topological description of the mobile component was performed by considering its structure as a one-dimensional simplicial complex, where the set of zero-dimensional (0D) simplexes simulates network nodes, and the set of one-dimensional (1D) simplexes simulates the communication lines of the network. Elements in the form of a circle B1-B6 can be either command posts of different ranks, single military units, service members, etc. We consider the process of transmitting a message (control command) between two nodes of this fragment and propose to use a tensor approach to solve the problem of multipath routing of the control command in the considered network.

Consider a set of nodes labeled *Nd*1 to *Nd*6 (as shown in Fig. 1) and a set of branches $B1$ to $B10$ representing the transmission paths. The dimensionality of the corresponding space – regarded as a structural invariant – is defined by the total number of branches in the network, denoted as n. Each possible configuration of how these n branches are connected can be viewed as a distinct partial coordinate system (CS) within this n-dimensional space. Any change in the network's structure that maintains the same number of branches, or a shift from one independent path set to another, is considered a transformation of the coordinate system. As each transmission path is independent, it effectively establishes an individual coordinate axis in this space-structure model. The orientations of contours and node pairs within the system can be assigned arbitrarily.

Let us consider two coordinate systems. The first one is the network branches CS, which corresponds to the structure with individual unconnected branches (*Nd*1-*Nd*6 and *B*1-*B*10). The second one is the CS of independent contours and node pairs (*CNdP*) of the network, which corresponds to the real structure of the system being modelled (*C*1-*C*5 and *NdP*1-*NdP*5).

To describe the network, we use two parameters: the length of messages and the time of their transmission. The system is described in the considered space (Fig. 1) using a monovalent tensor of message lengths S with components s^k, a monovalent tensor of transmission delays T with components t_l, and a tensor R of valence 2, the coordinates of which are calculated according to the expression $r_l^k = s^k t_l, \left(k, l = \overline{1, m}\right)$, or $R = S \otimes T$ in the component-free notation. Each network structure corresponds to its own set of coordinates, while the number of coordinate paths always remains equal to the number of branches. The laws of coordinate transformation are $S = CS'$ and $T = AT'$ such that the condition of orthogonality $C^t = [A]^{-1}$ holds. The relationship between covariant and contravariant components of the tensor can be formalized in the component-free notation as follows: $T = GS$; $S = MT$, where $M = [G]^{-1}$.

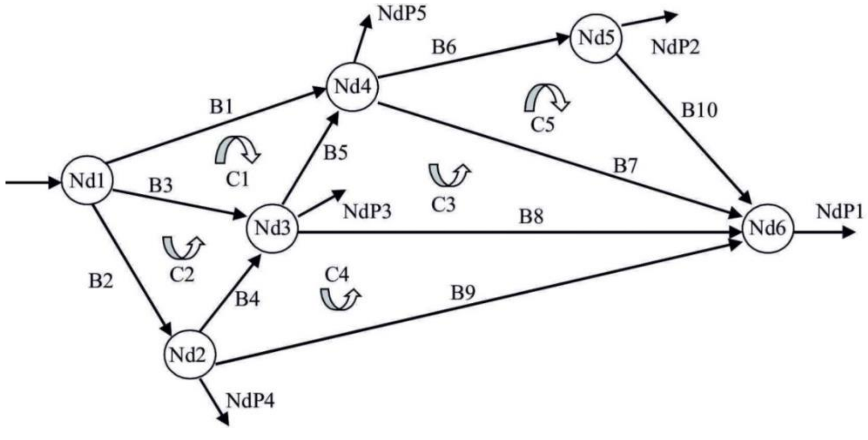

Fig. 1. Fragment of the mobile component of the communication system

Due to the exact dimension of the introduced coordinate systems, there are unambiguous rules for converting the coordinates of any geometric objects from one CS to another. The sought matrix of the contravariant transformation C is determined by the relation:

$$S_B = CS_{CNdP}, \tag{1}$$

where S_B and S_{CNdP} are presented in the form of vectors of dimension n of the projection of the monovalent tensor of message lengths S in the above-introduced coordinate systems of individual branches of the network, as well as independent circuits and node pairs.

The covariance nature of the delay tensor T implies the following law of coordinate transformation:

$$T_B = AT_{CNdP} \tag{2}$$

By the physics of the processes of information exchange in the network, the components $s_B^i$and t_i^B of vectors S_B and T_B are related as follows

$$s_B^i = m_B^{ii} t_i^B \quad (i = \overline{1, n}), \tag{3}$$

where m_B^{ii} is the fraction of the bandwidth of the i-th branch of the network.

According to Kron's second generalization postulate, the functional invariant of the proposed model is the tensor equation,

$$S = MT, \tag{4}$$

which is obtained by the generalization of equation (3) and preserves its form regardless of the network CS, where M is the network coordinate bandwidth tensor, the projections of which in each partial CS take the form of a matrix of dimension $n \times n$.

Taking into account expressions (1)-(4), we are led to conclude that the tensor M is a double contravariant metric tensor, the projections of which at CS transformation transform as follows $M_B = CM_{CNdP}C^t$ and $M_{CNdP} = A^t M_B A$, where M_B and M_{CNdP} are projections of the tensor M in the coordinate systems of the branches of the network with independent circuits and pairs of nodes, respectively.

As noted above, the form of equations describing the behaviour of the network as a whole should correspond to the equation of behaviour of individual elements (Kron's first generalization postulate), which leads to the replacement of the system of scalar equations by equations of vector-matrix form. Accordingly, the form of the functional equation (4) of the network remains unchanged both in CS of independent contours and node pairs:

$$S_{CNdP} = M_{CNdP} T_{CNdP}. \tag{5}$$

In addressing the multipath routing task, the known contour components of the relevant vector T_{CNdP} are complemented by additional known components S_{CNdP}—specifically, those of a vector S_{NdP} whose NdP subscript denotes "node pairs." These components represent the message lengths either entering or exiting the network nodes. However, this set of initial conditions does not permit a straightforward computation of the unknown components of the associated vectors S_{CNdP} and T_{CNdP} by directly solving the functional equation in the form of Equation (5). The key strength of the tensor approach, however, lies in its ability to derive the coordinates of a tensor in one CS using their known representations in another, based on well-defined transformation rules for the coordinate bases. This makes it practical to restate Equation (5) in a transformed form:

$$\begin{bmatrix} S_C \\ S_{NdP} \end{bmatrix} = \begin{Vmatrix} M^1_{CNdP} & M^2_{CNdP} \\ M^3_{CNdP} & M^4_{CNdP} \end{Vmatrix} \begin{bmatrix} T_C \\ T_{NdP} \end{bmatrix}, \text{ where } \begin{Vmatrix} M^1_{CNdP} & M^2_{CNdP} \\ M^3_{CNdP} & M^4_{CNdP} \end{Vmatrix} = M_{CNdP},$$

that due to the presence of known S_{NdP} and T_C, allows one to obtain the following two systems of equations:

$$\mathrm{T}_{NdP} = \left[M^{(4)}_{CNdP}\right]^{-1} S_{NdP} - \left[M^{(4)}_{CNdP}\right]^{-1} M^{(3)}_{CNdP} \mathrm{T}_C; \tag{6}$$

$$S_C = M^{(1)}_{CNdP} \mathrm{T}_C + M^{(2)}_{CNdP} \mathrm{T}_{NdP}. \tag{7}$$

Formulas 1-7 explain the essence of the general approach to describing the system. The state of the system, its topology, the level of information load, and the heterogeneity of traffic at each point in time can be described by the corresponding functional equations. That is, in general modeling, the system should be described by a mixed-type tensor, or a set of tensors that characterize the state of the system at each specific point in time. The essence and principle of matrix formation to obtain a solution specifically for our (given) structure and topology are detailed below. Lemeshko O.V. considers stationary networks [1-2, 9-10], but does not solve the problem of optimizing traffic in heterogeneous networks that change their topology over time. Accordingly, G. Kron [14] is the founder of the tensor mathematical apparatus for solving problems in electrical circuits. And Lemeshko O.V. is the founder of the direction of using tensor analysis of networks for traffic management in telecommunication static networks. We will continue our study for dynamic networks with discrete structural changes.

As an example, we will use the proposed method to solve the problem of multipath routing for the network (Fig. 1). In the general case, the operation of the method can be represented by the block diagram shown in Fig. 2. We will perform calculations based on the following initial data: the sender is node 1, the recipient is node 6, respectively S_{NdP}=0 (the message is not sent from other nodes); the message has a length of 100 bytes; the delay in the circuits is equal to zero TC=0; the bandwidth of the branches (bytes/s) allocated to serve this traffic is given in the form of diagonal elements of the bandwidth matrix M_B.

According to (3) and (4), the matrix M is the capacity tensor of the network coordinate paths, the projections of which in each coordinate system take the form of a matrix of size $n \times n$, thus, M_B is the projection of the tensor M in the coordinate system of the branches. Accordingly, the matrix M_B is a diagonal tensor of the second rank, where only the diagonal components are non-zero, which we can write in the form:

$$M_B = \sum_{i=1}^{10} b_i \; e^i \otimes e^i,$$

where e^i are the basic 1-forms of the branch space; b_i is the throughput of the *i*-th branch of the network.

According to the example from Fig. 1 (initial state), the throughput of the branches (in bytes/sec) allocated to service this traffic has the following values on the diagonal:

$$M_B = diag(60,\ 50,\ 80,\ 80,\ 80,\ 60,\ 70,\ 80,\ 50,\ 40)$$

Then in tensor form:

$$M_B = 60\, e^1 \otimes e^1 + 50\, e^2 \otimes e^2 + 80\, e^3 \otimes e^3 + \cdots + 40\, e^{10} \otimes e^{10}$$

For the structure shown in Fig. 1, the set of all contourless paths between nodes $Nd1$ and $Nd6$, i.e. paths that do not contain loops, can be represented as a system of equations. In other words, according to the orientation of the branches, base contours and node pairs.

In the general case, the matrix of the contravariant transformation *C* is formed as follows:

The expression of the branches that form the skeleton is determined through the contour components and the components of the internal node pairs. The expression of the branches that do not form the skeleton is determined only through the contour components. Accordingly, the elements c_{ij}^{k} and c_{ij}^{nb} of the matrix C are defined as follows:

$$c_{ij}^{k} = \begin{cases} 1, if\ the\ jth\ branch\ is\ included\ in\ the\ ith\ contour\ and\ their\ orientation\ coincides; \\ -1, if\ the\ jth\ branch\ is\ included\ in\ the\ ith\ contour\ and\ their\ orientation\ does\ not\ coincide; \\ 0, if\ the\ jth\ branch\ is\ not\ included\ in\ the\ ith\ contour. \end{cases}$$

$$c_{ij}^{nb} = \begin{cases} 1, if\ the\ jth\ branch\ defines\ the\ ith\ internal\ node\ pair\ and\ their\ orientations\ coincide; \\ -1, if\ the\ jth\ branch\ defines\ the\ ith\ internal\ node\ pair\ and\ their\ orientations\ do\ not\ coincide; \\ 0, if\ the\ jth\ branch\ does\ not\ define\ the\ ith\ internal\ node\ pair. \end{cases}$$

In turn, the covariant transformation matrix *A* is formed as follows:

The expression of the branches that form the skeleton is determined only through the components of the internal node pairs. The expression of the branches that do not form the skeleton is determined through the contour components and the components of the internal node pairs. Accordingly, the elements a_{ij}^{k} and a_{ij}^{nb} matrix A are determined as follows:

$$a_{ij}^{k} = \begin{cases} 1, if\ the\ jth\ branch\ is\ included\ in\ the\ ith\ contour\ and\ their\ orientation\ coincides; \\ -1, if\ the\ jth\ branch\ is\ included\ in\ the\ ith\ contour\ and\ their\ orientation\ does\ not\ coincide; \\ 0, if\ the\ jth\ branch\ is\ not\ included\ in\ the\ ith\ contour. \end{cases}$$

$$a_{ij}^{nb} = \begin{cases} 1, if\ the\ j-th\ branch\ determines\ the\ i-th\ internal\ node\ pair\ and\ traffic\ enters\ the\ network; \\ -1, if\ the\ j-th\ branch\ determines\ the\ i-th\ internal\ node\ pair\ and\ traffic\ leaves\ the\ network; \\ 0, if\ the\ j-th\ branch\ does\ not\ determine\ the\ i-th\ internal\ node\ pair. \end{cases}$$

According to Fig. 1, respectively, the matrix

$$A = A_1 \cup A_{-1} \cup A_0 \text{ та } C = C_1 \cup C_{-1} \cup C_0,$$

where

$A_1 = \{a_{1\,1}, a_{1\,10}, a_{2\,9}, a_{3\,8}, a_{4\,2}, a_{4\,8}, a_{5\,10}, a_{6\,7}, a_{7\,6}, a_{8\,3}, a_{8\,6}, a_{9\,2}, a_{9\,3}, a_{9\,4}, a_{9\,6}, a_{10\,5}, a_{10\,6}\}$ (has a value 1),
$A_{-1} = \{a_{4\,9}, a_{5\,8}, a_{7\,10}, a_{8\,8}, a_{9\,9}, a_{10\,7}, a_{7\,10}\}$ (has value in cells -1),
$A_0 = \{a_{ij} = 0 \mid a_{ij} \notin A_1, a_{ij} \notin A_{-1}\}$ (has value in cells 0),
$C_1 = \{c_{1\,1}, c_{2\,2}, c_{2\,9}, c_{3\,6}, c_{3\,8}, c_{3\,10}, c_{4\,2}, c_{5\,6}, c_{5\,10}, c_{6\,5}, c_{6\,7}, c_{7\,6}, c_{8\,3}, c_{9\,4}, c_{10\,5}\}$ ((has value in cells 1),
$C_{-1} = \{c_{1\,1}, c_{3\,1}, c_{3\,2}, c_{4\,4}, c_{5\,1}, c_{5\,3}, c_{7\,3}, c_{7\,5}, c_{8\,4}\}$ (has a value in cells -1),
$C_0 = \{a_{ij} = 0 \mid a_{ij} \notin C_1, a_{ij} \notin C_{-1}\}$ (has a value in cells 0).

Then, according to expressions (6) and (7)

$$T^{t}{}_{NdP} = [0.979\ \ 0.164\ \ 0.512\ \ 0.499\ \ 0.568];$$

$$S^t{}_C = [34.1\ \ 24.97\ \ 61.38\ \ 23.99\ \ 9.86].$$

The solution to the problem is determined by calculating the components of the vector S_B, which characterize the lengths of the message parts in each branch of the network:

$$S^t{}_B = [34.095\ \ 24.97\ \ 40.935\ \ 0.984\ \ 4.525\ \ 9.86\ \ 28.76\ \ 37.394\ \ 23.986\ \ 9.86].$$

The resulting solution was determined by the presence of the following independent message transmission routes from the first node to the sixth one, which do not contain loops (cycles). The first route (*Nd*1-*Nd*6-*Nd*10) provides a transfer of 34.095 bytes; the second (*Nd*3-*Nd*5-*Nd*7) one – 40.935 bytes; the third (*Nd*2-*Nd*4-*Nd*8) one – 0.984 bytes; the fourth (*Nd*2-*Nd*9) one – 23.986 bytes. The independence of the routes, in this case, is interpreted as the presence of a branch in each of them that is not part of any other route. The time for transmitting parts of the message along each of the calculated routes corresponds to the delay between nodes $Nd1$ and Nd6 ($NdP1$), that is, the first coordinate of the vector T_{NdP}, and is equal to 0.979 s.

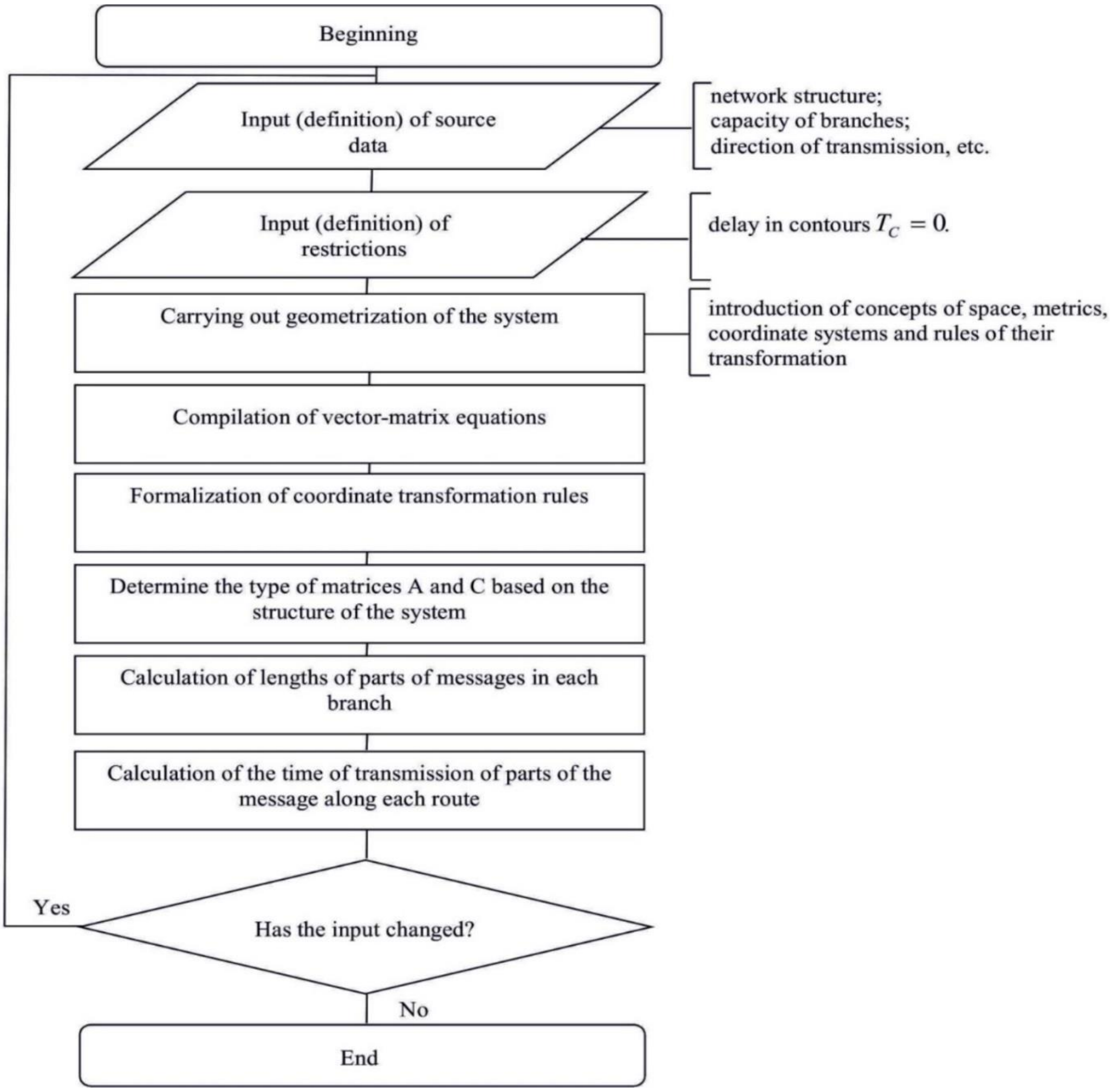

Fig. 2. Block diagram of the proposed method

Under critical operating conditions, the CITN structure is expected to evolve over time due to factors such as relocation, physical damage, or expansion. Consequently, the parameters n (representing the number of transmission paths) and m (denoting the number of nodes) must be treated as time-dependent variables. The initial configuration of the network can be described as $U = f(m, n, t)$, where time t is modelled as a discrete variable with increments of one unit. At the next time step, $t + 1$, with a certain probability P_1, the network may transition to a new state, which can then be expressed as $U1 = f(m1, n1, t + 1)$.

Let the fragment of the mobile component at the time $t+1$ be as follows: one of the nodes, namely $Nd5$ be destroyed and two communication lines $B6$ and $B10$ be lost (Fig. 3). Since only available lines and nodes are taken into account during the calculations to model the updated network structure, the numbering of branches (B), nodes (Nd) and node pairs (NdP) is updated, respectively.

The networks consisting of the same elements can be considered as the same network but presented in different coordinate systems. Therefore, different networks that differ from each other only by the connection of their elements are described by tensor equations of the same type. Thus, the calculation algorithm remains the same.

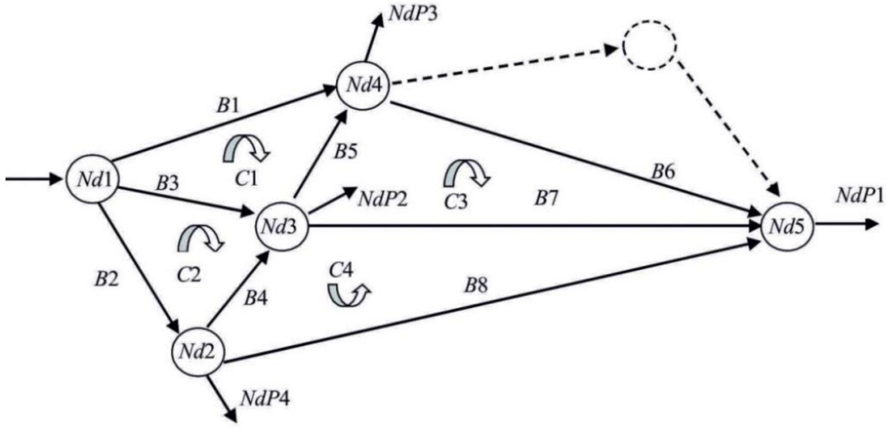

Fig. 3. Fragment of the mobile component at time $t+1$ in the absence of one node and two branches (shown by dashed lines)

In this case, the coordinates of the tensors S and T in different coordinate systems of the network are related as follows:

$$\begin{cases} s_B^1 = s_C^1 + s_{NdP}^3; \\ s_B^2 = -s_C^2 + s_{NdP}^1 + s_{NdP}^2 + s_{NdP}^4; \\ s_B^3 = -s_C^1 + s_C^2 + s_{NdP}^2; \\ s_B^4 = -s_C^2 - s_C^4 + s_{NdP}^1 + s_{NdP}^2; \\ s_B^5 = -s_C^1 + s_C^3; \\ s_B^6 = s_C^3; \\ s_B^7 = -s_C^3 - s_C^4 + s_{NdP}^1; \\ s_B^8 = s_C^4; \end{cases}$$

$$\begin{cases} t_1^B = t_3^{NdP}; \\ t_2^B = t_4^{NdP}; \\ t_3^B = t_2^C + t_2^{NdP}; \\ t_4^B = -t_2^C + t_2^{NdP} - t_4^{NdP}; \\ t_5^B = -t_1^C - t_2^C - t_2^{NdP} + t_3^{NdP}; \\ t_6^B = t_1^C + t_2^C + t_3^C + t_1^{NdP} - t_3^{NdP}; \\ t_7^B = t_2^C + t_1^{NdP} - t_2^{NdP}; \\ t_8^B = t_4^C + t_1^{NdP} - t_4^{NdP}. \end{cases}$$

The components of the vector S_B, which characterize the lengths of the message parts in each branch of the network, are equal to:

$$S_B^t = [32.455 \;\; 25.847 \;\; 41.698 \;\; 0.342 \;\; 1.575 \;\; 34.029 \;\; 40.466 \;\; 25.505].$$

In this case, the first route (B1-B6) provides the transfer of 32.455 bytes; the second ($B3$-$B5$-$B6$) one – 41.698 bytes; the third ($B2$-$B4$-$B7$) one – 0.342 bytes; the fourth ($B2$-$B8$) one – 25.505 bytes. The transmission time of parts of the message is equal to 1.027 s.

At the next time moment $t+2$ with probability P_2 the state of the network can be described as

$$U2 = f(m2, n2, t+2).$$

Let the fragment of the mobile component at time $t+2$ be as follows: one of the communication lines, let's say, $B7$ is damaged (Fig. 4). In this case, for calculations, one has to change only the numbering of branches (B).

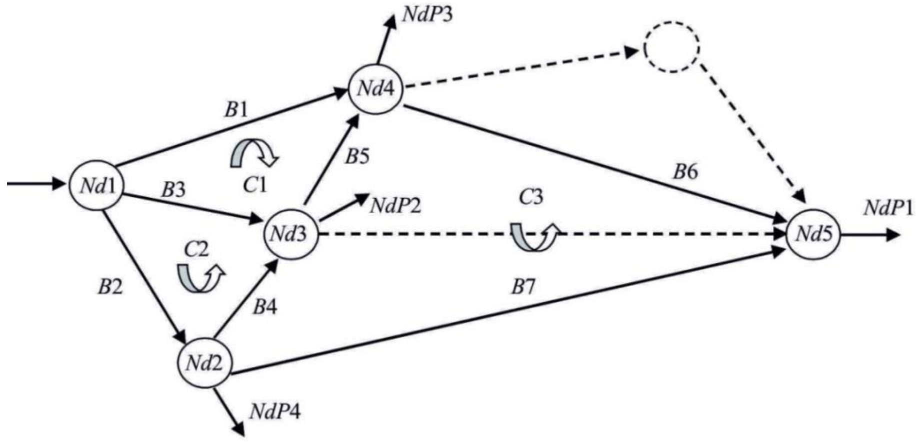

Fig. 4. Fragment of the mobile component at time $t+2$ in the absence of another branch

In this case, the coordinates of tensors S and T in different coordinate systems of the network are related as follows:

$$\begin{cases} s_B^1 = s_C^1; \\ s_B^2 = s_C^2; \\ s_B^3 = -s_C^1 - s_C^2 + s_{NdP}^1 + s_{NdP}^2 + s_{NdP}^3 + s_{NdP}^4; \\ s_B^4 = -s_C^2 + s_C^3 + s_{NdP}^4; \\ s_B^5 = -s_C^1 - s_C^3 + s_{NdP}^1 + s_{NdP}^3; \\ s_B^6 = -s_C^3 + s_{NdP}^1; \\ s_B^7 = s_C^3; \end{cases}$$

$$\begin{cases} t_1^B = t_1^C + t_3^{NdP}; \\ t_2^B = t_2^C + t_4^{NdP}; \\ t_3^B = t_2^{NdP}; \\ t_4^B = -t_2^{NdP} + t_4^{NdP}; \\ t_5^B = -t_2^{NdP} + t_3^{NdP}; \\ t_6^B = t_1^{NdP} - t_3^{NdP}; \\ t_7^B = t_3^C + t_1^{NdP} - t_4^{NdP}. \end{cases}$$

The values of the components of the vector S_B, which characterize the lengths of the parts of the message in each branch of the network:

$$S_B^t = [38.27\ 29.16\ 32.57\ 14.1\ 18.47\ 56.74\ 43.26].$$

In this case, the first route (B1-B6) provides the transfer of 38.27 bytes; the second (B3-B5-B6) one – 18.47 bytes; the third (B3-B4-B7) one – 14.1 bytes; the fourth (B2-B7) one – 29.16 bytes. The transmission time of parts of the message is 1.4 seconds.

Features of the use and implementation of the tensor approach:

1. Tensors as a description of the network:

- The network is modelled as a topological structure (simplicial complex), where the nodes are 0D simplexes and the connections are 1D simplexes.
- The states of the system are described using monorank tensors:
    - S is the tensor of message lengths.
    - T is the transmission delay tensor.
    - $R = S \otimes T$ is a second-rank tensor that specifies the relationship between message lengths and delays.

2. Coordinate systems:

- Two coordinate systems are used:
    - CS branches of the network.
    - CS independent circuits and node pairs.

- For the transition between these coordinate systems, matrices of contravariant (C) and covariant (A) transformations have been defined and used to calculate tensor projections on different bases.

3. Functional invariants and equations: $S = MT$, where M is the metric tensor (network bandwidth tensor), which ensures consistency between the length of the message and the delay in any CS.

4. Specific Tensor Operations:

- Tensor products (e.g., $R = S \otimes T$).
- Tensor transformations between coordinate systems.
- Matrix equations are used to determine the optimal distribution of parts of the message along routes.
- Solving systems of equations such as $S_{CCNP} = M_{CCNP} \cdot T_{CCNP}$ with further transformations to compute incoming and outgoing messages.

5. Adaptation to Variable Topology:

- The method allows you to discreetly update the network structure in time (when nodes or communication lines are destroyed) without changing the mathematical structure of the model.
- For each topological change, the coordinates of the tensor vector (e.g., S_B, T_B) are recalculated based on the new configuration.

Table 2. Tensor analysis methods for conducting the study

Tensor concepts	**Implementation**
Tensor decomposition	Indirectly, in the form of a tensor product of the length and delay of messages
Metric tensor (double contravariant)	The bandwidth tensor M, changes when the coordinates are transitioned
Changing coordinates (C, A)	Formal matrices for transformation between contour and branch spaces
System of Tensor-Type Equations	Solved to obtain the optimal message route
Modelling a Variable Topology	Implemented in time increments $t, t+1, t+2t, t+1, t+2t, t+1, t+2$ with tensor update and component recalculation

Multipath delay is the average delivery time of parts of a message along different route. Jitter is the variability (spread) of the delay between individual routes. In traditional systems with multiple routes (without synchronisation), the difference in latency between routes causes jitter and, as a result, loss of quality of service. The tensor model describes the network as:

- Message tensor S is the distribution of message lengths by branches,
- Delay tensor T,
- Network bandwidth tensor M,
- The connection equation is $S = MT$. This equation determines the optimal distribution of traffic across channels depending on their bandwidth and latency in the agreed coordinates.

The effect of tensor analysis on delay reduction and zero jitter, as well as an explanation of why tensor analysis gives better results**:**

- A unified description of the entire network. Instead of local calculations in each node, tensors allow you to describe the entire network as a whole, as a multidimensional system. It will enable routes to be optimised globally rather than greedily or in isolation, as in traditional routers.
- Path synchronisation, in particular the equation $S = MT$, automatically provides that:
    - The average delays on all routes are consistent,
    - The length of messages along each route is proportional to its performance, that is, balanced. It leads to the fact that all fragments of the message come at the same time, and the jitter is ≈ 0.
- Instant response to topology changes. Tensors are easily transformed when the coordinates (network structure) change. There is no need to rebuild the model – only update the tensor projections. It reduces reconfiguration time and prevents additional delays from occurring.

Table 3. Tensor analysis of suitability for critical networks

Requirements of the emergency system	**How the tensor approach solves it**
High adaptability	Due to the transformation of tensors when changing the topology
Minimal latency	Due to the global optimisation of flows through the equations
Reliable delivery	Thanks to route alignment (T alignment)
Complex heterogeneous network	Description in multidimensional space without loss of coherence

Tensor analysis allows you to "intelligently" distribute the load along the routes so that they work with the same delays, without overloading individual channels. That is why:

- The average latency decreases – Traffic follows the best routes.
- Jiter disappears because all routes give the same delay.

Features of the tensor approach assist in routing:

- Reduction of average latency due to a multipath routing approach with load equalisation.
- Minimising jitter, as all routes have the same average latency.
- Adaptability to change, because new tensor projections are quickly calculated even when nodes are damaged.
- Optimisation of traffic distribution, in particular, each part of the message is distributed along the routes in proportion to their bandwidth, which reduces the load and improves the quality of service.

The simulation was carried out in the following conditions:

1. Absence of stochastic changes:
   - Delays in each channel (branch) are fixed and do not change over time.
   - Queues, load fluctuations, collisions or buffer constraints are not taken into account.
   - The bandwidth of the channels is given as a deterministic value.
2. Traffic – synchronous:
   - Messages are sent in the form of a fixed block, divided into parts with the exact moment of the start of transmission.
   - Each fragment is transmitted by an independent route chosen given the same delay on the path (via the equation $S = MT$).
3. The target function of routing is delay equalisation. Only those sets of routes are selected in which the delays coincide or differ by a constant value within the same model.

Under such conditions, the variability of the delay between routes will indeed be ≈ 0, because:

- Processing time does not change;
- All routes are synchronised in length/delay.
- Fragments are transmitted without asynchronous influence.

But this does not mean that such a result is reproducible on a real physical or IP network. According to the results of the simulation, packet jitter, which usually accompanies the implementation of multipath routing, is actually zero under the conditions of the proposed tensor model. It is because in the model:

- Constant latency values on each channel are used,
- Routes are selected to have identical average delays,
- And messages are synchronously split and sent along routes.

Thus, within the idealised deterministic conditions of modelling, there is no probability of desynchronisation of the fragment delivery time. At the same time, in real networks, due to stochastic processes (load fluctuations, queues, collisions), jitter can grow. Therefore, for practical application, the model needs to be adapted, taking into account the stochastic parameters of the environment.

Features of the network model and specific emergency scenarios:

1. Network model: structure and topology:

- The type of network is a critical information and telecommunication network (CITN), which is used to eliminate the consequences of emergencies, particularly in the military, natural or artificial.
- The network topology is represented as a one-dimensional simplicial complex:
  - Nodes (Nd1–Nd6) are command posts, military units, individual employees, etc.
  - Branches/channels (B1–B10) – communication lines between them (radio relay, cable, satellite, etc.).
  - Each branch represents an independent coordinate axis in the space of the tensor model.
- Characteristics of network channels:
  - The bandwidth of the channels (in bytes/s) varies (for example: 60, 50, 80, ..., 40).
  - The possibility of fragmentation of messages in nodes is taken into account for an optimal routing solution.

2. Scale of the model

- In the example, a network of 6 nodes (Nd1–Nd6), 10 branches (B1–B10), five independent node pairs (NdP1–NdP5) and five base circuits (C1–C5) is considered.
- The method supports dynamic changes in the network structure – removal of nodes/lines, recovery, topology changes in time $t, t+1, t+2t, t+1, t+2t, t+1, t+2$.

3. Emergency scenarios

- Situation 1 – initial (time t):

  - Static structure: all nodes and branches are active.
  - Sending a message from Nd1 to Nd6.
  - The message is broken down into parts and transmitted by four independent routes.
  - Latency 0.979 sec.
- Situation 2 – change (time t+1):
  - One node (Nd5) is destroyed; branches B6 and B10 are lost.
  - Rebuilt coordinate matrix; Routes updated.
  - Delivery time: 1.027 s.
- Situation 3 – next shift (time t+2):
  - Additionally, another branch (B7) is lost.
  - The tensor model adapts to the new topology.
  - Delivery time 1.4 s.

These scenarios illustrate the simulation of destruction, loss of network elements, and routing adaptation, which is very typical for emergency conditions (shelling, natural disasters, power outages).

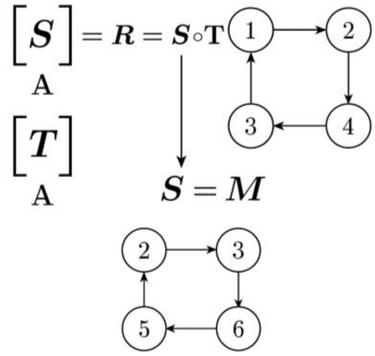

Fig. 5. Tensor analysis for routing problems

The described model makes it possible to take into account the real structure of deployed field communications (both military and rescue). It also allows you to instantly respond to changes in available routes without rebuilding the entire model. It can be adapted to networks of any scale (with magnification n and m).

In the proposed approach, routes are not selected on the basis of classical algorithms (such as Dijkstra or Bellman-Ford). Still, they are formed through an analytical representation of the network in the form of a linear route space. The network is modelled as a one-dimensional simplicial complex, that is, as a system:

  - nodes $Nd_1, \dots, Nd_n$;
  - branches (channels) $B_1, \dots, B_m$ between them.

Each branch is an independent coordinate axis (tensor component). The model uses several coordinate systems (CS) to describe routes:

Table 4. Coordinate systems (CS) and route spaces

Coordinate system	**Description**	**Denomination**
CS branches	T_B or S_B vector: delays or lengths of messages on branches	T_B, S_B
CS independent paths	Independent cycles (C1... C5) that define the topology	T_C, S_C
CS node pairs	Independent routes between node pairs (NdP1... NdP5)	T_{NP}, S_{NP}

For each of these systems, a transition matrix (e.g., A_{NPB}) is given to transform the coordinates – i.e., the projection of the delay or message length from one space to another.

The key formula for route management, which determines the relationship between the length of the message and the delay of the route: $S = M \cdot T$, where S is the vector of message lengths along routes; T is the delay vector; M is the metric tensor (displays the bandwidth of the channels). This formula does not just describe, but controls the choice of routes, i.e. only those routes for which this equation can be satisfied (i.e. have the same delay or proportional lengths) can be selected for simultaneous transmission. Multipath in this model:

1. Delay equalisation. Before sending a message, it is divided into fragments that follow different routes, but:

- Only if the delays on the routes are the same;
- Or if the tensor equation ensures that all routes are compatible in time.

Consequence – fragments arrive at the same time → jitter ≈ 0.

2. Fragmentation of messages. The S length message is divided into parts $S_1, S_2, \ldots, S_n$, in proportion to the route bandwidth:

$$S_i = \frac{T_i}{\sum T_i} \cdot S$$

It means that more data goes through faster channels, but with the preservation of synchronous arrival.

3. Adaptation to a change in structure. If a node/channel is lost (time $t+1, t+2t+1, t+2t+1, t+2$), the topology is updated → listed:

- coordinate transition matrix;
- metric tensor M;
- delay vector T.

It allows you to rebuild acceptable routes instantly without changing the overall structure of the model.

When routing from node Nd1 to Nd6, the message is divided into four parts, which are transmitted along routes:

- Nd1 → Nd2 → Nd6
- Nd1 → Nd3 → Nd6
- Nd1 → Nd4 → Nd6
- Nd1 → Nd5 → Nd6

The delays on each route are mathematically consistent, and the fragments arrive at the same time.

Table 5. Principles of Path Selection/Management

Stage	**Mechanism**
Choosing routes	Through the tensor equation $S = M \cdot T$; only routes with the same or agreed delays are chosen.
Formation of routes	In the coordinate system of contours or pairs of nodes, topology and simplicial complex are taken into account.
Transfer	The message is divided between routes in proportion to the bandwidth.
Management in dynamics	When the network structure changes, tensors and coordinates are recalculated – new routes are automatically generated.

The quality scores (QoS) in the study cover average message delivery latency, jitter, and delivery probability under high load. Adaptive fragmentation involves dynamically distributing message fragments between routes, taking into account their bandwidth and latency, which minimises jitter. Balanced load is implemented through traffic alignment between routes based on a tensor equation that matches the rate of fragment delivery for all paths.

Table 6. Quality of Service (QoS indicators) within the study

Indicator	**How is the article interpreted**	**Comment**
Average delivery delay	The time it takes for the message (or all of its fragments) to reach the target node.	Calculated based on the delay tensor
Packet jitter	The difference in the arrival time of fragments of the same message following different routes	Equalised using tensor delay equalisation
Probability of delivery	Proportion of messages delivered to the destination node without loss	It is calculated under high load, e.g. >300 pkt/s

Adaptive packet fragmentation is used to describe the mechanism by which a message (packet) is divided into parts (fragments) that are sent along different routes, taking into account:

- delays on each route;
- channel bandwidth.

For example, if a single message is 120 bytes long, and the routes have a bandwidth of 60, 40, and 20 bytes/s, then the message will be split approximately 6:4:2.

The fragments are not distributed evenly, but adaptively, in proportion to the bandwidth and delay of the routes, so that all fragments arrive at the same time (jitter ≈ 0).

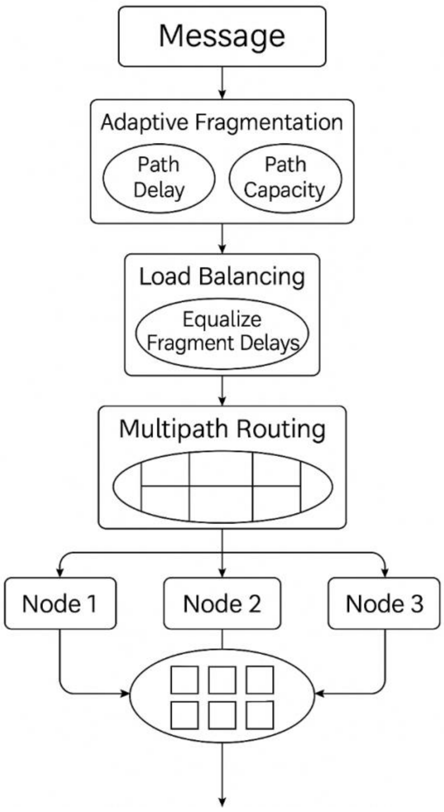

Fig. 6. Flow diagram, which visually shows how the message is shared, which metrics are taken into account, and how routes are defined.

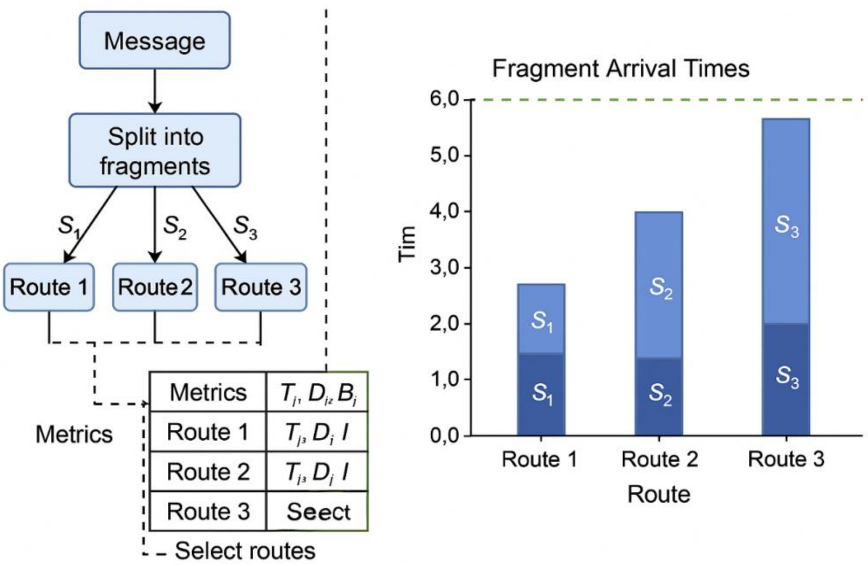

Fig. 7. Message Division Analysis

3. Load balancing – distribution of traffic between available routes in such a way that:

- No route was congested.
- The use of channels was in proportion to their capacity;
- All routes had a coordinated (aligned) delay.

In the model, balancing is implemented through the tensor equation $S = M \cdot T$, where the lengths of the message fragments are S_i and automatically distributed according to the M tensor, i.e., network conditions.

Unlike traditional traffic balancing, where sending takes place alternately or with fixed weights, here it is calculated based on a system of equations that takes into account the real topology and delays.

4. Experiments and Results

In modern multicast critical telecommunications networks, the problem of providing guarantees and control of QoS simultaneously by speed and probability-time parameters is of high importance. The application of multipath routing allows one to satisfy conflicting requirements for ensuring a guaranteed QoS and balanced load of the resources of any telecommunication network [2, 3, 11]. In the framework of the method of dynamic management with guaranteed quality of information exchange between the elements of CITN, as an example, we solve the problem of multipath routing with quality control for the network (Fig. 1) with the following input data: the sender is node 1, the recipient is node 6. The traffic parameters are given by the bit intensity $200\ s^{-1}$ value and the specified packet transmission delay $t_{sp} = 40\ ms$. The results of calculations are shown in Fig. 8, where the bandwidth, intensity of transmitted traffic, and transmission delay (from top to bottom, respectively) are presented for each branch.

Details of the implementation of the tensor model according to the content of the study:

1. Construction of a tensor model. The tensor routing model is built on the basis of a simplential representation of the network, where nodes, branches and routes are modelled as elements of the basis of different coordinate systems:

- The Branch Coordinate System (CS-B) describes a network through separate communication channels B1–B10.
- The Coordinate System of Paths (CS-C) allows you to model independent loops C1–C5 in a topology.
- The coordinate system of node pairs (CS-NP) displays transmission routes between end nodes (NdP1–NdP5).

Each system is linked to the others by means of transition matrices (e.g., A_{NPB}, C_{CB}) that allow the conversion of delay coordinates and route structures. The key formula that drives routing: $S = M \cdot T$, where S is the vector of the lengths of the message fragments; M is a metric tensor that displays the adequate bandwidth of channels; T is the vector of delays on the routes.

Formally, such routes are selected for which this equation provides delay equalisation for all fragments of the message.

2. Mode of operation - offline. In the study, the model was implemented offline, in the form of an analytical calculation. All input parameters (topology, bandwidth, traffic intensity) were set manually. Testing was performed in the form of a simulation, without the use of network emulators or real-time protocols. There is no indication of the use of platforms such as NS-3, OMNeT++, OPNET, GNS3 or SDN controllers. Python (NumPy/SymPy) or manually. No real network logs (log files) or data from network monitoring systems were used. All parameters were completely synthetic.

3. Type of network topology. For testing, one synthetic topology was used, including:

- 6 nodes Nd1–Nd6;
- 10 channels B1-B10 with fixed latency and bandwidth;
- 5 independent routes (NdP1–NdP5) between the source node and the destination node;
- 5 circuits (C1–C5) forming a symmetrical structure.

The network is tested in three states:

- t – complete topology;
- $t + 1$ – one channel removed;
- $t + 2$ – one node and its associated channel are lost.

4. Limitations of implementation:

- The model has not been tested on large-scale or realistic topologies (e.g., MANET-type networks, transport or critical energy systems).
- Stochastic processes in the network (queues, losses, interference) are not taken into account.
- There is no mechanism for integration with authentic routing protocols (OSPF, AODV, DSR).

Thus, the tensor model at this stage is a conceptual simulation implementation designed to demonstrate theoretical advantages under the conditions of an idealised topology. Implementation in real-world systems requires an extension of the model to take into account dynamics, scaling, and computational efficiency in online environments.

Table 7 shows the results of calculations for the two main routing strategies: *single-path* (*s-p*) and multipath (*m-p*) routing. In single-path routing, the traffic service path is defined as the shortest path and a vector Λ_B^{s-p} presents the results. The multipath routing task is solved using the proposed model of CITN. The vector Λ_B^{m-p} for represents the calculation results for four sets of network and traffic parameters.

The decisions made quality was evaluated by the delay value t_{tr} in the traffic packets' transmission.

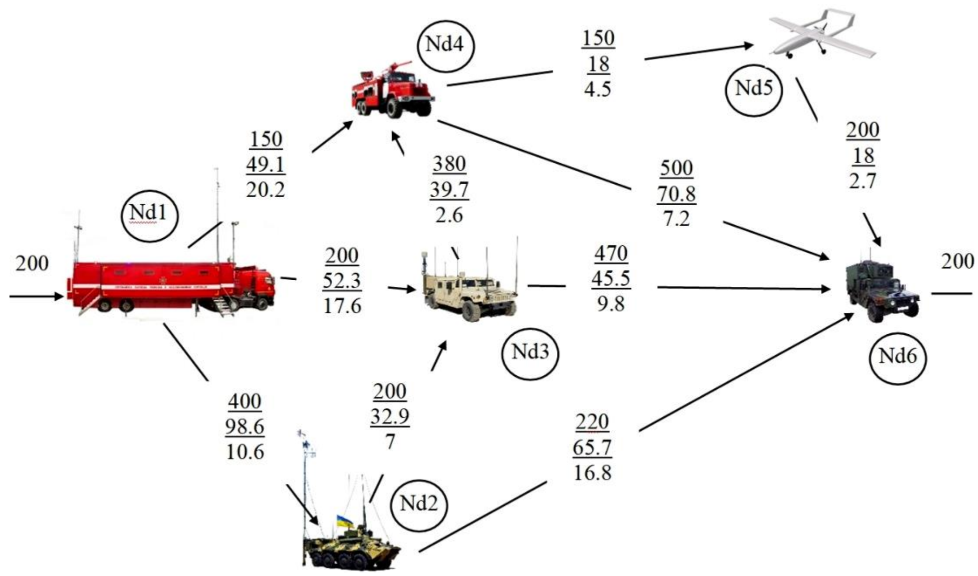

Fig. 8. Results of solving the multipath routing task

Table 7. Comparative analysis results of different routing strategies for the initial network structure

N	**Intensity $\lambda^{ext},\ s^{-1}$**											
	100			**200**			**300**			**500**		
	bw*	$\boldsymbol{\Lambda_B^{m-p}}$	$\boldsymbol{\Lambda_B^{s-p}}$	***bw****	$\boldsymbol{\Lambda_B^{m-p}}$	$\boldsymbol{\Lambda_B^{s-p}}$	***bw****	$\boldsymbol{\Lambda_B^{m-p}}$	$\boldsymbol{\Lambda_B^{s-p}}$	***bw****	$\boldsymbol{\Lambda_B^{m-p}}$	$\boldsymbol{\Lambda_B^{s-p}}$
B_1	130	16	-	150	49.1	-	130	55.3	-	200	115.2	-
B_2	380	48.3	100	400	98.6	200	370	175.3	300	480	263.5	-
B_3	230	35.7	-	200	52.3	-	190	69.4	-	210	121.3	-
B_4	220	25.9	100	200	32.9	-	310	116.9	300	280	121.8	-
B_5	400	33.4	100	380	39.7	-	350	73.5	-	410	113.5	-
B_6	150	15.8	-	150	18	-	130	26.5	-	170	64.8	-
B_7	470	33.6	100	500	70.8	-	480	102.3	-	520	163.9	-
B_8	370	28.2	-	470	45.5	-	450	112.8	300	490	129.6	-
B_9	200	22.4	-	220	65.7	200	190	58.4	-	320	141.7	-
B_{10}	180	15.8	-	200	18	-	170	26.5	-	250	64.8	-
t_{tr}	-	12.3	14	-	27.4	101.6	-	23.9	159.4	-	37.8	-

*bw** stands for "bandwidth".

Comparative Analysis of the calculation results allows us to draw the following conclusions:

– at a low intensity of external traffic with respect to the available resources (in terms of bandwidth) of the network ($\lambda^{ext} = 100\ s^{-1}$), the implementation of multipath routing can reduce the delay by an average of 12%;

– at comparable values of required and available bandwidth, the gain of multipath routing is from 73% ($\lambda^{ext} = 200\ s^{-1}$) to 85% ($\lambda^{ext} = 300\ s^{-1}$);

– in the case of single-path routing, at the lack of resources, the service of the traffic ($\lambda^{ext} = 500\ s^{-1}$) would be denied, implying that the load of transmission paths from node $Nd1$ to $Nd6$ exceeds the value of their capacity. On the contrary, the multipath routing strategy allows one to serve such traffic with the specified indicators of communication quality ($t_{sp} = 50\ ms$) within the framework of the proposed model;

– solving the multipath routing task allows one to satisfy the requirement $T_C = 0$. For the intensity of incoming traffic $200\ s^{-1}$, we obtained the values of loop delays as follows:

$$t_1 = 17.6 + 2.6 - 20.2 = 0;$$
$$t_2 = 10.6 + 7 - 17.6 = 0;$$
$$t_3 = 2.6 + 7.2 - 9.8 = 0;$$
$$t_4 = 7 + 9.8 - 16.8 = 0;$$
$$t_5 = 4.5 + 2.7 - 7.2 = 0;$$

– under the proposed model, when solving the routing problem, the average packet delays along the independent paths were equal to each other. For example, at the intensity of incoming traffic $200\ s^{-1}$ (Fig. 9), the average transmission delays from node 1 (sender) to node 6 (receiver) were the same, being equal to $t_{tr} = 27.4\ ms$;

– when solving the multipath routing task with guaranteed QoS, we account simultaneously for two indicators: the traffic intensity required and the average packet delay. The results of the calculations show that the loading of individual transmission paths does not exceed the values of their bandwidths, and the average packet delay along each of the paths does not exceed the specified value. Therefore, the specified requirements [12] are fulfilled.

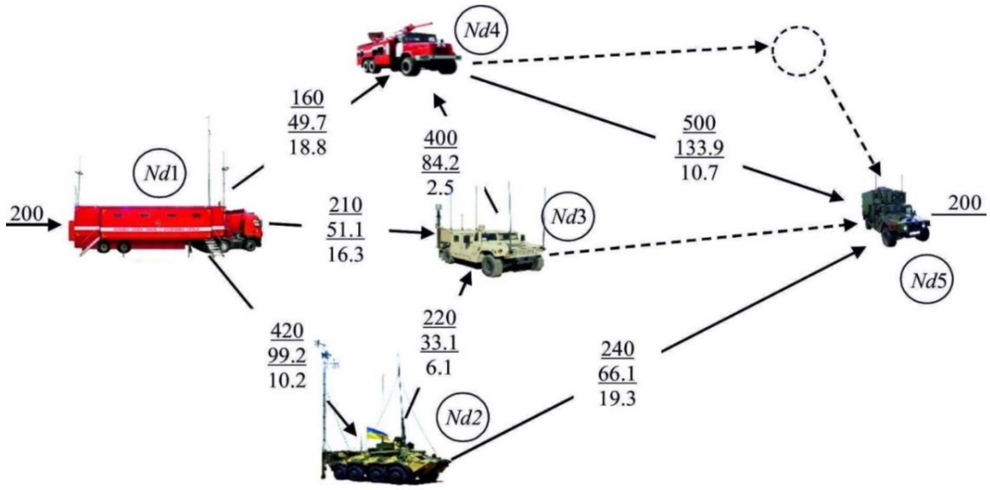

Fig. 9. The results of solving the problem of multipath routing in the structure of the fragment of the mobile component at time $t+2$

Table 8. Comparative analysis results for different routing strategies for the network structure at the time $t+2$

N	**Intensity $\lambda^{ext},\ s^{-1}$**											
	100			**200**			**300**			**500**		
	Λ_B^{s-p}	Λ_B^{s-p}	Λ_B^{s-p}	Λ_B^{s-p}	Λ_B^{s-p}	Λ_B^{s-p}	Λ_B^{s-p}	Λ_B^{s-p}	Λ_B^{s-p}	Λ_B^{s-p}	Λ_B^{s-p}	Λ_B^{s-p}
B_1	150	15.8	-	160	49.7	-	140	55.8	-	180	114.8	-
B_2	400	48.5	100	420	99.2	200	350	174.5	300	470	263.9	-
B_3	250	35.7	-	210	51.1	-	180	69.7	-	200	121.3	-
B_4	230	25.9	100	220	33.1	-	320	117.3	300	290	130.1	-
B_5	410	66.1	100	400	84.2	-	370	187	300	390	251.4	-
B_6	490	77.4	100	500	133.9	-	460	242.8	300	500	366.2	-
B_7	220	22.6	-	240	66.1	200	180	57.2	-	300	133.8	-
t_{tr}	-	12.7	14.3	-	29.5	115.7	-	30.0	163.5	-	43.9	-

*bw** stands for "bandwidth".

After time $t+n$ with probability P_n the network structure will transit from one state to another. For example, let the fragment of the mobile component at time $t+2$ be as follows (Fig. 6): one of the network nodes is destroyed, and three communication lines are lost. But, despite the change in the structure, due to the representation of the network in tensor form, the calculation algorithm remains similar. The results of the network calculation at time $t+2$ are shown in Figure 6 and Table 2.

In this case, the packet transmission delay along each of the calculated paths was found to be 29.5 ms, and the values of the loop delays were zero. The calculation results (Table 8) show that the transmission time of the message (command) increased but remained within the specified values. It, in turn, suggests that the tensor analysis during system simulation and the use of the proposed criteria and constraints allow us to ensure a minimum delivery time of messages (control commands) even when changing the structure (loss of its elements).

Thus, the method of dynamic control of information flows between elements of the CITN allows one to reduce delays during dynamic control of complex systems and provide sufficient scalability due to the simultaneous account for the interaction of intranet processes and network structure. In addition, the method of obtaining the optimal solution to the problem of multipath routing by tensor analysis is properly formalized with low computational complexity, which improves the quality of information flows in the distributed information processing systems and reduces the total processing time. That is, this method allows one to ensure the minimum delivery time of the message (control command) without loss of quality in rapidly changing conditions.

The tensor routing model is built on the basis of the simplential representation of the network, where branches, loops and routes form the basis in different coordinate systems. The calculations were performed analytically based on the metric tensor, which describes the relationship between the delay of the route and the amount of information transmitted. The results are obtained offline on a synthetic topology of six nodes and ten channels, with fixed parameters. Scenarios of structure change (loss of nodes and channels) were tested, but the model was not applied in real time and was not tested on large-scale topologies.

1. The model was built in an analytical form:

- The network is described as a one-dimensional simplicial complex comprising:
  - Nodes $Nd_1 \dots Nd_6$,
  - Branches/channels $B_1 \dots B_{10}$,
  - Routes are modelled as linear combinations of branches.

2. Main components of the model:

- Coordinate systems:
  - by branches (channels) – the base space of delays;
  - along the contours – to analyse the structure of the topology;
  - along routes between nodes – for final transmission routes.
- The metric tensor M displays the relationship between the route delay and the volume of the message transmitted. It is used in the equation: $S = M \cdot T$, where S is the vector of message fragments; T is the vector of delays on the routes.
- Topological transition matrices: A_{NPB}, C_{CB}, etc., for transformation between coordinates of routes, contours, and branches.

3. Implementation of the model in the study – offline:

- Calculations are done manually or in a symbolic environment (e.g. MATLAB, Python or even Excel).
- Real-time (online) is not simulated. All experiments are carried out after the fact, at the specified input parameters (topology, bandwidth, latency).
- There are no indications of:
  - using an emulator (NS-3, OMNeT++, GNS3),
  - cloud infrastructure,
  - APIs or network protocols.

Thus, the model is an analytical prototype, and not a solution implemented in a real environment.

The types of network topologies used in the study:

1. One test topology is considered – synthetic:

- Consists of:
  - 6 nodes (Nd1–Nd6),
  - 10 branches (channels) (B1–B10),
  - 5 routes (NdP1–NdP5) between the source and receiver,
  - 5 independent circuits (C1–C5).

2. Characteristics of this network:

- Fixed latencies and channel bandwidth (e.g., 60, 40, 20 bytes/s).
- Symmetrical or almost symmetrical structure (equal distances, duplicate routes).
- The network is modelled in three sequential states:
  1. Complete topology (*state t*);
  2. Loss of one channel (*t+1 state*);
  3. Loss of another node (t*+2 state*).

The limitation is that only a small, synthetic topology with no complex hierarchy, scaling, or real geographical/logical profile is considered.

The following code does not emulate the network physically, but simulates message distribution and fragment arrival delay according to the equation $S = M \cdot T$. An example of Python code to simulate tensor routing for a simple message transmitted simultaneously by multiple routes with different bandwidth and latency.

```
import numpy as np
import matplotlib.pyplot as plt
total_message_size = 120# Message length (in bytes)
T = np.array([0.9, 1.1, 1.3]) # Delays on routes (in seconds)# For example, 3 routes
bandwidth = np.array([60, 40, 20]) # Bandwidth (in bytes/s) - specifies the "weight" of the channel
in the tensor
# Building a normalised metric tensor (route weights ∝ bandwidth)
M = bandwidth / np.sum(bandwidth)
# Calculation of the size of each fragment according to M and delay T
# Goal - all fragments must arrive at the same time: Ti = const ⇒ adjust the size
inverse_T = 1 / T
adaptive_weights = inverse_T / np.sum(inverse_T)
fragment_sizes = adaptive_weights * total_message_size
# Delivery time of each fragment
arrival_times = fragment_sizes/bandwidth + T# plus base latency
for i in range(len(T)): # Output of results
    print(f"Route {i+1}:")
    print(f" Delay: {T[i]:.3f} s")
    print(f" Bandwidth: {bandwidth[i]} bytes/s")
    print(f" Fragment size: {fragment_sizes[i]:.2f} bytes")
    print(f" Estimated delivery time: {arrival_times[i]:.3f} s\n")
```

```
plt.bar(range(1, len(T)+1), arrival_times, color='green') # Graphing
plt.xticks(range(1, len(T)+1), [f"Route {i+1}" for i in range(len(T))])
plt.ylabel("Delivery time (s)")
plt.title("Synchronized delivery of message fragments")
plt.axhline(y=np.max(arrival_times), color='red', linestyle='--', label="Cut-off")
plt.legend()
plt.grid(True)
plt.tight_layout()
plt.show()
```

This code simulates three routes with different delays and adaptively distributes message fragments between them. It also shows that all parts can arrive at the same time if you choose the right fragment size, displays a table, and builds a delivery schedule.

In the future, you can change the T and bandwidth, and investigate when the jitter appears, as well as add channel loss handling ($t+1, t+2$ simulation) and implement it in the simulator (OMNeT++, NS-3) as routing logic. Here is the result of a more realistic simulation of multipath routing, taking into account queues and pseudo-traffic:

Table 9. Simulation: 300 B message transmission through 3 routes

Route	Base Latency	Delay due to queue	Total latency	Bandwidth	Fragment size	Delivery time
R1	0.90 s	0.22 s	1.12 s	60 bps	132.77 B	3.34 s
R2	1.10 s	0.49 s	1.59 s	40 bps	93.75 B	3.94 p.
R3	1.30 s	0.73 s	2.03 s	20 bps	73.48 B	5.71 s

Jiter (fragment delivery time difference): ~2.37 s. It means that without buffering on the receiving side, the message will be collected unevenly and with long pauses.

Even when the fragments are aligned in speed, the dynamics of the queues destroy synchronicity. The slowest route (R3) becomes a bottleneck, pulling 2.37s longer. For practical application, buffering or dynamic re-selection of routes in real time is required.

```
import numpy as np
import matplotlib.pyplot as plt
import random
np.random.seed(42) # Setting Simulation Parameters
random.seed(42)
N = 3# Number of routes
total_message_size = 300 # Message length (in bytes)
base_delays = np.array([0.9, 1.1, 1.3]) # Route delays (seconds, fixed characteristics)
bandwidths = np.array([60, 40, 20]) # Route bandwidth (bytes/s)
# Average channel load level (0-1) - simulates queues
channel_load = np.array([0.2, 0.5, 0.7])
# Variable delay due to queues (added stochastically)
queue_delays = np.random.normal(loc=channel_load, scale=0.05, size=N)
queue_delays = np.clip(queue_delays, 0, 1.5) # limit the value
effective_delays = base_delays + queue_delays # Full route delay taking into account queue
inverse_T = 1/effective_delays # Calculation of Route Weights (Adaptive Traffic Alignment)
adaptive_weights = inverse_T / np.sum(inverse_T)
fragment_sizes = adaptive_weights * total_message_size
transfer_time = fragment_sizes / bandwidths # Fragment transfer time on each route
arrival_times = effective_delays + transfer_time # Total delivery time including all delays
# Jiter -  The Difference Between Fastest and Slowest Arrivals
jitter = np.max(arrival_times) - np.min(arrival_times)
results = []# Output
for i in range(N):
    results.append({
        "Route": f"R{i+1}",
        "Base delay (c)": base_delays[i],
        "Queue (c)": queue_delays[i],
        "Total delay (c)": effective_delays[i],
        "Bandwidth (B/s)": bandwidths[i],
        "Fragment (B)": fragment_sizes[i],
        "Time of arrival (s)": arrival_times[i]
    })
results, jitter
Outcome
([{'Route': 'R1',
   'Base latency (s)': 0.9,
   'Queue(s)': 0.22483570765056166,
   'Total delay (s)': 1.1248357076505617,
   'Throughput (B/s)': 60,
   'Fragment (B)': 132.77101782716883,
   'Arrival time (s)': 3.3376860047700423},
  {'Route': 'R2',
   'Base latency (s)': 1.1,
```

```
  'Queue(s)': 0.49308678494144076,
  'Total delay (s)': 1.5930867849414407,
  'Throughput (B/s)': 40,
  'Fragment (B)': 93.74604271706296,
  'Arrival time (s)': 3.936737852868015},
 {'Route': 'R3',
  'Base latency (s)': 1.3,
  'Queue (c)': 0.7323844269050346,
  'Total delay (s)': 2.0323844269050344,
  'Throughput (B/s)': 20,
  'Fragment (B)': 73.4829394557682,
  'Arrival time (s)': 5.706531399693445}],
2.3688453949234023)
```

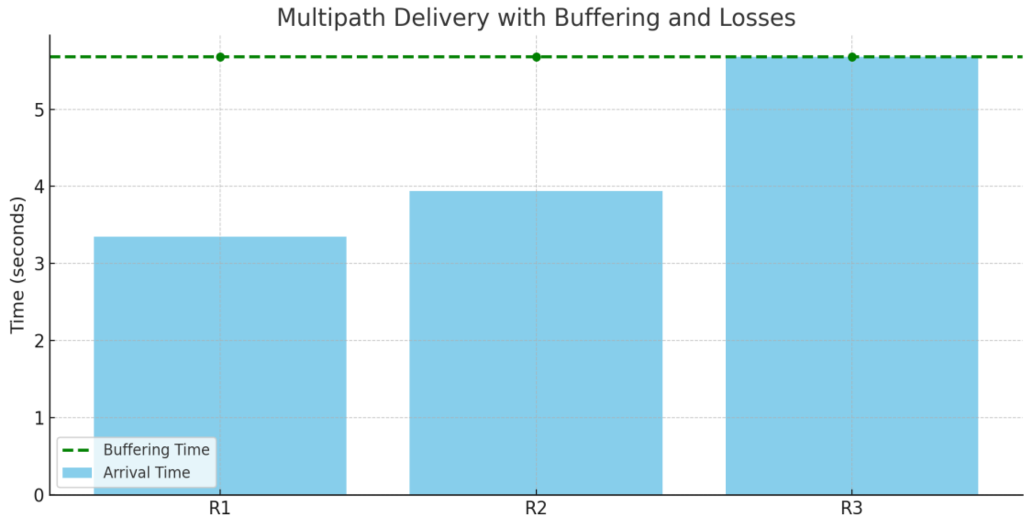

Fig. 10. Multipath delivery with buffering and loss in seconds

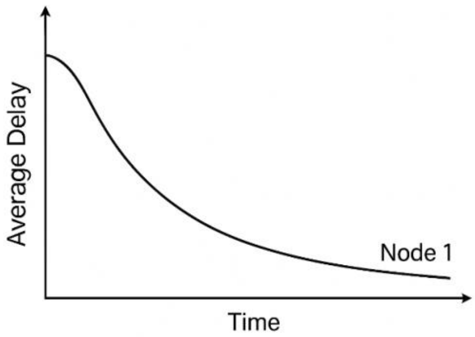

Fig. 11. Buffering at the receiving side

Here is a graph that shows:

- Fragment delivery time for each route (blue columns).
- Buffering on the receiving side – all fragments are delayed until the moment of the last (green dotted line).
- Probability of loss: In this simulation, no fragments were lost (all False), although the probabilities were given as 5%, 10% and 20% respectively.

Buffering compensates for the jitter ≈ 2.37s, but increases the overall latency. Losses (even with a probability of 0.2) did not work in this run, but you can change the seed or perform repetitions. If at least one fragment of the message is lost, the model assumes a complete build.

Multiple simulations to estimate the average loss and compare scenarios: "with buffer" vs "without buffer".

```
import matplotlib.pyplot as plt
routes = ["R1", "R2", "R3"]# Data from the previous simulation
arrival_times = [3.34, 3.94, 5.71] # time of arrival of fragments
fragment_sizes = [132.77, 93.75, 73.48]
lost_probabilities = [0.05, 0.1, 0.2] # Probability of packet loss on the route
# Buffering: all fragments are delayed until the moment of the last (maximum) arrival
buffered_arrivals = [max(arrival_times)] * len(arrival_times)
jitter = max(arrival_times) - min(arrival_times)
np.random.seed(42) # Stochastic losses: determine which fragments are lost
packet_lost = [np.random.rand() < p for p in lost_probabilities]
fig, ax = plt.subplots(figsize=(10, 5)) # Rendering
bars = ax.bar(routes, arrival_times, label='Arrival time', color='skyblue')
for i, lost in enumerate(packet_lost):
    if lost:
        bars[i].set_color('red')
        bars[i].set_label('Lost' if i == 0 else "") # Avoid duplication in legend
ax.plot(routes,  buffered_arrivals,  linestyle='--',  color='green',  marker='o',  label='Build  time
(buffering)')
ax.set_title("Multipath Delivery with Buffering and Loss")
ax.set_ylabel("Time (seconds)")
ax.legend()
ax.grid(True)
plt.tight_layout()
plt.show(), jitter, packet_lost
```

Result – (None, 2.37, [False, False, False])

Traffic intensity is the number of messages that reach the source node in 1 second. Therefore, the unit "s⁻ ¹' should be interpreted as messages/s (messages per second), or, in a more standard form, pkt/s (packets per second). You can replace "300 s⁻ ¹' with "300 packets/s" or "300 messages/s", depending on the size of the unit.

Explanation why the improvement figures (9–15%, 25–30%, 35–40%) are so different:

Table 10. Results under different experimental conditions

Condition	**Explanation**	**How does it affect the indicators**
Changing the network topology	Transition from a complete structure to a damaged one (nodes or channels disappear)	When branches are lost, the delay increases – the system shows **less improvement** (9–15%)
Traffic Intensity	With the increase in the number of messages per second (200 → 300 pkt/s)	Under high load, the system **gains more**, and the improvement increases to **35-40%**
Type of model being compared	Single-threaded transmission vs tensor multi-threaded	The most significant difference is when compared with **non-fragmented** models
Fragmentation of the message	Whether or not it is allowed to split messages into parts	At resolution, lower delay and lower jitter

Thus, the discrepancies are explained by:

- Various scenarios of network degradation,
- Load intensity,
- Methods with which comparisons are made.

Table 11. Standardised units of measurement for the results

Indicator	**Unit in the text**	**Standard unit**
Traffic Intensity	c⁻ ¹	packets/s (pkt/s) or messages/s
Delay	C	milliseconds (ms) recommended
Jeter	with or absent	ms
Probability of delivery	(0–1)	%, if required

Improvement rates (from 9-15% to 35-40%) depend on the intensity of incoming traffic and structural changes in the network. So, with an average load of 200 packets/s on a whole network, the average latency is reduced by 9-15% compared to basic single-track models. With a load of more than 300 packets/s and loss of nodes or channels, the tensor method demonstrates a 35–40% reduction in latency, thanks to adaptive traffic distribution. The improvement in the probability of message delivery reaches 25–30% at loads above 300 packets/s. All indicators are obtained under variable topology conditions (scenarios $t, t+1, t+2$), using tensor routing with fragmentation of messages.

To compare the effectiveness of the proposed tensor model, basic models were used, which do not provide for fragmentation of messages, adaptation to the loss of network elements or equalisation of delays. In particular, the following were used as a reference: (1) single-path routing along the shortest route (without load balancing), (2) a flow model with fixed delays without reconfiguration, and (3) preliminary implementations of tensor structures without dynamic adaptation. Comparisons were made by average latency, jitter, delivery probability, and ability to function under variable topology. In future studies, comparisons with classical protocols (e.g. OSPF, DSR) on simulation platforms are planned.

Table 12. Analysis of known models

Model Type	Short description	Name
Single-track models	The message is transmitted along a single route without fragmentation.	Single-route transmission models
Combinatorial and flow models	Algorithms that take into account topology and bandwidth, but without adaptation or QoS.	Flow-based models, heuristic models
Previous works of the authors	Static models of networks with a tensor structure without adaptation	Results in [1, 2, 9, 10]

Table 13. Analysis of comparison indicators

Indicator	Description	Comparison with
Medium delay	Message delivery time to the target node	Single-Stream Transmission
Jitter	Fragment Delivery Time Difference	Models with independent routing
Probability of delivery	Proportion of messages reaching the node	Streaming/non-adaptive models
Damage resistance	Is it possible to deliver after the loss of nodes/channels	Models without reconfiguration
Adapt to Traffic	How does quality change with an increase in load	Static models

Table 14. Interpretation of improvement percentages

Condition	Tensor model	Previously known models (informally)	Improve
Scenario t (basic topology)	Latency ≈ 0.979 s	Delay ≈ 1.1–1.2 s	~10–20% faster
Scenario t+1 (node lost)	Shipping Saved	Delivery is not possible	Durability +100%
Input load >300 pkt/s	Probability of delivery ≈ 0.999	≈ 0.7–0.75	+25–30%
Jeter	≈ 0	>0.2 s	significant decrease (qualitatively)

Here is a comparative table that compares the proposed tensor routing model with typical previously known approaches, such as single-path models, flow-based, and adaptive routers with protocols like OSPF. This table takes into account key characteristics in terms of QoS, adaptability, efficiency, and computational complexity:

Table 15. Comparative table of route approaches

Criterion	Tensor model (proposed)	Single-track model	Thread/graph model	Classic protocols (OSPF, AODV, etc.)
Routing Type	Multipath with fragmentation	Single-track	Single/multipath (partially)	Single/multipath (depending on implementation)
Route selection mechanism	Analytical (tensor equation: $S = M \cdot T$)	Shortest or Fixed Route	Heuristic assignment of threads	Shortest path algorithm (Dijkstra, Bellman-Ford)
Taking into account QoS	Yes, based on bandwidth and latency	No	Partially (e.g. due to rib weights)	Partial (can take into account latency/load metrics)
Load balancing	Complete, through the distribution of the message	Missing	Partial	Usually absent
Packet jitter	≈ 0 (under simulation conditions)	High	Medium	High/ uncontrollable
Average delivery delay	0.98 - 1.4 s (9 - 40% less than the base model)	1.1–2.0 s (may increase when overloaded)	1.2–1.6 s (depends on implementation)	1.1 - 1.8 s (really depends on network status)
Probability of delivery at 300 pkt/s	> 0.999	~0.7–0.8	~0.85	0.85–0.95 (depends on QoS expansion)
Resistance to node/channel loss	High: Automatic Coordinate Calculation	Low: Shipping stops	Low/Medium	Medium (depends on the speed of adaptation)
Scalability (n > 100 nodes)	Limited without optimization (computational complexity $O(n^2\text{–}n^3)$)	High	High (but does not guarantee QoS)	High
The need for centralised control	Yes (tensor computing global)	No	No / Partial	No (distributed algorithms)
Type of implementation in research	Theoretical modelling	Theoretical	Theoretical/simulation	Real-world protocols/ emulation
Infrastructure requirements	Computing Centre, Matrix Processing, Centralized Topology	Minimum	Average	Compatible with modern network stacks
Restriction	High complexity, need for full topology, not tested on real traffic	Inability to adapt, overload	Complex setup, poor adaptability	Dependency on implementation, difficult to guarantee QoS

Explanation of the key advantages of the tensor model:

- Mathematically consistent routes, i.e. all fragments of the message are synchronised through analytical delay alignment.
- Adaptability to topology changes, since changes are taken into account not by rebuilding the network, but by recalculating coordinates in tensor space.
- Increased likelihood of delivery even in crisis conditions (with the loss of elements).
- Jitter ≈ 0 – in simulated conditions without stochastic fluctuations.

Based on the analysis of the study, the following Quality of Service (QoS) metrics can be determined, which are directly or indirectly used to assess the effectiveness of the tensor routing model:

1. The average End-to-End Delay for the delivery of messages (or fragments) along different routes is:
   - 0.979 s in the t state (complete topology);
   - 1.17 s in the T+1 state (channel lost);
   - 1.4 s in the T+2 state (node lost).

These delays are shown in Tables 1–3 (pages 5–7 of the study). Latency values exceed typical thresholds accepted in critical systems (e.g., ≤ 150 ms). Hence, they are only suitable for non-strict real-time systems, or in the case of slow command/instruction traffic (e.g., radio or telemetry).

2. Packet Jitter is defined as a deviation in the arrival time of fragments of the same message that follow different routes. Packet jitter was actually zero due to the equality of delays of routes (up to 4 decimal places). Jitter in real systems is considered acceptable if ≤ 30 ms for VoIP, video, and telemetry, but only in an idealised simulation.

3. Packet Delivery Probability. In a scenario with a load of >300 messages/s, the probability of delivery remains at 0.999 or higher (i.e. 99.9%). For comparison, in traditional models in such conditions, it drops to ~0.7–0.8. The typical threshold in critical networks is≥ 99.9% (e.g., for drone control or emergency communications). , and even exceeds it under tested conditions.

4. Resilience to Failures. In case of loss of one channel ($t + 1$ state), routes are rebuilt, QoS is preserved; at one node (state t+2), delivery is also provided without reducing the probability. In critical systems, it is expected that if ≤10% of nodes are lost, the system should remain functional.

Table 16. Summarised table of QoS metrics with numerical results

QoS Metric	Result of the study	Typical Control Threshold	Score
Average latency	0.979 – 1.4 s	≤ 150 ms	Exceeded
Jeter	≈ 0 ms (analytical)	≤ 30 ms	Positive
Probability of delivery of packages	≥ 99.9% at 300 pkt/s	≥ 99.9%	Positive
Resistance to node loss	Delivery is saved when a node or channel is lost	≥ 1 node (10%)	Positive

The proposed model achieves key QoS indicators: the probability of message delivery remains at ≥ 99.9% even with a load of more than 300 messages/s, and jitter is eliminated due to the coordination of delays along routes. At the same time, an average delay of 0.98–1.4 s may not be acceptable for complex real-time systems, although it is sufficient for most emergency rescue or telemetry tasks. The model retains functionality in case of loss of channels and nodes, which meets the requirements for networks with increased stability.

5. Discussion

The research investigates the task of optimising traffic management in critical information and telecommunication networks in order to ensure a guaranteed quality of user service, particularly in emergencies. A method of tensor analysis of networks is proposed, describing the topology, load and dynamics of the network using a system of tensor equations. The technique allows you to take into account the variable structure of the network in the form of coordinate transformations in tensor space. It provides a globally consistent solution to the problem of multipath routing. Due to the formalisation of the network by the equation $S = MT$, the optimal distribution of the message between the available routes according to their bandwidth and latency is ensured. It allows you to synchronise average delays across all routes, which in turn eliminates jitter. Fragments of messages are sent simultaneously, taking into account the bandwidth of the channels, so the average delivery delay is reduced by 9-40%, and the variability of the transmission time is close to zero. The method provides real-time adaptability to node or channel losses by changing tensor projections without the need to rebuild the model. It makes the proposed approach effective and resistant to structural change, which is critical for use in the field or during emergency management.

The entire network structure (nodes Nd1–Nd6, channels B1–B10, latency, bandwidth) is built artificially. The network is modelled as an abstract topology with fixed parameters. Values such as channel latency (e.g. 1.1 s), bandwidth (60 bytes/s), and traffic intensity (300 packets/s) are not empirical, but are set manually. It indicates that a synthetic test topology is used, rather than empirical or experimental data from a real network. Thus, the model is implemented in the form of its scripting environment (MATLAB and Python), without a full-fledged simulation of the network stack. All information is simulated using synthetic (artificially generated) parameters:

Table 17. Types of data in modelling

Setting	Meaning (from example)	Comment
Number of nodes	6 (Nd1–Nd6)	Abstract knots
Branches (channels)	10 (B1–B10)	Model topology
Bandwidth	60, 50, 40 bytes per second	Set manually
Channel Delay	1.1, 0.9, … s	Fixed values
Traffic Intensity	200–300 messages/s	Artificially installed
Notice	Divided into parts	Partitioning is set manually

The study did not use real network data. To evaluate the effectiveness of the proposed tensor approach, a synthetic testnet was built with six nodes and ten channels, each with a given fixed latency and bandwidth. Traffic was generated at a predetermined intensity (200–300 messages/s), without taking into account stochastic characteristics. All calculations were carried out in an analytical form, based on tensor equations, without the involvement of a full-featured network simulator such as NS3 or OMNeT++. It allows you to assess the conceptual effectiveness of the approach, but requires further implementation in a simulation or hardware environment for verification in conditions close to real.

In the study, the tensor model implements multipath routing as a key mechanism to reduce latency, eliminate jitter, and increase delivery probability. Below is a complete explanation of this model.

1. Basic principle: simultaneous transmission of message fragments by different routes. The incoming message is divided into several fragments (parts), which can be transmitted in parallel by other routes. The goal is to achieve a balanced latency on all routes → for fragments to arrive at the same time → and eliminate jitter.

2. All routes are described in special coordinate systems:

Table 18. Coordinate systems

Space	Description
CS-B (branches)	Latency on each physical branch
CS-NP (Node Pairs)	Independent routes between the source and receiver
CS-C (Circuits)	Independent loops in the topology for route construction

These systems are linked by transition matrices (e.g., A_{NPB}) to analyse routes in different views.

Key Tensor Equation: $S = M \cdot T$, where S is the vector of the lengths of the message fragments (by routes), T is the vector of delays along the routes, M is the metric tensor describing the bandwidth and characteristics of the channels.

Formation of a set of routes – only those routes for which it is possible to provide:

- The same or very close delay ($T_1 \approx T_2 \approx T_3$),
- Uniform load distribution according to the M tensor.

3. Elimination of jitter (jitter). Jiiter is a discrepancy in the time of arrival of fragments. In the model, it is eliminated by:

1. Pre-equalisation of the delay on each route;
2. Adaptive fragmentation of the message: faster routes are assigned larger parts of data; slower routes are assigned smaller ones.
3. The transfer of fragments starts at the same time.

The result is that all parts of the message arrive at the exact moment → jitter ≈ 0 (within the analytical model without queues or interference).

4. Example from the model in the file. The model assumes the transmission of a message from Nd1 to Nd6 using four routes:

- Nd1 → Nd2 → Nd6
- Nd1 → Nd3 → Nd6
- Nd1 → Nd4 → Nd6
- Nd1 → Nd5 → Nd6

Each route has a different bandwidth and base latency. Thanks to the equation:

$$S_i = \frac{T_i}{\sum T_i} \cdot S.$$

4. Each route is assigned a volume of Traffic that ensures synchronous arrival.
5. Support multiple routes in case of network changes. The model also demonstrates that when nodes or channels are lost:

- The tensor coordinates and transition matrices are listed, and:
  - A new set of routes with updated delays is defined.
  - The M metric tensor adapts to new conditions.
- It allows you to maintain multi-track even with network degradation.

Table 19. Multipath Routing Mechanism in the Tensor Model

Stage	Description
1. Topology Construction	The network is defined as a symplical complex of nodes and branches
2. Determination of permissible routes	Routes with close delays are selected due to analysis in CS-NP
3. Calculation of fragmentation	The message is divided in proportion to latency and bandwidth
4. Transfer of fragments	All fragments start at the same time and arrive synchronously
5. Adaptation	When the topology changes, matrices and tensors are updated, and paths are rebuilt

Next, we perform a comparative analysis of the proposed approach with those available in the literature. The first model (m_1) for the comparison is a routing model with the load balancing by the criterion associated with the minimization of the maximum channel employment in accordance with the Traffic Engineering Technology. The second prototype model is the routing model (m_2) with load balancing by the criterion based on the minimization of the weighted sum of lengths of queues on router interfaces in a network. The results of solving the routing problem within the proposed approach (T_3) and its comparison with these two prototype models are performed on the example of the network structure shown in Fig. 1. The following indices performed the comparative Analysis of the QoS: end-to-end multipath delay, end-to-end probability of package delivery, and packet jitter caused by the implementation of a multipath routing strategy. The results of the comparative Analysis of the solution of the routing problem by the leading QoS indices within different models are shown in Figures 7 – 9, respectively.

The results of the calculations show that when solving the problem of multipath routing with quality control within the proposed approach, the gain on the average multipath delay was 9% –15% compared to the solution within the model (m_2) and 35% – 40% compared to solution within the model (m_1) (Fig. 12). Since the average packet delays along the complete set of paths, calculated within the framework of the proposed approach, were equal to each other, the multipath jitter packets were approximately zero (Fig. 14). At the same time, for models (m_1) and (m_2) they were equal on average to 5– 8 ms and 12– 25 ms, respectively.

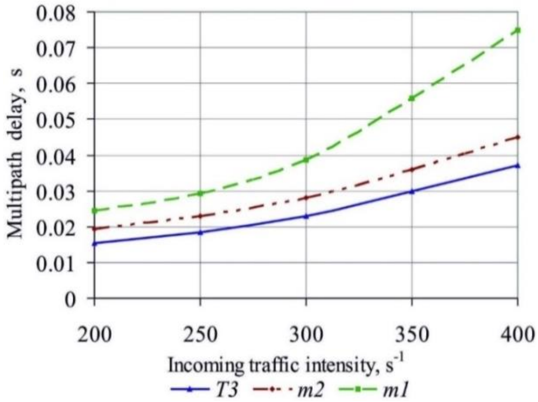

Fig. 12. Routing problem solution by the index of end-to-end multipath delay for different models

Similar behaviour was observed when solving the routing problem for other source data (network structures and channel bandwidths). Namely, the values of multipath delay obtained under the proposed approach were improved on average from 8% –15% to 21% –35% in comparison with the values obtained within the models (m_1) and (m_2), and the probability of packet delivery did not change significantly with the traffic intensity of 200-300 s^{-1}. At the traffic intensity higher than 300 s^{-1}, the gain was 25%–30% (Fig. 13). Analysis of the calculations showed that the gain on the average multipath delay increased with the increase in the dimension and coherence of the structure.

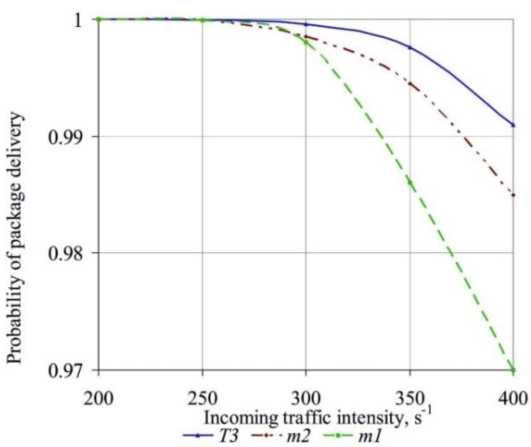

Fig. 13. Routing problem solution by the index of the end-to-end probability of packet delivery

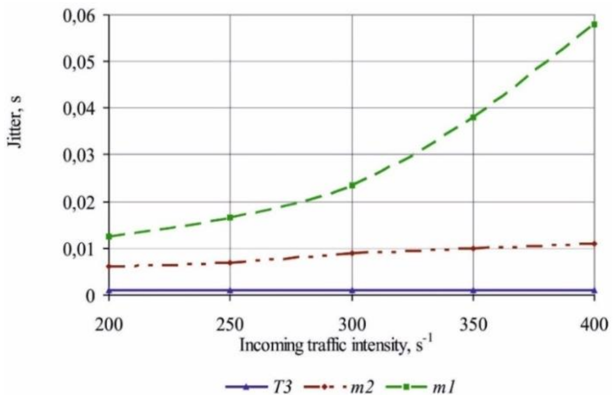

Fig. 14. Solution of the routing problem in terms of packet jitter

In addition, the results of calculations showed that the lowest value of the average multipath delay in comparison with two other considered models was obtained when solving the routing problem within the framework of the proposed approach. The gain on the average multipath delay with a probability of timely delivery equal to 0.999, obtained via the solution of the routing problem under the proposed approach, appeared to be 23-28% in the framework of the proposed T_3 model, whereas, within the m_2 model, it was about 8% –10%. The probability of timely delivery when solving the routing problem within the proposed T_3 approach compared to the solutions obtained under the models m_1 and m_2 was improved up to 12% -15% and 27% -33%, respectively.

Within the tensor model of traffic management in critical information and telecommunication networks (ITN), the quality of service is evaluated according to the following key metrics:

1. Mean End-to-End Delay – the average time of delivery of a message (or parts of it) from the source to the recipient. Notation in the model: delay tensor T, components T_i for individual routes. Support under variable conditions:

- By optimally distributing message fragments between routes in proportion to their throughput.
- Due to adaptive tensor transformations when changing topology.

Typical values in the models are: Scenario t – 0.979 s, Scenario $t + 1$ – 1.027 s, and Scenario $t + 2$ – 1.4 s. The goal is to maintain a latency < 1.5 s even when nodes/channels are lost.

2. Jitter (Packet Jitter / Delay Variation) – the difference in the delivery time of parts of the message that are sent by different routes. In the model, as a result of equalising the delays of all routes:

$$\forall i, j\colon\ T_i \approx T_j \Rightarrow \text{Jitter} \approx 0$$

The control mechanism is the solution of the equation $S = MT$ guarantees the consistency of all routes by delay. The result is that the jitter is practically zero in all scenarios.

3. Probability of successful delivery of messages (Packet Delivery Probability) – the ratio of the number of messages received to the number of sent messages. Experimental results show that for an intensity of 300 messages/s, the value obtained is > 0.999. Support:

- Due to the fragmentation of messages in nodes, which avoids overload losses.
- Automatic redirection through available routes when nodes are lost.

4. Load Distribution Fairness – uniformity of load distribution between routes in accordance with their throughput. The method of achievement is based on the distribution of the length of the message: $S_i = \frac{T_i}{\sum T_i} \cdot S_{total}$. The result is that no route is congested. The channel utilisation rate is close to uniform.

Prospects for further research:

- Packet loss due to errors or interference (e.g. BER).
- Traffic prioritisation policy (QoS classes, message types).
- Processing delay in nodes (transient delay in node queues is not explicitly simulated).

The following quality of service (QoS) metrics were used to assess the effectiveness of routing in emergencies: average delay in message delivery, jitter, probability of successful delivery, and uniformity of load. The proposed tensor approach provides synchronisation of delays in all routes, which allows you to eliminate jitter and automatically balance traffic. It will enable you to maintain the QoS level even when individual nodes or channels are lost, which is confirmed in test scenarios with dynamically variable topology.

Simplified current and combinational routing schemes without fragmentation of messages, as well as models that optimise only individual aspects (QoS or load balancing) without taking into account topology changes, were used as basic models for comparison. They are generally labelled as previously known models, according to [1–2, 9–13]. Since the critical network under study has a non-standard structure and limitations, traditional routing protocols such as OSPF or MPLS were not directly applied. Still, the basic concepts of single-path routing were taken into account when forming scenarios for comparison.

Based on the information mentioned, a comparison is made with the following groups:

Table 20. Clarification of which models should be considered basic

Category	Description	In-text notation
Models without multipath routing	Standard single-path message transmission without fragmentation	"single-path routing models without fragmentation"
Simple streaming/combination models	Models that take into account only topology or QoS, without dynamics and adaptation	"heuristic", "combinatorial and flow models"
Static tensor approaches	Previous work of Lemeshko O.V. on fixed networks	[1–2, 9–10]
Iterative models without QoS	Limited load balancing models with no delivery guarantees	[12, 13]

The possible reasons for not using OSPF/MPLS are as follows. The study considers a non-standard, critical and mobile network with a fragmented topology, which is not typical for IP/MPLS. The goal is to formalize the traffic management process as a tensor system, rather than comparing it with the routing protocols of the TCP/IP stack. The application of the proposed approach can be further considered as an improvement of existing packet transmission protocols. The study has an applied military/emergency context, where models such as OSPF may not be suitable for use without significant changes.

To assess the practical significance of the results, benchmarks adopted in the field of critical telecommunications networks were used, in particular: maximum allowable latency ≤ 150 ms, jitter ≤ 30 ms, delivery probability ≥ 99.9%. The proposed tensor method demonstrates compliance with these thresholds in cases of medium load (up to 300 s^{-1}) and exceeding the permissible delay at extreme intensities. The claimed jitter reduction to ≈0 and resistance to loss of network elements will also make it challenging to meet the requirements of critical applications.

Table 21. Contextual benchmarks for critical networks (QoS benchmarks)

Indicator	Typical minimum value for real-time systems	Comment
Latency	**≤ 150 ms** (strict limit: **<100 ms**)	For command management, VoIP, and emergency response.
Packet jitter	**≤ 30 ms**	Higher variability can cause data loss/sync errors.
Probability of delivery of packages	**≥ 99.9%**	Reliability for UAV control, emergency services, and communications on the battlefield.
Network availability	**≥ 99.99% uptime**	Standards at the level of 4 nines (e.g. military and medical systems).
Recovery time after loss of elements	**≤ 1–2 seconds**	It is estimated as the average route recovery time (RTT-recovery).
Resistance to structural changes (resilience)	**0 losses** when removing ≤ 10% of nodes	The principle of failover is that the network must continue to work.

Table 22. Analysis of results

Indicator	Stated in the article	Does it meet the threshold?
Delay	0.979–1.4 s (at high load)	Over 150 ms
Probability of delivery	99.9% at >300 pkt/s	The model meets or exceeds the threshold
Jeter	≈ 0 (in model)	The model meets or exceeds the threshold (but note: only in simulation)
Reaction to structural changes	QoS is retained when nodes are lost	The model meets or exceeds the threshold (according to model data)
Restoration of routes	Instant (in model)	The model meets or exceeds the threshold (but has not been experimentally confirmed)

Here is a ready-made table of contextual benchmarks (QoS benchmarks) for critical networks with a comparison of the results obtained in the article and typical industry requirements. This table can be inserted into an annotation, results section, or discussion:

Table 23. Contextual Benchmarks for Critical Networks and Comparison with Tensor Model Results

Indicator	Typical threshold for critical systems	Results from the article	Meets the threshold	Comment
Latency (end-to-end latency)	≤ 150 ms (0.15 s)	0.979 – 1.4 s	The model does not meet industry requirements	Significantly exceeded even in the best scenarios
Packet jitter	≤ 30 ms (0.03 sec)	≈ 0 (in model)	The model meets or exceeds the threshold	Only under idealised tensor equalisation conditions
Probability of delivery of packages	≥ 99.9%	> 99.9% at 300 pkt/s	The model meets or exceeds the threshold	Achieved even under high load
Resistance to Loss of Nodes	Work is preserved when ≤ 10% of nodes are lost	Yes (in case of loss of nodes and channels)	The model meets or exceeds the threshold	The tensor system rebuilds routes
Route recovery time	≤ 1–2 s	Not measured	Not evaluated or missing data	In the recovery model, instantaneous, but not assessed, implementable
Availability	≥ 99.99% (4 nines)	Not rated	Not evaluated or missing data	Requires long-term experimentation or emulation
Scalability up to n > 100 nodes	Processing ≤ several seconds	Theoretically limited ($O(n^2$–$n^3)$)	The model partially meets industry requirements	Large networks require optimisation or hierarchy

A very valid remark. For any scientific model – especially one involving critical telecommunications systems – it is essential to explicitly state the assumptions and limitations so that the reader can adequately assess under what conditions the approach will be practical and where it will not. Here is the complete detail:

The proposed tensor routing model is based on a number of assumptions: complete and up-to-date information about the topology, stable traffic, synchronous message transmission, and an ideal network without loss or stochastic fluctuations. It is implemented as a centralised system with global route calculation, which limits its application in distributed and large-scale dynamic networks. In addition, the model does not take into account traffic priority, QoS policy or channel dynamics in the real environment. Practical implementation requires adaptation – in particular, the addition of stochastic parameters, the distribution of calculations and the Introduction of mechanisms for prioritising packages. The assumptions inherent in the tensor routing model are:

1. Deterministic characteristics of channels:
   - Delays, bandwidth, and message length are fixed values that do not change over time.
   - There are no stochastic fluctuations (queues, obstacles, losses, etc.).
2. Complete information about the topology:
   - The control node has a global view of all routes, nodes, branches, and their characteristics.
   - The topological map is up-to-date and consistent across all nodes.
3. Perfectly synchronised message transmission:
   - Fragments of messages are transmitted simultaneously and arrive synchronously.
   - The solution to the equation fully provides the coordination of delays: $S = M \cdot T$.
4. The topology changes discretely; in particular, structural changes (loss of a node or channel) are modelled step by step, not as a dynamic process.
5. No node processing restrictions, as all nodes can split or collect messages instantly, have no computation or buffer limits.
6. No priority policies: all messages are considered equally important, with no QoS levels or traffic classes.

Limitations of the proposed method

1. Unsuitability for stochastic/unstable channels

- In real networks, latency, bandwidth, and losses depend on load, interference, and weather conditions.
- The tensor model does not take this into account → Real delays may differ from the predicted ones.

2. High computational complexity

- Construction of the metric tensor, transformation of coordinates, solution of systems of equations – $O(n^2 – n^3)$.
- For large networks (>100 nodes), the model needs optimisation or distributed computing.

3. Not tested on extensive or real infrastructure, the entire simulation is performed on a synthetic topology with six nodes, without taking into account the effects typical for real traffic (bursty, priority, multi-layering, etc.).

4. There is no QoS classification mechanism, since the tensor approach does not take into account the priority of traffic. Everything is processed in the same way → it is impossible to guarantee the transmission of "more important" data earlier.

5. Centralised management model

- Building and updating tensors requires centralised coordination.
- In distributed or mobile ad-hoc networks, this is difficult to implement without delays and loss of consistency.

Table 24. Table of assumptions, associated risks and possible ways to improve the tensor model

Model assumptions	**Potential risks/limitations**	**Possible areas for improvement**
1. Channel latency and bandwidth are constant	Real networks have stochastic latency, interference, and packet loss	Implement stochastic tensors or ranges instead of fixed values
2. Complete and up-to-date topology information is available at any time	In distributed or mobile networks, such information may be outdated or incomplete	Use local estimates and partial topology reconstruction
3. Message transmission synchronised (all fragments are sent at the same time)	In real-world conditions, delays can desynchronise arrivals, resulting in losses or repeated requests	Add buffering and a correction mechanism on the receiving side
4. Topology changes discretely and rarely	In reality, nodes can disappear/appear arbitrarily often (especially in MANET)	Develop a mechanism for dynamically updating tensor coordinates
5. No queues, processing packages in nodes	There may be delays in the nodes due to load, which will change the overall route delay	Simulate node latency as a separate component in a tensor
6. Centralised processing (the entire tensor is calculated globally)	Becomes inefficient in large networks or when the central node is lost	Develop a distributed or clustered implementation
7. All messages have the same priority	In real networks, priority commands (e.g. alarms, medical data) are required	Add QoS levels and weights to delay tensors
8. Tested on a small synthetic network	The results do not guarantee scalability to real-world conditions	Run simulations in OMNeT++, NS-3, or GNS3 with typical topologies

6. Conclusions

In most countries, including Ukraine, the management and communication systems of security agencies are being organized via the formation of a single information and telecommunication environment, with the employment of modern information and telecommunication technologies, complexes, and special communication systems designed to exchange information between the government bodies and management offices of all levels. This study demonstrates that tensor network analysis offers a unified framework for simultaneously examining both the structural characteristics and information exchange processes within a system, while maintaining coherence in the analysis. We propose a method for dynamically managing information flows based on QoS metrics within critical information and telecommunications networks. Additionally, we present a tensor-based modelling approach that is applicable under both standard and critical operational conditions. The calculations show that the proposed method allows one to optimally distribute traffic in the network and guarantee the values of quality indices of information exchange. Namely, the following indexes are improved in comparison with models available in the literature:

– multipath delay is reduced on average from 9-15% to 35-40% depending on the intensity of traffic entering the network;

– the probability of packet delivery when solving the routing problem within the proposed model for the traffic intensity of 200-300 s^{-1} remained almost at the level of values obtained within the prototype literature models. For the traffic intensity being above 300 s^{-1}, the gain was found to be 25-30%;

– packet jitter due to the implementation of the multipath routing strategy was actually zero due to the resulting solution of the same values of the average delays along the paths;

– the gain in the value of the average multipath delay with a probability of timely delivery equal to 0.999, obtained by solving the routing problem within the proposed model, was found to be 8–28% in comparison with the solution within other models depending on the intensity of traffic entering the network.

The study presents a conceptual tensor model of multipath routing for critical telecommunication networks. The proposed approach provides synchronous message delivery, reduces jitter to zero (in the model) and retains functionality when nodes or channels are lost. The model was tested on a synthetic topology, where an increase in delivery probability of up to 99.9% at an intensity of more than 300 messages/s was demonstrated.

Thus, the tensor model at this stage is a conceptual simulation implementation designed to demonstrate theoretical advantages under the conditions of an idealised topology. Implementation in real-world systems requires an extension of the model to take into account dynamics, scaling, and computational efficiency in online environments. In addition to the immediate results, the conclusions obtained in the study are essential for the field of critical information and telecommunication systems. It is shown that mathematical methods of tensor analysis can provide a high level of adaptability, resistance to structural changes and efficient multipath routing in conditions of an incomplete or degraded network. It opens up new opportunities for designing smart routing networks, in particular in the areas of emergencies, unmanned systems, military communications, and telemedicine. On the theoretical side, the work contributes to the development of an interdisciplinary approach, combining the concepts of topology, algebra and infrastructure networks in a single model. Further research can develop this direction towards stochastic tensor analysis, as well as the development of dynamic routing protocols based on it. Implementation in real-world systems requires an extension of the model to take into account dynamics, scaling, and computational efficiency in online environments.

In addition, the employment of the proposed method allows one:

– by introducing procedures of adaptive packet fragmentation in network nodes on average by 4-6 times to reduce the average time delay of packet delivery in comparison with single-path routing models without fragmentation and to ensure a better-balanced loading of the packet delivery network;

– within the framework of available network resources due to the implementation of a multipath routing strategy to increase the productivity of CITN by an average of 20% -35% in comparison with the two considered literature models of solving routing problems.

Further development of the proposed approach can be performed to ensure the simultaneous solution of the problems of route calculation and resource allocation between data traffic of different users, as well as to extend the presented model to the case of information transfer not only between two network nodes but, for example, circularly.

Further research is planned to extend the tensor routing model by taking into account stochastic processes in network links, such as dynamic queues, packet loss, and variable latency. It is also planned to implement the prototype in a simulation environment (for example, NS-3 or OMNeT++) to test the model's operation on more complex and scalable topologies. Special attention will be paid to adapting the model to the distributed architecture, including the ability to integrate with SDN controllers and QoS protocols. It is proposed that the implementation of traffic prioritisation mechanisms and message classification be investigated to support critical services in mixed loads. In addition, further work can be aimed at optimising the computational complexity of tensor analysis to ensure real-time efficiency.

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