

Coalition Tolerance to Multiple Failures of Its Members

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Abstract: The paper deals with coalitions whose members are altruistic. Coalition members do their best to complete the arising tasks, and do not expect to receive a reward. A coalition member can be an entity such as a social or governmental organization, a military unit, or a complex technical device such as an autonomous robot, or any other entity that has the capabilities, willingness, and ability to cooperate. The paper considers the non-redundant coalitions, which have only those coalition members without whom they cannot perform the tasks. In the paper, we consider only situations where substitution of failed coalition members is impossible. A coalition tolerates the failure of its members by using the surplus of its capabilities. In our research, coalition capabilities are understood as resources and services (e.g., materials, energy, power). During the execution of tasks, one or more members of the coalition may fail. The paper uses the probability of the event that the coalition tolerates the failures of its members to evaluate the coalition fault tolerance. A method is proposed for determining the coalition fault tolerance in the case of multiple member failures. The complexity of the proposed method and its applicability are assessed.

Index Terms: Alliance, Coalition, Coalition Capabilities, Coalition Failure, Fault Tolerance.

1. Introduction

Recently, alliances and coalitions have been attracting great attention from researchers [1,2,3]. In the paper, an alliance is considered as a group of entities that agree to cooperate and form coalitions to accomplish the tasks that arise. An entity can be a social organization, a military unit, a robot, a software agent, etc. An alliance can be characterized from different points of view, such as duration of cooperation, stability, expandability, preferences of alliance members, etc. [4]. In the paper, we consider long-term alliances with a fix number of unselfish members. When task arises, the alliance members form a coalition. In our research, coalition is considered as a short-term cooperation among the coalition members that have a single goal (i.e., perform a mission/tasks). To achieve this goal the coalition members do all their best selflessly. Coalition formation problem was studied in many researches [5,6,7]. Among the main problems of coalition formation there could be named the manner of coalition formation (centralized or decentralized), communications among the alliance members, modeling coalition formation, determining the “best” coalition among the possible ones, etc. In our research, we suppose that the “best” coalition is the coalition which provides the following objectives: minimal number of coalition members; minimal excess of coalition capabilities; maximum value of coalition fault tolerance. Since some of these objectives are in contradiction, the appropriate trade-off between them should be performed.

A coalition can perform a task if it has capabilities that are greater than or equal to the required capabilities (i.e., sufficient to perform the task). In general, capabilities are perceived as resources and services. Examples of resources include energy, materials, power, information, etc. [8].

During the execution of a task, coalition members may fail. If the alliance has members who can substitute failed coalition members, the coalition can tolerate the failures of its members. In this case, the coalition fault tolerance can be evaluated by the maximum number of coalition members that can be substituted (i.e., the total number of failed members that can be tolerated). If substitution is impossible, a coalition can tolerate the failure of its members only if it has a surplus of capabilities available. It is assumed that the substitution of failed coalition members may be impossible for various reasons. For example, when there is insufficient time to perform the substitution or when there is no alliance member who could perform the substitution. There are also situations where an alliance member is unable to substitute a failed coalition member within the designated time due to unforeseen obstacles. In this case, the coalition fault tolerance can be evaluated by the probability that the coalition tolerates the failure of its members [9]. In [9], only

situations where at most one coalition member can fail were considered. This paper considers situations where several coalition members may fail during task execution.

The failure of each coalition member results in a loss of its capabilities. Depending on the degree of loss of capabilities, the failure of a coalition member can be classified into several levels. For example, the failure of a coalition member can be graded into substantial if the loss of its capabilities is less than 100%, and complete if the loss of its capabilities is equal to 100%. Coalition failure can also be graded into more than two levels. In this paper, we consider four levels of coalition member failure: minor, partial, substantial, and complete.

In the paper, we consider that the probability of a minor failure is greater than the probability of a partial, substantial, and complete failure. The probability of a partial failure is greater than the probability of a substantial and complete failure. The probability of a substantial failure is greater than the probability of a complete failure.

When solving the problem of determining the “best” coalition that alliance members can form, the coalition fault tolerance should be taken into account. In general, determining the “best” coalition is based on several criteria, which may conflict with each other. Therefore, many-objective optimization is usually used to determine the “best” coalition [10]. The rest of the paper is organized as follows. Section 2 provides related work. Section 3 describes the coalition’s tolerance to failure of its members when only one coalition member may fail. Section 4 presents a method for determining the coalition’s tolerance to multiple failures of its members. Section 5 presents the application of the proposed method to robot coalitions. Section 6 presents conclusions and future work.

2. Related Work

In different areas, the terms alliance and coalition are interpreted differently. For example, in ethology, a coalition is usually defined as a joint aggression between at least two individuals against a common target, whereas an alliance is reserved for long-term relationships within which coalitions can arise [11]. In sociology, for example in [12], a coalition is defined as the joint use of resources by two or more social units. Some authors see a difference between coalitions and alliances in their duration. For example, in [13] an alliance is a long-term relationship within which coalitions may occur, and the key difference is that alliances can overlap, whereas coalitions are mutually exclusive. In [14], alliance members are considered as different social entities (e.g., people, social organizations and institutions, military units). Entities agree to cooperate and form coalitions. In multi-agent computing, a coalition is understood as cooperation between autonomous agents, which can be mutually beneficial. Mutual benefits can arise from resource sharing and task redistribution [15]. In our research, we follow the definitions of alliance and coalition that were proposed in [14]. In [14], a coalition is formed preferably within a single alliance.

In general, a coalition member can be selfish (focused on self-interest) [16,17] or selfless [4]. In this paper, we consider only selfless entities that do not expect any rewards for their participation in the coalition.

The formation of coalitions by selfish agents has been addressed in many studies [18,19,20], while the formation of coalitions by non-selfish agents is less studied. The same applies to the modeling of coalition formation. The modeling of coalition formation by selfish agents usually utilizes cooperative game theory and structural network analysis [21]. The modeling of alliances and coalitions formed by non-selfish entities was examined in [22].

Along with organizations, companies, individuals, and software agents, robots are also viewed as entities when they possess high entitativity. Coalitions formed by robots were mainly considered within the framework of multi-agent systems. In this case, robots are considered selfish entities [23, 24]. It should be noted that there are many situations in which robots can behave like selfless entities (e.g., rescue operations, operations to mitigate the consequences of a disaster).

During the formation of a coalition, selfish entities may leave the coalition for various reasons, including their own failure, or they may join the forming coalition. If a member of an already formed coalition fails, a reconstruction procedure can be carried out [25]. In [9], coalitions of non-selfish entities remain unchanged after their members fail when substitutions of the failed members are impossible. Substituting failing coalition members is considered one of the main methods of coalition recovery [26]. In [27], the problem of replacing a failing robot with another robot with similar capabilities was examined. In this paper, we assume that substitution of the failing coalition members is not possible.

There is limited research concerning the problem of quantitative assessment of a coalition fault tolerance. For example, in [27], a fault tolerance coefficient was proposed. This coefficient can be incorporated into an algorithm that determines the best coalition. To evaluate coalition fault tolerance, in this paper, we use the metric proposed in [9].

3. Coalition Fault Tolerance

The term “coalition fault tolerance” refers to the coalition’s tolerance to the failure of its members. This means that despite the failure of some coalition members during task execution, the coalition can successfully carry out the assigned tasks. The coalition can perform the assigned tasks if it has capabilities that are greater than or equal to the required capabilities, i.e.,

$$C_{COAL} \geq C_r \quad (1)$$

Excess of coalition capabilities, i.e., $\Delta C = C_{COAL} - C_r$, provides the coalition fault tolerance. Failure of a coalition member results in a reduction of its capabilities. The initial capabilities of a coalition member are denoted as C_{m_i} , where m_i is the i -th member of the coalition.

If the reduction in a coalition member's capabilities is less than or equal to ΔC , the coalition can tolerate the failure. Otherwise, the coalition fails to fulfill the assigned tasks. In order to describe the relationship between a coalition member's loss of capabilities and the levels/types of its failures, we use the failure function, which was introduced in [9]. Fig. 1 shows the failure function $f(L)$.

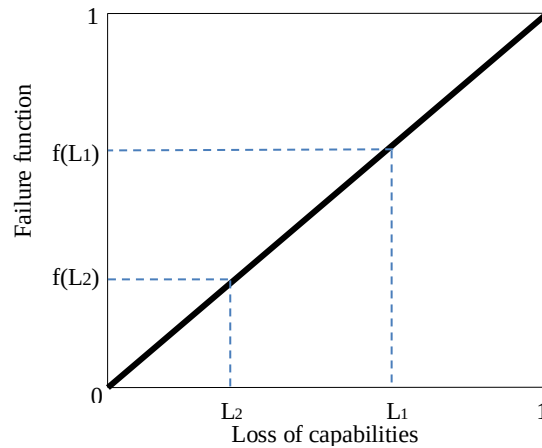


Fig. 1. The failure function of a coalition member.

The failure function takes values from the interval $[0, 1]$. The values of the loss of capabilities of a coalition member can be presented either in percentages or in absolute values of a specific domain. We assume that the failure function $f(L)$ is a linear function. It should be noted that the type of failure function is not critical. This function should only be monotonically increasing and continuous. What is important is how the values of the failure function are interpreted. The linear function was chosen because it is clear and the simplest method to map capability losses to real numbers in the interval $[0,1]$. In the given case, the coalition's tolerance to the failure of one of its members m_i can be determined by using the a priori probabilities of the states of a coalition member. The hypotheses correspond to the states (failures) of a coalition member. This means that the failure of a coalition member can be either minor (hypothesis H_1), or partial (hypothesis H_2), or substantial (hypothesis H_3), or complete (hypothesis H_4). Fig. 2 depicts hypotheses H_i , conditional probabilities $P(A/H_i)$, $i=1,...,4$, and event A that the coalition tolerates the failure of its member m_i .

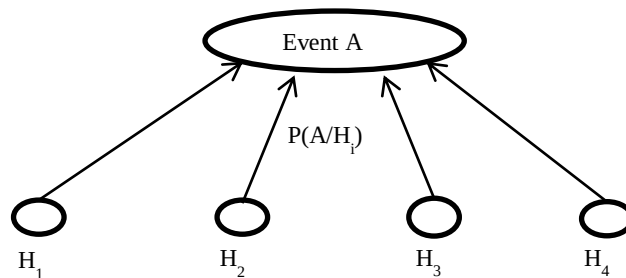


Fig. 2. Probability of the event A .

In the given case, the probability $P(A)$ that the coalition tolerates the failure of its member m_i is calculated as

$$P(A) = \sum_{i=1}^4 P(A/H_i) P(H_i) \quad (2)$$

Conditional probabilities $P(A/H_i)$ can be determined based on the value ΔC . The loss of capabilities of the coalition can be divided into intervals $[0, L_1]$, $[L_1, L_2]$ and $[L_2, C_{m_i}]$. Each interval corresponds to a specific failure state of a coalition member. Table 1 shows the relationship between the intervals and the failure states of a coalition member.

The paper considers the case where the probabilities of failure states follow the expression $P_{minor} < P_{partial} < P_{substantial} < P_{complete}$. The maximum value of the loss of the capabilities of member m_i that the coalition can tolerate is equal to ΔC , i.e., $L_{ma} = \Delta C$ (in absolute units) or $L_{ma} = (\Delta C/C_{mi}) * 100\%$ (in percentages). The loss of capabilities L_{ma} corresponds to the value $f(L_{ma})$ of the failure function of member m_i . If value $f(L_{ma})$ is greater than or equal to values $f(L_i)$, $i=1,2$, then the conditional probability $P(A/H_i)$ is equal to 1. If value $f(L_{ma})$ is less than value $f(L_i)$,

Table 1. Failure states.

Interval	Failure state
$0 < L \leq L_1$	minor
$L_1 < L \leq L_2$	partial
$L_2 < L < C_{mi}$	substantial
$L = C_{mi}$	complete

then the conditional probability $P(A/H_1)$ is determined as

$$P(A/H_1) = \frac{f(L_{ma})}{f(L_1)} \quad (3)$$

If value $f(L_{ma})$ is greater than value of function $f(L_1)$ but lesser than value $f(L_2)$, then conditional probability $P(A/H_2)$ is determined as

$$P(A/H_2) = \frac{f(L_{ma}) - f(L_1)}{f(L_2) - f(L_1)} \quad (4)$$

If value $f(L_{ma})$ is greater than value of function $f(L_2)$ but lesser than C_{mi} , then conditional probability $P(A/H_3)$ is determined as

$$P(A/H_3) = \frac{f(L_{ma}) - f(L_2)}{C_{mi} - f(L_2)} \quad (5)$$

And finally, if $f(L_{ma}) = 1$, then $P(A/H_4) = 0$.

It is assumed that within each interval $[0, L_1]$, $[L_1, L_2]$ and $[L_2, C_{mi}]$, the loss of capabilities can take any value with equal probability. This means that the random value of the loss of capabilities has a uniform distribution. If there is only one interval $[0, C_{mi}]$ of possible values of the loss of capabilities, probability $P(A)$ is determined as

$$P(A) = \frac{\Delta C}{C_{mi}} \quad (6)$$

Probability $P(A)$ is determined for each coalition member. For a coalition composed of N members, the coalition fault tolerance is determined as

$$P_{CFT} = \frac{1}{N} \sum_{i=1}^N P_{CFT}^i \quad (7)$$

Where P_{CFT}^i is the probability that the coalition tolerates the failure of its i -th member.

Determining coalition fault tolerance becomes more complicated when more than one coalition member may fail during task execution.

4. Coalition Tolerance to Multiple Failures

4.1. Coalitions Consisting of Two Members

First, we consider coalitions composed of two members. Let us denote them as m_1 and m_2 . The members of the coalition have capabilities C_{m1} and C_{m2} , respectively. Let C_r denote the capabilities that are needed to perform the assigned tasks. In this case, the excess of coalition capabilities is equal to

$$\Delta C = C_{m_1} + C_{m_2} - C_r \quad (8)$$

It is assumed that for each coalition member, the failure function and values P_{minor} , $P_{partial}$, $P_{substantial}$, and $P_{complete}$ are known. The probability of the event that the coalition tolerates the failure of both of its members can be determined as the total probability

$$P(A) = \sum_{i=1}^n P(A/S_i)P(S_i) \quad (9)$$

where

n is the total number of possible states;

$P(S_i)$ is the probability of state S_i ;

$P(A/S_i)$ is the conditional probability that the coalition tolerates the failure of its members when state S_i takes place.

Table 2 lists possible faulty states of the coalition.

Table 2. Coalition faulty states.

$S_1: 1m - 2m$	$S_5: 1p - 2m$	$S_9: 1s - 2m$	$S_{13}: 1c - 2m$
$S_2: 1m - 2p$	$S_6: 1p - 2p$	$S_{10}: 1s - 2p$	$S_{14}: 1c - 2p$
$S_3: 1m - 2s$	$S_7: 1p - 2s$	$S_{11}: 1s - 2s$	$S_{15}: 1c - 2s$
$S_4: 1m - 2c$	$S_8: 1p - 2c$	$S_{12}: 1s - 2c$	$S_{16}: 1c - 2c$

The following shortcuts are used in Table 2. 1 and 2 stand for member m_1 and member m_2 , respectively, m stands for minor failure, p stands for partial failure, s stands for substantial failure, and c stands for complete failure. The conditional probabilities $P(A/S_4)$, $P(A/S_8)$, $P(A/S_{12})$, $P(A/S_{13})$, $P(A/S_{14})$, $P(A/S_{15})$, and $P(A/S_{16})$ are equal to 0 because the coalition under consideration is not redundant (i.e., the coalition does not include members without which it can perform a task). The other conditional probabilities can be determined on the basis of the following analysis.

A coalition can tolerate the failure of its members if the total (aggregated) loss of capabilities of the failing members is less than the surplus of the coalition's capabilities. i.e.,

$$L^1 + L^2 \leq \Delta C \quad (10)$$

where

L^1 is the loss of capabilities of member m_1 ;

L^2 is the loss of capabilities of member m_2 .

The superscripts in (10) denote the numbers of coalition members. Condition (10) is met when the point with coordinates (L_x^1, L_y^2) is located under the line $L = \{(L_x^1, L_y^2) | L_x^1 + L_y^2 = \Delta C\}$ or on this line (Fig. 3).

4.1.1. Determining Conditional Probability $P(A/S_1)$

In the given case, the loss of capabilities of members m_1 and m_2 takes values from the intervals $[0, L_1^1]$ and $[0, L_1^2]$, respectively. These intervals form a rectangle whose sides correspond to these intervals. Fig. 4 shows some locations of the rectangle relative to the line.

Conditional probability $P(A/S_1)$ can be determined as geometric probability

$$P(A/S_1) = \frac{S_{desired}}{S_{total}} \quad (11)$$

where

$S_{desired}$ is the area of the part of the rectangle that lies under the line;

S_{total} is the area of the rectangle (total area).

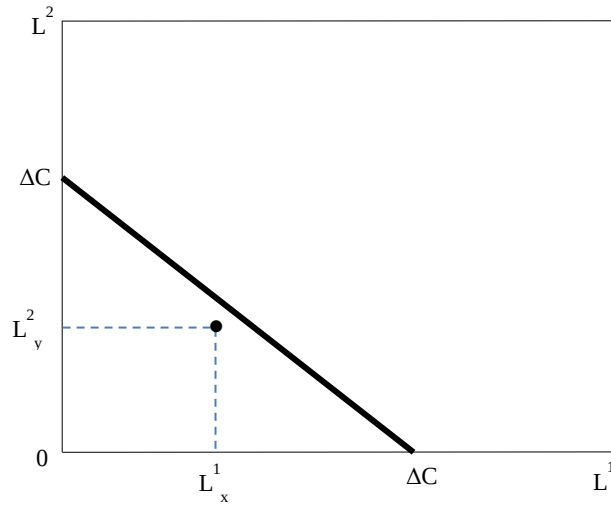


Fig. 3. Total loss of the coalition.

If $L_1^1 + L_1^2 \leq \Delta C$ (see Fig. 4 case 2), then $P(A/S_1)=1$.

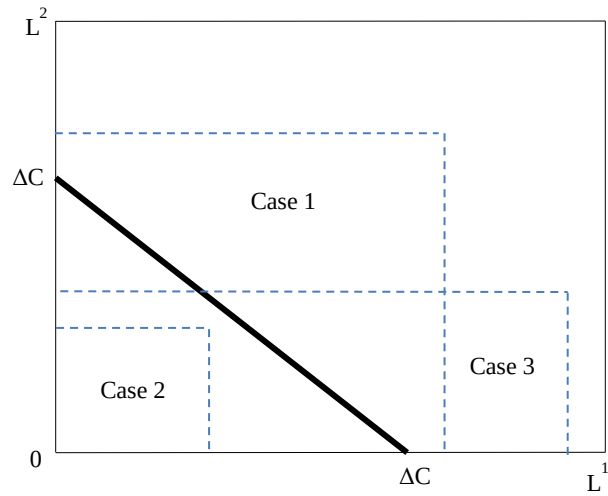


Fig. 4. Possible locations of the rectangle relative to the line.

Otherwise

1) if $L_1^1 > \Delta C$ and $L_1^2 > \Delta C$ (case 1), then

$$P(A/S_1) = \frac{(\Delta C)^2}{2L_1^1 L_1^2} \quad (12)$$

2) if $L_1^1 < \Delta C$ and $L_1^2 < \Delta C$, then

$$P(A/S_1) = \frac{2L_1^1 L_1^2 - (L_1^1 + L_1^2 - \Delta C)^2}{2L_1^1 L_1^2} \quad (13)$$

3) if $L_1^1 > \Delta C$ and $L_1^2 < \Delta C$ (case 3), then

$$P(A/S_1) = \frac{2L_1^2 \Delta C - (L_1^2)^2}{2L_1^1 L_1^2} \quad (14)$$

4) if $L_1^1 < \Delta C$ and $L_1^2 > \Delta C$, then

$$P(A/S_1) = \frac{2L_1^1 \Delta C - (L_1^1)^2}{2L_1^1 L_1^2} \quad (15)$$

4.1.2. Determining Conditional Probability $P(A/S_2)$

In the given case, the loss of capabilities of members m_1 and m_2 takes values from intervals $[0, L_1^1]$ and $[L_1^2, L_2^2]$, respectively. These intervals form a rectangle whose sides correspond to these intervals. Similarly to determining the conditional probability $P(A/S_1)$, several locations of the created rectangle relative to the line are possible (Fig. 5). Conditional probability $P(A/S_2)$ is determined using geometric probability.

If $L_1^2 > \Delta C$ (see Fig. 5 case 1), then $P(A/S_2) = 0$.

If $L_2^2 + L_1^1 < \Delta C$ (case 2), then $P(A/S_2) = 1$.

Otherwise

1) if $L_2^2 < \Delta C$, then the following two options are possible

- if $L_1^2 + L_1^1 > \Delta C$ (case 3), then

$$P(A/S_2) = \frac{2(\Delta C - L_1^1) - (L_2^2 - L_1^2)}{2L_1^1} \quad (16)$$

- else if $L_1^2 + L_1^1 < \Delta C$ (case 4), then

$$P(A/S_2) = \frac{2L_1^1(L_2^2 - L_1^2) - [L_1^1 - (\Delta C - L_2^2)]^2}{2L_1^1(L_2^2 - L_1^2)} \quad (17)$$

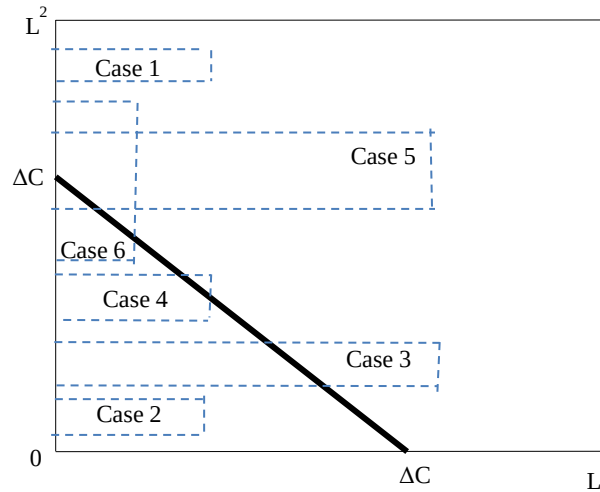


Fig. 5. Possible locations of the rectangle relative to the line when intervals $[0, L_1^1]$ and $[L_1^2, L_2^2]$ considered.

2) if $L_2^2 > \Delta C$, then the following two options are possible

- if $L_1^2 + L_1^1 > \Delta C$ (case 5), then

$$P(A/S_2) = \frac{(\Delta C - L_1^1)^2}{2L_1^1(L_2^2 - L_1^2)} \quad (18)$$

- else if $L_1^2 + L_1^1 < \Delta C$ (case 6), then

$$P(A/S_2) = \frac{2(\Delta C - L_1^1) - L_1^1}{2(L_2^2 - L_1^2)} \quad (19)$$

4.1.3. Determining Conditional Probability $P(A/S_3)$

In the given case, the loss of capabilities of members m_1 and m_2 takes values from the intervals $[0, L_1^1]$ and $[L_2^2, C_{m_2}]$, respectively. These intervals form a rectangle whose sides correspond to these intervals. Several locations of this rectangle relative to the line are possible (Fig. 6). Conditional probability $P(A/S_3)$ can be determined using geometric probability.

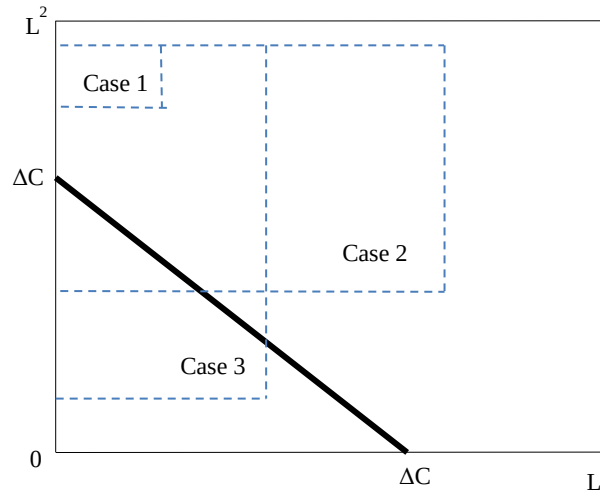


Fig. 6. Possible locations of the rectangle relative to the line when intervals $[0, L_1^1]$ and $[L_2^2, C_{m_2}]$ are considered.

If $L_2^2 > \Delta C$ (see Fig. 6 case 1), then $P(A/S_3) = 0$

Otherwise

if $L_2^2 + L_1^1 > \Delta C$ (case 2), then

$$P(A/S_3) = \frac{(\Delta C - L_2^2)^2}{2L_1^1(C_{m_2} - L_2^2)} \quad (20)$$

else if $L_2^2 + L_1^1 < \Delta C$ (case 3), then

$$P(A/S_3) = \frac{2L_1^1(\Delta C - L_2^2) - (L_1^1)^2}{2L_1^1(C_{m_2} - L_2^2)} \quad (21)$$

4.1.4. Determining Conditional Probability $P(A/S_5)$

In the case of state S_5 , the loss of capabilities of members m_1 and m_2 can take values from intervals $[L_1^1, L_2^1]$ and $[0, L_1^2]$, respectively. These intervals form a rectangle whose sides correspond to these intervals. Several locations of this rectangle relative to the line are possible. Conditional probability $P(A/S_5)$ can be determined using geometric probability.

If $L_1^1 > \Delta C$, then $P(A/S_5) = 0$.

If $L_1^1 + L_2^1 < \Delta C$, then $P(A/S_5) = 1$.

Otherwise

1) $L_2^1 < \Delta C$. In the given case, two options are possible

- if $L_1^1 + L_2^1 > \Delta C$, then

$$P(A/S_5) = \frac{2(\Delta C - L_1^1) - (L_2^1 - L_1^1)}{2L_1^2} \quad (22)$$

- else if $L_1^1 + L_2^1 < \Delta C$, then

$$P(A/S_5) = \frac{2L_1^2(L_2^1 - L_1^1) - [L_1^1 - (\Delta C - L_2^1)]^2}{2L_1^2(L_2^1 - L_1^1)} \quad (23)$$

2) $L_2^1 > \Delta C$. In the given case, two options are possible

- if $L_1^1 + L_2^1 < \Delta C$, then

$$P(A/S_5) = \frac{2L_1^2(\Delta C - L_1^1) - (L_1^1)^2}{2L_1^2(L_2^1 - L_1^1)} \quad (24)$$

- else if $L_1^1 + L_2^1 > \Delta C$, then

$$P(A/S_5) = \frac{(\Delta C - L_1^1)^2}{2L_1^2(L_2^1 - L_1^1)} \quad (25)$$

4.1.5. Determining Conditional Probability $P(A/S_6)$

In the given case, the loss of capabilities of members m_1 and m_2 can take values from intervals $[L_1^1, L_2^1]$ and $[L_1^2, L_2^2]$, respectively. These intervals create a rectangle whose sides correspond to these intervals. Several locations of this rectangle relative to the line are possible. Conditional probability $P(A/S_6)$ can be determined using geometric probability.

If $L_1^1 + L_2^2 > \Delta C$, then $P(A/S_6) = 0$.

If $L_2^1 + L_2^2 < \Delta C$, then $P(A/S_6) = 1$.

Otherwise

1) $L_2^2 < \Delta C$. In the given case, there are two options

- if $L_2^1 + L_2^2 > \Delta C$, then

$$P(A/S_6) = \frac{2(\Delta C - L_1^1) - (L_2^1 + L_2^2)}{2(L_2^1 - L_1^1)} \quad (26)$$

- else if $L_2^1 + L_2^2 < \Delta C$, then

$$P(A/S_6) = \frac{2(L_2^1 - L_1^1)(L_2^2 - L_1^2) - [L_2^1 - (\Delta C - L_2^2)]^2}{2(L_2^2 - L_1^2)(L_2^1 - L_1^1)} \quad (27)$$

2) $L_2^2 > \Delta C$. In the given case, there are two options

- if $L_2^1 + L_2^2 > \Delta C$, then

$$P(A/S_6) = \frac{(\Delta C - L_1^2 - L_1^1)^2}{2(L_2^2 - L_1^2)(L_2^1 - L_1^1)} \quad (28)$$

- else if $L_2^1 + L_2^2 < \Delta C$, then

$$P(A/S_6) = 1 - \frac{(L_2^1 - L_1^1 - \Delta C + L_2^2)^2}{2(L_2^2 - L_1^2)(L_2^1 - L_1^1)} \quad (29)$$

4.1.6. Determining Conditional Probability $P(A/S_7)$

In the given case, the loss of capabilities of members m_1 and m_2 can take values from intervals $[L_1^1, L_2^1]$ and $[L_2^2, C_{m_2}]$, respectively. These intervals form a rectangle whose sides correspond to these intervals. Several locations of this rectangle relative to the line are possible. Conditional probability $P(A/S_7)$ can be determined using geometric probability.

If $L_1^1 + L_2^2 > \Delta C$ then $P(A/S_7) = 0$.

Otherwise

$$P(A/S_7) = \frac{(\Delta C - L_2^2 - L_1^1)^2}{2(L_2^1 - L_1^1)(C_{m_2} - L_2^2)} \quad (30)$$

4.1.7. Determining Conditional Probability $P(A/S_9)$

In this case, the loss of capabilities of members m_1 and m_2 can take values from intervals $[L_2^1, C_{m_1}]$ and $[0, L_1^2]$, respectively. These intervals form a rectangle whose sides correspond to these intervals. Several locations of the formed rectangle relative to the line are possible. Conditional probability $P(A/S_9)$ can be determined using geometric probability.

If $L_2^1 > \Delta C$, then $P(A/S_9) = 0$.

Otherwise

- if $L_2^1 + L_1^2 > \Delta C$, then

$$P(A/S_9) = \frac{(\Delta C - L_2^1)^2}{2L_1^2(C_{m_1} - L_2^1)} \quad (31)$$

- else if $L_2^1 + L_1^2 < \Delta C$, then

$$P(A/S_9) = \frac{2L_1^2(\Delta C - L_2^1) - (L_1^2)^2}{2L_1^2(C_{m_1} - L_2^1)} \quad (32)$$

4.1.8. Determining Conditional Probability $P(A/S_{10})$

In the given case, the loss of capabilities of members m_1 and m_2 can take values from intervals $[L_2^1, C_{m_1}]$ and $[L_1^2, L_2^2]$, respectively. These intervals create a rectangle whose sides correspond to these intervals. Several locations of the created rectangle relative to the line are possible. Conditional probability $P(A/S_{10})$ can be determined using geometric probability.

If $L_2^1 + L_1^2 > \Delta C$ then $P(A/S_{10}) = 0$.

Otherwise

$$P(A/S_{10}) = \frac{(\Delta C - L_2^1 - L_1^2)^2}{2(L_2^2 - L_1^2)(C_{m_1} - L_2^1)} \quad (33)$$

4.1.9. Determining Conditional Probability $P(A/S_{11})$

In the given case, the loss of capabilities of members m_1 and m_2 can take values from intervals $[L_2^1, C_{m_1}]$ and $[L_2^2, C_{m_2}]$, respectively. These intervals form a rectangle whose sides correspond to these intervals. Several locations of this rectangle relative to the line are possible. Conditional probability $P(A/S_{11})$ can be determined using geometric probability.

If $L_2^1 + L_2^2 > \Delta C$ then $P(A/S_{11}) = 0$.

Otherwise

$$P(A/S_{11}) = \frac{(\Delta C - L_2^2 - L_2^1)^2}{2(C_{m_1} - L_2^1)(C_{m_2} - L_2^2)} \quad (34)$$

4.2. Coalitions Consisting of More than Two Members

In the case of a coalition consisting of three members, there are sixty-four different coalition faulty states possible. Thirty-seven of these states involve at least one complete failure of one of the coalition members. Therefore, conditional probabilities $P(A/S_j)$ related to these states are equal to 0. The remaining twenty-seven conditional probabilities can be determined using Monte Carlo method.

First, random numbers r_i with a uniform distribution in the interval $[0, 1]$ are generated. Then the random numbers are converted to random numbers y_i with a more complex distribution. Using random numbers y_i , estimates of the characteristics of the random variable L (loss of capabilities) are either calculated directly or calculated using an appropriate algorithm. In the paper, we consider the case when $r_i = y_i$ (i.e., uniform distribution). In general, various distributions of the random variable L can be considered.

For example, the conditional probability $P(A/S_j)$, $j \in \{1, 2, \dots, 27\}$, when the state S_j is described as a minor failure of members m_1 and m_2 and a partial failure of member m_3 , can be determined using the following Algorithm 1.

Algorithm 1.

Input: $C_{m1}, C_{m2}, C_{m3}, L_1^1, L_2^1, L_1^2, L_2^2, L_1^3, L_2^3, C_r, k$

1. int i, z
 2. $\Delta C = (C_{m1} + C_{m2} + C_{m3}) - C_r$
 3. $z \leftarrow 0$
 4. for $i \leftarrow 1$ to k
 5. $r_1 \leftarrow \text{random}(1, 100)$
 6. $r_2 \leftarrow \text{random}(1, 100)$
 7. $r_3 \leftarrow \text{random}(1, 100)$
 8. $L^1 = \frac{r_1 L_1^1}{100}$
-

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9.  $L^2 = \frac{r_2 L_1^2}{100}$ 
10.  $L^3 = \frac{r_3 (L_2^3 - L_1^3)}{100}$ 
11. if  $(L^1 + L^2 + L^3) < \Delta C$  then
12.  $z \leftarrow z + 1$ 
13. end if
14. end for
15.  $P^* = z/k$ 
16. return  $P^*$ 
Output: conditional probability  $P(A/S_i) = P^*$ 

```

At the second step of Algorithm 1, the surplus of capabilities of the coalition is calculated. At Step 8, the loss of capabilities of member 1 caused by its minor failure is simulated. The obtained value L^1 of loss of capabilities corresponds to minor failure of member 1. At Step 9, the loss of capabilities of member 2 caused by its minor failure is simulated. The obtained value L^2 corresponds to minor failure of member 2. At Step 10, the loss of capabilities of member 3 caused by its partial failure is simulated. The obtained value L^3 corresponds to partial failure of member 3. Step 11 checks if the coalition tolerates the above mentioned failures of its members. Step 12 counts the number of successful cycles z , where the coalition tolerates the failure of its members. At Step 15, estimation of the probability that the coalition tolerates the failure of its members P^* is calculated.

Error of calculation of probability $P(A/S_i)$ is equal to

$$\sqrt{\frac{P^*(1-P^*)}{k}} \quad (35)$$

For a 95% confidence level, the margin of error is equal to

$$1,96 \times \sqrt{\frac{P^*(1-P^*)}{k}} \quad (36)$$

Thus, there is a 95% probability that true value P lies within

$$P^* \pm 1,96 \times \sqrt{\frac{P^*(1-P^*)}{k}} \quad (37)$$

If the precision of calculation of the conditional probability $P(A/S_i)$ is unsatisfactory, then the number of cycles k must be increased.

In general, the total number of faulty situation is equal to m^n , where m is the total number of failure states, n is the total number of coalition members. The number of conditional probabilities that should be calculated is equal to the number of faulty situations, i.e., m^n . Thus, the computational complexity in this case is exponential $O(m^n)$.

It should be noted that for faulty situations, which include at least one complete failure of a coalition member, the conditional probabilities should not be calculated since they are equal to 0. Thus, computational complexity reduces to $O((m-1)^n)$.

In general, a coalition fault tolerance (i.e., the probability of event A that a coalition tolerates the failure of its members) can be computed by using the following Algorithm 2.

Algorithm 2.

1. Based on the available information, decide on the number of possible failure states (i.e., states such as minor, partial, etc.).
 2. Determine conditional probabilities $P(A/S_i)$ for all possible faulty situations S_i , $i = 1, \dots, m^n$.
 3. For computing the coalition tolerance to the failure of any coalition member, use the method presented in Section 3 which allows determining the conditional probabilities $P(A/S_i)$.
 4. For computing the coalition tolerance to any two failures of its members, use the geometric probability method presented in Section 4 which allows determining the conditional probabilities $P(A/S_i)$.
 5. For computing the coalition tolerance to more than two failures of its members, use the mathematical modeling (the Algorithms similar to Algorithm 1) to determine the estimations of conditional probabilities $P(A/S_i)$, $i = 1, \dots, m^n$.
 6. If the estimations of the conditional probabilities $P(A/S_i)$ are not acceptable, increase the number of trials (cycles) k and repeat Step 5 until the desired precision is achieved.
 7. Using the probabilities of faulty situations S_i and the conditional probabilities $P(A/S_i)$, calculate the total marginal probability of event A (i.e., coalition fault tolerance).
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5. Fault Tolerance of Coalition of Robots

Let the alliance consist of four robots (R_1 , R_2 , R_3 , and R_4). The parameters of the robots' failure functions are given in Table 3.

Table 3. Parameters of the failure function of robots.

	L_1	L_2	C_{mi}	P_M	P_P	P_S	P_C
robot 1	35%	55%	150	0.6	0.2	0.16	0.04
robot 2	25%	40%	120	0.4	0.3	0.2	0.1
robot 3	30%	50%	130	0.5	0.25	0.15	0.1
robot 4	40%	60%	170	0.7	0.2	0.07	0.03

One of the tasks of the alliance is removing the obstacles after building collapses. The capabilities of the robots are evaluated (i.e., measured) as the weight of the load they can lift and move.

Let, for example, robot 1 be able to lift 150kg, robot 2 – 120kg, robot 3 – 130kg, and robot 4 – 170kg, respectively. The load weighs 200kg. The task can be performed by any two robots. In this case, these two robots form a coalition. In total, six different coalitions can be formed. Each of these coalitions can be assessed based on the value of its tolerance to the failure of its members. The surplus of capabilities of the coalition provides compensation for the loss of robot capabilities caused by their failure.

The results of the calculation of the coalition fault tolerance using the methods presented in the previous sections are shown in Table 4.

Table 4. Coalition fault tolerance.

coalition	coalition capabilities	excess of capabilities	tolerance to failure of one robot	tolerance to failures of both robots
R_1, R_2	270	70	0.74	0.356
R_1, R_3	280	80	0.783	0.389
R_1, R_4	320	120	0.905	0.627
R_2, R_3	250	50	0.655	0.219
R_2, R_4	290	90	0.823	0.467
R_3, R_4	300	100	0.860	0.534

The obtained results show that in this case the dependence of coalition fault tolerance on the surplus of capabilities can be evaluated as exponential. However, this conclusion needs further confirmation. This dependence should be taken into account when solving the problem of selecting the “best” coalition based on multi-criteria evaluation.

Table 5 contains the values of coalition fault tolerance for the same six coalitions. In this case, however, it is assumed that for each coalition member m_i , the random value of the loss of its capabilities has a uniform distribution on the interval $[0, C_{mi}]$.

In Table 5, P_{CFT}^i and P_{CFT}^j denote the coalition fault tolerance in case of failure of members i and j , respectively, $i, j \in \{1, 2, 3, 4\}$, $i < j$. In the second case, where there is only one interval for random variable L (Table 5), the values of coalition fault tolerance are smaller than the values given in Table 4. The division of the interval $[0, C_{mi}]$ into several subintervals according to their probability not only reflects real faulty situations, but also allows grouping of collected data concerning coalition member failures. These data can be used to determine the probabilities P_{minor} , $P_{partial}$, $P_{substantial}$, and $P_{complete}$.

Table 5. Coalition fault tolerance for the second case.

	excess of capabilities	P_{CFT}^i	P_{CFT}^j	tolerance to failure of one robot	tolerance to failures of both robots
R_1, R_2	70	0.467	0.583	0.525	0.130
R_1, R_3	80	0.533	0.616	0.574	0.158
R_1, R_4	120	0.800	0.706	0.753	0.274
R_2, R_3	50	0.417	0.385	0.401	0.077
R_2, R_4	90	0.750	0.529	0.639	0.191
R_3, R_4	100	0.769	0.588	0.678	0.223

6. Conclusion

The paper considers coalitions with selfless members. It is assumed that the capabilities of the coalition are the sum of the capabilities of its members. In general, each member of the coalition has a multidimensional vector of real

non-negative capabilities. In this paper, we consider only one dimension of capabilities. A multidimensional vector can also be considered. However, such reasoning is more complicated.

During the execution of a task, one or more members of the coalition may fail. The coalition can tolerate the failure(s) of its members either by substituting the failing members or by using the surplus capabilities of the coalition. In this work, we use the second option. The key distinguishing feature of our research is that the coalition remains the same even after the failure of its members.

In the paper, we dealt with cases where any number of coalition members fails during task execution. For cases where only one coalition member fails, a method for determining the coalition fault tolerance using the total marginal probability of the event that the coalition tolerates member failures is described. Cases where two members of a coalition fail are considered for coalitions with two members. Nevertheless, the proposed method for determining coalition fault tolerance can also be applied to coalitions with more than two members. The proposed method for determining coalition fault tolerance largely depends on the probabilities of random values of a member's loss of capabilities.

The entire range of possible values of a member's capability loss can be divided into several subintervals according to the probabilities of these values. These subintervals correspond to minor, partial, and substantial failures of a coalition member. The greater the number of subintervals, the higher the precision of calculating the coalition fault tolerance. However, the computational complexity also increases with the growing total number of subintervals. Computational complexity also exponentially depends on total number of coalition members. In view of this, the proposed method for determining coalition fault tolerance can be used if the coalition consists of a small number of members.

In the paper, a uniform distribution of the random value of coalition member capability loss was considered. In the future, we plan to consider other possible distributions of this random value, as well as considering more dimensions of the multidimensional coalition capability vector.

Coalition fault tolerance is only one of the characteristics (properties) of the coalition. It should be taken into account when selecting the "best" coalition from among those available that will be able to perform the tasks that arise.

All the Declarations and Statements

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The author independently conducted the research and independently wrote the manuscript.

Conflict of Interest Statement

The author declares no conflicts of interest.

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This study analyzed only publicly available datasets.

Ethical Declarations

This study did not involve human participants or animals. The research used publicly available and aggregated data. Ethical approval was not required.

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Declaration of Generative AI in Scholarly Writing

Generative AI tools were not used.

Abbreviations

None.

Appendix A/B/C..., with appendix title

None.

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