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New Optimal Observer Design Based on State Prediction for Linear Systems

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Abstract: In this study, a new optimal observer design is presented for linear systems. First, the system states are predicted in their receding horizon, and then, based on the error of state prediction, a quadratic cost function is defined such that its minimization results in an optimal gain for the proposed observer. Since the present analytic approach uses off-line optimization, it can overcome the related difficulties that may arise in non-convex optimization problems. In addition, the condition ensuring the asymptotic stability of the proposed optimal observer is developed, thus guaranteeing the asymptotic convergence of the state estimation error to zero. The estimation of the optimal state based on the prediction of system states, achieved through the asymptotic stability of the estimation error, is among the main features of the proposed method. Furthermore, the proposed observer design is independent of the type of system controller. Finally, three examples are provided to demonstrate the effectiveness of the proposed method in state estimation.

Index Terms: Linear System, Optimal Observer, State Prediction, Luenberger Observer, Observer Gain Vector

1. Introduction

Since the application of state estimation in real-world applications is widespread [1-3], the observer design has been the focus of many researchers in recent decades. The first observer was proposed by Luenberger [4-6], in which identity observer was determined by specifying observer gain vector. Basically, gain vector of the Luenberger observer is determined such that, its corresponding characteristic equation has roots with negative real part. On the other hand, in high gain observers [7,8], the gain is determined based on the stability of error dynamics in the presence of uncertainties or disturbances. However, in all these mentioned observers, the observer gain is not designed based on optimization technique.

During the 1970s, development of techniques for linear observer design continued and changed from classical deterministic to stochastic well-known Kalman filter [9-11]. In addition, the stochastic Luenberger observer (SLO) called as suboptimal estimator was introduced solving the problem of general state estimation in linear discrete time stochastic systems [12] and [13]. In this regard, many researchers have tried to improve optimal performance of the reduced-order stochastic observers using some optimization methods considering stability of estimation error dynamics [14-20].

Considering optimal state estimation, the family of Kalman filters and the moving horizon estimators (MHEs) are popular approaches in which the observer parameters such as observer gains, are determined by minimization of the defined cost function.

In addition, estimation by MHE is based on optimization of a finite horizon problem, using defined estimation window to handle noises, disturbances or constraints [21-23].

Although, the MHE can provide a degree of robustness in the presence of uncertainties [24,25], but tuning of weight parameters and minimization technique of the defined cost function are among debatable points in its design.

Furthermore, they have some main troubles in the implementation. In general, in these observers, the optimization is non-convex and might be terminated at a local minimum. In addition, online optimization requires heavy computational operations, which it requires fast calculations and short sampling time. This issue can be considered a challenge for systems with fast dynamics.

Moreover, some studies have developed the concept of Kalman filter for various linear or non-linear systems. For instance, an optimal state estimation was designed for non-linear system using state-dependent Riccati Equation [26], which has the structure similar to the continuous steady-state linear Kalman filter.

Moreover, an optimal reduced-order observer was proposed for non-observable-form-based linear system based on evolutionary optimization [27]. Similarly, a limited number of studies have been published in the field of optimal state estimation, such as the studies by [28, 29].Although, in these studies, cost functions have been defined based on minimization of estimation error, but regarding optimal state prediction, there is no considerable number of studies in this context. In this regard, there are also researches on the use of prediction algorithm to estimate the states of systems with input delay, whose idea was first presented by [30]. Also by [31,32] in order to avoid using the integrator term in controllers, a larger delay was developed. In this idea, a chain of predictors are used so that each of them predicts a fraction of the system delay. This method is known as sequential predictors, which has been studied by many other researchers [33-37].

In this regard, the present study was done to investigate an optimal observer for linear systems, in which optimization focuses on prediction of states in the future, with selectable time horizons. Furthermore, since online optimization problems involved in future prediction of states are generally non-convex, the offline analytical approach presented in this paper can overcome the related problems that may occur in non-convex optimization problems in online methods.

In this paper, a new optimal observer is designed based on state prediction for linear systems, which can significantly improve state estimation in linear observers. Herein, estimation errors are predicted in receding horizon by their Taylor series expansion. Then, defining a quadratic cost function, including error of state prediction and then minimizing it, an optimal observer is designed. Minimizing the proposed cost function is achieved by choosing an optimal gain vector. In addition, the condition ensuring asymptotic stability of the proposed observer is developed. Hence, the main contributions of this paper are as follows:

I. In the proposed observer design, the focus is on minimizing estimation error for selectable time horizons in the future (predictive property), which, as simulation results show, effectively improves estimation quality. In contrast, the popular optimal observer design method emphasizes minimizing estimation error at the current time.
II. In general, the online optimization problem, especially concerning future estimation errors for time horizons, is non-convex. The present offline analytic approach can overcome the related difficulties that may arise in non-convex optimization problems.
III. Since the horizon of prediction can be chosen arbitrarily, the observer gain can be optimally chosen among gains obtained from the analytic solution of the prediction error problem that satisfy the stability condition. Thus, the proposed observer presents various gains for different horizons of prediction
IV. The stability of the proposed optimal observer is proven independently of the controller design strategy. In other words, the type of controller does not affect the optimal selection of the observer gain.

The rest of the paper is organized as follows: In Sections 2 and 3, mathematical preliminaries and the optimal linear predictive observer are presented in canonical and complete forms, respectively. The simulation results are presented in Section 4 to demonstrate the effectiveness of the proposed observer, and conclusions are given in the final section.Notations: Throughout this paper, λ(A) denotes eigenvalues of matrix A.

2. Linear Optimal Observer in Canonical Form

Consider the following linear multi-input and multi-output (SIMO) system described in a canonical form by

$$\begin{aligned}
&\dot{x}_1(t) = x_2(t) \\
&\dot{x}_2(t) = x_3(t) \\
&\vdots \\
&\dot{x}_n(t) = x_1^{[n]}(t) = a_{n1}x_1(t) + \ldots + a_{nn}x_n(t) + bu \\
&y(t) = Cx(t)
\end{aligned} \tag{1}$$

This linear system can be presented as

$$\begin{aligned}&\dot{x}(t)=Ax(t)+Bu\\&y(t)=Cx(t)\\&A=\begin{bmatrix}0&1&\dots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\dots&1\\0&0&0&0\end{bmatrix},B=\begin{bmatrix}0\\\vdots\\0\\b\end{bmatrix},C=[c_1,\dots,c_n]\end{aligned} \tag{2}$$

where, $x \in R^n, u \in R$ and $y \in R^l$ are state, input and output, respectively.
Assumption1. Assume that pair (A, C) is observable.
Approximating the state $x_1(t)$ in receding horizon by its Taylor series expansion

$$\begin{aligned}&x_1(t+\tau)\cong x_1(t)+\tau\dot{x}_1(t)+\dots+\frac{\tau^r}{r!}x_1^{[r]}(t),\\&r=n,T_1\le\tau\le T_2\end{aligned} \tag{3}$$

Or

$$x_1(t+\tau)\cong\tau_1^T X_1(t) \tag{4}$$

where, $\tau_1=\left[1,\tau,\dots,\frac{\tau^n}{n!}\right]^T$ and

$$X_1(t)=\left[x_1(t),x_1^{[1]}(t),\dots,x_1^{[n]}(t)\right]^T=\left[x_1(t),x_2(t),\dots,\dot{x}_n(t)\right]^T \tag{5}$$

Then, the proposed observer is given as

$$\begin{aligned}&\dot{\hat{x}}(t)=A\hat{x}(t)+Bu+B_o u_o\\&\hat{y}(t)=C\hat{x}(t)\end{aligned} \tag{6}$$

Or

$$\begin{aligned}&\dot{\hat{x}}_1(t)=\hat{x}_2(t)\\&\dot{\hat{x}}_2(t)=\hat{x}_3(t)\\&\vdots\\&\dot{\hat{x}}_n(t)=\hat{x}_1^{[n]}(t)=a_{n1}\hat{x}_1+\dots+a_{nn}\hat{x}_n+bu+u_o\\&\hat{y}(t)=C\hat{x}(t)\end{aligned} \tag{7}$$

where u_o is yield in continuation.
Similar to (3), prediction of $\hat{x}_1(t)$ is given as

$$\begin{aligned}&\hat{x}_1(t+\tau)\cong\hat{x}_1(t)+\tau\dot{\hat{x}}_1(t)+\dots+\frac{\tau^r}{r!}\hat{x}_1^{[r]}(t),\\&r=n,T_1\le\tau\le T_2\end{aligned} \tag{8}$$

Or

$$\begin{aligned}&\hat{x}_1(t+\tau)\cong\tau_1^T\hat{X}_1(t)\\&\hat{X}_1(t)\cong\left[\hat{x}_1(t),\hat{x}_1^{[1]}(t),\dots,\hat{x}_1^{[n]}(t)\right]^T=\left[\hat{x}_1(t),\hat{x}_2(t),\dots,\hat{x}_n(t)\right]^T.\end{aligned} \tag{9}$$

Then, prediction of the estimation error can be expressed as

$$e(t+\tau)=\hat{x}_1(t+\tau)-x_1(t+\tau)\cong\tau_1^T\left[\hat{X}_1(t)-X_1(t)\right]=\tau_1^T E(t) \tag{10}$$

where, $E(t)=\left[\hat{X}_1(t)-X_1(t)\right]$.
Now, let introduce the following quadratic cost function:

$$J = \frac{1}{2}\int_{T_1}^{T_2} e(t+\tau)^2\, d\tau \tag{11}$$

where, T_1 and T_2 are lower and upper prediction times, respectively. The cost function (11)
Using Eq. (10), the defined cost function Eq. (11) can also be approximated as

$$\begin{aligned} J &\cong \frac{1}{2}\int_{T_1}^{T_2} \left(\hat{X}_1(t) - X_1(t)\right)^T \tau_1\tau_1^T \left(\hat{X}_1(t) - X_1(t)\right) d\tau \\ &= \frac{1}{2}\left(\hat{X}_1(t) - X_1(t)\right)^T \Lambda \left(\hat{X}_1(t) - X_1(t)\right) \end{aligned} \tag{12}$$

where,

$$\begin{aligned} \Lambda &= \int_{T_1}^{T_2} \tau_1\tau_1^T\, d\tau = \begin{bmatrix} \tau_{11} & \tau_{12} \\ \tau_{12}^T & \tau_{22} \end{bmatrix}, \tau_{11} \in R, \tau_{12} \in R^n \times R, \tau_{22} \in R \\ &= \begin{bmatrix} T_2 - T_1 & \dots & \frac{T_2^{n+1} - T_1^{n+1}}{(n+1)!} \\ \vdots & \vdots & \vdots \\ \frac{T_2^{n+1} - T_1^{n+1}}{(n+1)!} & \dots & \frac{T_2^{2n+1} - T_1^{2n+1}}{n!n!(2n+1)} \end{bmatrix}, \Lambda \in R^{n+1} \end{aligned} \tag{13}$$

and consequently, the necessary condition for minimization of the defined cost function is as follows:

$$\frac{\partial J}{\partial u_o} = 0. \tag{14}$$

As already mentioned, since the online optimization problems for cost function (11) are generally non-convex, the offline analytical approach in Theorem 1 can overcome the related issues that may occur in non-convex optimization problems in online methods.

Theorem1. Consider the linear system Eq. (2) and the defined quadratic cost function Eq. (11). Then, based on prediction of the estimation error defined in (10), u_o as predictive term of the optimal observer in (6) is obtained as

$$u_o = -(a + K)E_{n-1} \tag{15}$$

where,

$$E_{n-1} = \begin{bmatrix} \hat{x}_1(t) - x_1(t) \\ \mathrm{M} \\ \hat{x}_1^{[n-1]} - x_1^{[n-1]} \end{bmatrix} = \begin{bmatrix} e_1(t) \\ \mathrm{M} \\ e_1^{[n-1]} \end{bmatrix} \tag{16}$$

and the elements of $a = [a_{n1}, \dots, a_{nn}]$ are defined in Eq. (2) and also $K \in R^n$ is calculated as

$$K = [k_1, \dots, k_n] = \tau_{22}^{-1}\tau_{12}^T \tag{17}$$

and

$$\begin{aligned} \tau_{22} &= \frac{T_2^{2n+1} - T_1^{2n+1}}{n!n!(2n+1)} \\ \tau_{12}^T &= \left[\frac{T_2^{n+1} - T_1^{n+1}}{n!(n+1)}, \dots, \frac{T_2^{2n} - T_1^{2n}}{(n-1)!n!(2n)}\right] \end{aligned} \tag{18}$$

Proof. The necessary condition Eq. (14) can be written as

$$\frac{\partial J}{\partial u_o}=\frac{\partial\left(\hat{X}_1(t)-X_1(t)\right)^T}{\partial u_o}\Lambda\left(\hat{X}_1(t)-X_1(t)\right)=0 \tag{19}$$

where,

$$\left[\hat{X}_1(t)-X_1(t)\right]=\begin{bmatrix}E_{n-1}\\ H\end{bmatrix} \tag{20}$$

and

$$H=aE_{n-1}+u_o. \tag{21}$$

Hence, according to (20), Eq. (19) is rewritten as

$$\begin{bmatrix}0_{1\times n} & \frac{\partial H}{\partial u_0}\end{bmatrix}\Lambda\left(\hat{X}_1(t)-X_1(t)\right)=\begin{bmatrix}0_{1\times n} & \frac{\partial H}{\partial u_0}\end{bmatrix}\begin{bmatrix}\tau_{11} & \tau_{12}\\ \tau_{12^T} & \tau_{22}\end{bmatrix}\begin{bmatrix}E_{n-1}\\ H\end{bmatrix}=0 \tag{22}$$

where according to Eq. (21), $\frac{\partial H}{\partial u_o}=1$. implying that

$$H=-\tau_{22}^{-1}\tau_{12}^T E_{n-1}=-KE_{n-1} \tag{23}$$

and K is defined by Eq. (17).

Then, invoking Eq. (23) into Eq. (21) leads to

$$aE_{n-1}+u_o=-KE_{n-1} \tag{24}$$

and finally,

$$u_o=-\left(a+K\right)E_{n-1}. \tag{25}$$

This completes the proof of Theorem 1.

Then, subtracting Eq. (2) from Eq. (7) leads to error dynamic

$$\begin{aligned}\dot{e}_1(t)&=e_2(t)\\ \dot{e}_2(t)&=e_3(t)\\ &\vdots\\ \dot{e}_n(t)&=aE_{n-1}+u_o\end{aligned} \tag{26}$$

or

$$\dot{E}=AE+B\left(aE_{n-1}+u_o\right). \tag{27}$$

Invoking Eq. (25) into error dynamic Eq. (27) gives

$$\dot{E}=AE+B_o(-KE_{n-1})=A_cE \tag{28}$$

where,

$$A_c=\begin{bmatrix}0 & 1 & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \dots & 1\\ -k_1 & -k_2 & \dots & -k_n\end{bmatrix} \tag{29}$$

Assumption 2. According to Assumption 1, there exists a vector L such that,

$$\lambda\left(A+LC\right)=\lambda\left(A_c\right) \tag{30}$$

where, based on Theorem 1, A_c is determined optimally.

Hence, defining

$$\bar{A}_c = A+LC \tag{31}$$

and output error as

$$e_y = \hat{y}\left(t\right)-y\left(t\right)=CE_{n-1}, \tag{32}$$

we can redefine error dynamic as

$$\dot{E} = AE+Le_y = \bar{A}_c E. \tag{33}$$

Consequently, it implies that optimal observer dynamic is

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t)+Le_y(t) \\ \hat{y}(t) &= C\hat{x}(t) \end{aligned} \tag{34}$$

where, L is computed based on Assumption 2.

For brief totalization, Algorithm 1 sums the proposed linear optimal observer in this section.

ALGORITHM 1 Proposed optimal observer for canonical form
solving the optimal observer problem.
Step 1 Check assumption 1.
Step 2 Select appropriate prediction times T_1, T_2.
Step 3 Calculate matrix Λ from equation (13).
Step 4 Solve equation (17) for K.
Step 5 Check stability of A_c in (29).
Step 6 Solve equation (30) for L.
Step 7 $\boldsymbol{L}$ is the optimal gain vector for the observer (34).

2.1 Stability analysis for canonical form

In this section, for evaluating stability, let consider the error dynamic Eq. (28), implying that

$$e_1^{[n]}+k_n e_1^{[n-1]}+k_{n-1}e_1^{[n-2]}+\ldots+k_1 e_1 = 0. \tag{35}$$

Also, τ_{22} and τ_{12} in (18) can be rewritten as

$$\tau_{22} = \frac{T_2^{n+1}}{n!}\left(\frac{1-\frac{T_1}{T_2}^{2n+1}}{\left(2n+1\right)}\right)\frac{T_2^n}{n!} \tag{36}$$

Now, let define

$$M_n = \left[\frac{1-\frac{T_1}{T_2}^{n+1}}{(n+1)}, \ldots, \frac{1-\frac{T_1}{T_2}^{2n}}{(2n)} \right], \tag{37}$$

$$M_\mu = \left[\left(\frac{1-\frac{T_1}{T_2}^{2n+1}}{(2n+1)} \right) \right] \tag{38}$$

Then, one can redefine K in Eq. (17) as

$$K = \tau_{22}^{-1}\tau_{12}^{T} = T_2^{-n}\mathrm{n}!M_\mu^{-1}M_n \mathrm{diag}(1, T_2, \ldots, \frac{T_2^{n-1}}{(n-1)!}) \tag{39}$$

in which

$$M_\mu^{-1}M_n = [m_1, \ldots, m_n]. \tag{40}$$

Considering Eq. (17) , it follows Eq. (39) such that,

$$k_i = m_i T_2^{-n-1+i} \frac{n!}{(i-1)!} \quad i = 1, \ldots, n. \tag{41}$$

Substituting k_i from Eq. (41) into the error dynamic Eq. (35) yields

$$e_1^{[n]} + m_n T_2^{-1} n e_1^{[n-1]} + m_{n-1} T_2^{-2} n(n+1) e_1^{[n-2]} + \ldots + m_1 \mathrm{T}_2^{-\mathrm{n}} \mathrm{n}! e_1 = 0. \tag{42}$$

Hence, it can be concluded that stability of the error dynamic is determined by the polynominal

$$s^{[n]} + m_n T_2^{-1} n s^{[n-1]} + m_{n-1} T_2^{-2} n(n+1) s^{[n-2]} + \ldots + m_1 T_2^{-n} n! = 0 \tag{43}$$

Or

$$T_2^n s^{[n]} + m_n T_2^{n-1} n s^{[n-1]} + m_{n-1} T_2^{n-2} n(n+1) s^{[n-2]} + \ldots + m_1 n! = 0 \tag{44}$$

Using the transform $p = T_2 s$, the polynomial Eq. (44) becomes

$$p^{[n]} + m_n n p^{[n-1]} + m_{n-1} n(n+1) p^{[n-2]} + \ldots + m_1 \mathrm{n} = 0. \tag{45}$$

Consequently, the error dynamic Eq. (35) is stable if and only if all roots of the polynomial in Eq. (45) have negative real parts. Therefore, stability of the error dynamic is determined by the coefficients $m_i,\ i = 1, \ldots, .$ i.e., stability of the proposed optimal observer Eq. (6) depends on prediction times T_1 and T_2.

Furthermore, if T_1 is selected as zero, it follows Eq. (38) such that, the coefficient in polynomial Eq. (45) does not depend on prediction time T_2. In this case, stability depends only on n.

3. Linear Optimal Observer in Complete Form

In this section, the linear system Eq. (2) is considered to be in the complete form where matrix A and B are considered as:

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}, b = [b_1, ..., b_n] \tag{46}$$

Then, considering Assumption1, approximating the state $x_i(t)$ in receding horizon by its Taylor series expansion up to order r is as follows

$$\begin{aligned} & x_i(t+\tau) = x_i(t) + \tau \dot{x}_i(t) + \ldots + \frac{\tau^r}{r!} x_i^{[r]}(t) \ , \\ & i = 1, \ldots, n, T_1 \le \tau \le T_2 \end{aligned} \tag{47}$$

Then, assuming that τ is small enough, terms with higher-order derivatives can be ignored as below:

$$\begin{aligned} & x_i(t+\tau) \cong \tau_i^T X_i(t), \\ & i = 1, \ldots, n, \quad T_1 \le \tau \le T_2 \end{aligned} \tag{48}$$

where,

$$\tau_i = [1, \tau]^T \tag{49}$$

and according to (2) and (46)

$$X_i(t) = \left[x_i(t), \dot{x}_i(t) \right]^T = \left[x_i(t), b_i u + \sum_{j=1}^{n} a_{ij} x_j \right]^T, i = 1, ..., n. \tag{50}$$

Let define the proposed observer as

$$\begin{aligned} & \dot{\hat{x}}(t) = A\hat{x}(t) + Bu + B_o u_o \\ & \hat{y}(t) = C\hat{x}(t) \end{aligned} \tag{51}$$

where, $B_o = [0, \ldots, 1]^T$and u_o is yield in continuation. Similar to Eq. (47), the observer state is defined as

$$\begin{aligned} & \hat{x}_i(t+\tau) \cong \hat{x}_i(t) + \tau \dot{\hat{x}}_i(t) + \ldots + \frac{\tau^r}{r!} \hat{x}_i^{[r]}(t) \ , \\ & i = 1, \ldots, n, T_1 \le \tau \le T_2 \end{aligned} \tag{52}$$

Or

$$\begin{aligned} & \hat{x}_i(t+\tau) \cong \tau_i^T \hat{X}_i(t) \\ & i = 1, \ldots, n, \quad T_1 \le \tau \le T_2 \end{aligned} \tag{53}$$

where, acording to to (51) and (46)

$$\begin{aligned} & \hat{X}_i(t) = \left[\hat{x}_i(t), \hat{x}_i^{[1]}(t) \right]^T = \left[\hat{x}_i(t), b_i u + \sum_{j=1}^{n} a_{ij} x_j \right]^T, i = 1, ..., n-1 \\ & \hat{X}_n(t) = \left[\hat{x}_n(t), \hat{x}_n^{[1]}(t) \right]^T = \left[\hat{x}_n(t), b_n u + u_o + \sum_{j=1}^{n} a_{ij} x_j \right]^T. \end{aligned} \tag{54}$$

Hence, prediction of estimation error can be expressed as

$$e(t+\tau) \cong \left[\hat{x}_1(t+\tau) - x_1(t+\tau), \ldots, \hat{x}_n(t+\tau) - x_n(t+\tau)\right]^T \tag{55}$$

Invoking Eq. (48) and Eq. (53) into Eq. (55) results in

$$e(t+\tau) \cong \left[\tau_i^T\left[\hat{X}_1 - X_1\right], \ldots, \tau_i^T\left[\hat{X}_n - X_n\right]\right]^T \tag{56}$$

hence,

$$e^T(t+\tau) \cong \left[\left[\hat{X}_1 - X_1\right]^T, \ldots, \left[\hat{X}_n - X_n\right]^T\right]\tau_o \tag{57}$$

where,

$$\tau_o = \left[\tau_i, \ldots, \tau_i\right]^T. \tag{58}$$

For minimizing error of state prediction, let define a quadratic cost function as follows:

$$J = \frac{1}{2}\int_{T_1}^{T_2} e(t+\tau)^T e(t+\tau)\, d\tau \tag{59}$$

where, T_1 and T_2 are lower and upper prediction times, respectively. Based on Eq. (57), the defined cost function can be approximated as

$$J \cong \frac{1}{2}\int_{T_1}^{T_2}\left(\left[\hat{X}_1 - X_1\right]^T, \ldots, \left[\hat{X}_n - X_n\right]^T\right)\tau_o\tau_o^T\left(\left[\hat{X}_1 - X_1\right]^T, \ldots, \left[\hat{X}_n - X_n\right]^T\right)^T d\tau \tag{60}$$

Or

$$J \cong \frac{1}{2}\left(\left[\hat{X}_1 - X_1\right]^T, \ldots, \left[\hat{X}_n - X_n\right]^T\right)\int_{T_1}^{T_2}\tau_o\tau_o^T d\tau\left(\left[\hat{X}_1 - X_1\right]^T, \ldots, \left[\hat{X}_n - X_n\right]^T\right)^T \tag{61}$$

and consequently, we have

$$J \cong \frac{1}{2}\left(\left[\hat{X}_1 - X_1\right]^T, \ldots, \left[\hat{X}_n - X_n\right]^T\right)\Lambda d\tau\left(\left[\hat{X}_1 - X_1\right]^T, \ldots, \left[\hat{X}_n - X_n\right]^T\right)^T \tag{62}$$

where,

$$\Lambda = \int_{T_1}^{T_2}\tau_o^T\tau_o\, d\tau = \begin{bmatrix} T_2 - T_1, \dfrac{T_2^2 - T_1^2}{2}, & \cdots, T_2 - T_1, & \dfrac{T_2^2 - T_1^2}{2} \\ \vdots \quad\quad \vdots & \vdots & \vdots \\ \dfrac{T_2^2 - T_1^2}{2}, \dfrac{T_2^3 - T_1^3}{3}, & \cdots, \dfrac{T_2^2 - T_1^2}{2}, & \dfrac{T_2^3 - T_1^3}{3} \end{bmatrix}$$
$$= \begin{bmatrix} \Lambda_{1,1}, \Lambda_{1,2}, & \cdots, \Lambda_{1,2n-1}, & \Lambda_{1,2n} \\ \vdots \quad\quad \vdots & \vdots & \vdots \\ \Lambda_{2n,1}, \Lambda_{2n,2}, & \cdots, \Lambda_{2n,2n-1}, & \Lambda_{2n,2n} \end{bmatrix} \tag{63}$$

Theorem2. Consider the defined linear system Eq. (46) and quadratic cost function Eq. (59). Then, optimal predictive term of the proposed observer u_o in Eq. (51) minimizing the defined cost function is obtained as

$$u_o = -\sum_{i=1}^{n} k_i e_i = -KE \tag{64}$$

where,

$$K = \left[\frac{\Lambda_{2n,1}}{\Lambda_{2n,2}} + \sum_{j=1}^{n} a_{1j}, \ldots, \frac{\Lambda_{2n,1}}{\Lambda_{2n,2}} + \sum_{j=1}^{n} a_{nj} \right] = \left[k_1, \ldots, k_n \right] \tag{65}$$

And

$$E = \left[e_1, \ldots, e_n \right]^T \tag{66}$$

Proof. According to (62), the necessary condition for optimization is given as

$$\frac{\partial J}{\partial u_o} = \frac{\partial \left(\left[\hat{X}_1 - X_1 \right]^T, \ldots, \left[\hat{X}_n - X_n \right]^T \right)}{\partial u_o} \Lambda \left(\left[\hat{X}_1 - X_1 \right]^T, \ldots, \left[\hat{X}_n - X_n \right]^T \right)^T = 0 \tag{67}$$

where, using Eq. (50) and Eq. (54) , we have

$$\begin{aligned} \left(\left[\hat{X}_1 - X_1 \right]^T, \ldots, \left[\hat{X}_n - X_n \right]^T \right) &= [\hat{x}_1(t) - x_1(t), \sum_{j=1}^{n} a_{1j} \left(\hat{x}_1(t) - x_1(t) \right), \\ &\ldots, \hat{x}_n(t) - x_n(t), \sum_{j=1}^{n} a_{nj} \left(\hat{x}_n(t) - x_n(t) \right) + u_o] \end{aligned} \tag{68}$$

and a_{ij} is element of matrix A in Eq. (46).

As can be seen from (68) and (63), the necessary condition Eq. (67) can be written as

$$\begin{aligned} &\left[\Lambda_{2n,1}, \Lambda_{2n,2}, \ldots, \Lambda_{2n,2n-1}, \Lambda_{2n,2n} \right] \\ &\left[\hat{x}_1(t) - x_1(t), \sum_{j=1}^{n} a_{1j} \left(\hat{x}_1(t) - x_1(t) \right), \ldots, \hat{x}_n(t) - x_n(t), \sum_{j=1}^{n} a_{nj} \left(\hat{x}_n(t) - x_n(t) \right) + u_o \right]' = 0 \end{aligned} \tag{69}$$

Note that, according to (63), in Eq. (69)

$$\left[\Lambda_{2n,1}, \Lambda_{2n,2} \right] = \left[\Lambda_{2n,3}, \Lambda_{2n,4} \right] = \ldots = \left[\Lambda_{2n,2n-1}, \Lambda_{2n,2n} \right] \tag{70}$$

So, Eq. (69) can be written as follows

$$\Lambda_{2n,1} \sum_{i=1}^{n} e_i + \Lambda_{2n,2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} e_i + \Lambda_{2n,2} u_o = 0 \tag{71}$$

where,

$$e_i = \hat{x}_i - x_i. \tag{72}$$

Finally, based on Eq. (71)

$$u_o = -\frac{\Lambda_{2n,1}}{\Lambda_{2n,2}}\sum_{i=1}^{n} e_i - \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} e_i = -\sum_{i=1}^{n} k_i e_i = -KE \tag{73}$$

where,

$$K = \left[\frac{\Lambda_{2n,1}}{\Lambda_{2n,2}} + \sum_{j=1}^{n} a_{1j}, \dots, \frac{\Lambda_{2n,1}}{\Lambda_{2n,2}} + \sum_{j=1}^{n} a_{nj}\right] = \left[k_1, \dots, k_n\right] \tag{74}$$

and E is defined in Eq. (66). This completes the proof of Theorem 2.

According to Eq. (1), Eq. (51), and Eq. (64), the error dynamic can be found as

$$\dot{E} = AE + B_o u_o \tag{75}$$

Invoking Eq. (73) into error dynamic Eq. (75) results in

$$\dot{E} = A_c E \tag{76}$$

where,

$$A_c = A - B_0 K = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \vdots & \vdots \\ a_{n1} - k_1 & \dots & a_{nn} - k_n \end{bmatrix} \tag{77}$$

Similar to the Assumption 2 presented in Section 2 and according to the defined matrix A for the complete form in Eq. (46), there is a vector L such that, it holds for Eq. (30) and consequently, one can present the proposed optimal observer for the complete form as Eq. (34).

3.1 Stability analysis for complete form

Invoking Eq. (74) into A_c matrix in error dynamic Eq. (77) gives

$$A_c = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \vdots & \vdots \\ a_{n1} - \frac{\Lambda_{2n,1}}{\Lambda_{2n,2}} - \sum_{i=1}^{n-1} a_{1i} & \dots & a_{nn} - \frac{\Lambda_{2n,1}}{\Lambda_{2n,2}} - \sum_{i=1}^{n-1} a_{ni} \end{bmatrix} \tag{78}$$

where,

$$\frac{\Lambda_{2n,1}}{\Lambda_{2n,2}} = \frac{3}{2}\left(\frac{T_2^2 - T_1^2}{T_2^3 - T_1^3}\right) \tag{79}$$

Then, as can be easily seen, the error dynamic Eq. (75) is asymptotically stable if and only if all the eigenvalues of $\boldsymbol{A_c}$ in Eq. (78) have negative real parts. Therefore, stability of the proposed observer depends on prediction times $\boldsymbol{T_1}$, $\boldsymbol{T_2}$ and also on the elements of system matrix $\boldsymbol{A}$.

For brief totalization, Algorithm 1 sums the proposed linear optimal observer in this section.

ALGORITHM 2 Proposed optimal observer for complete form
solving the optimal observer problem.
Step 1 Check assumption 1.
Step 2 Select appropriate prediction times T_1, T_2.
Step 3 Calculate matrix Λ from equation (63).
Step 4 Solve equation (65) for K.
Step 5 Check stability of A_c in (78).
Step 6 Solve equation (30) for L.
Step 7 $\boldsymbol{L}$ is the optimal gain vector for the observer (34).

4. Illustrative Examples

Example 1. For evaluating performance of the optimal observer, consider the pendulum system, where its dynamics is described by the following equation

$$ml^2\ddot{\theta} + b\theta + mglsin\theta = 0$$

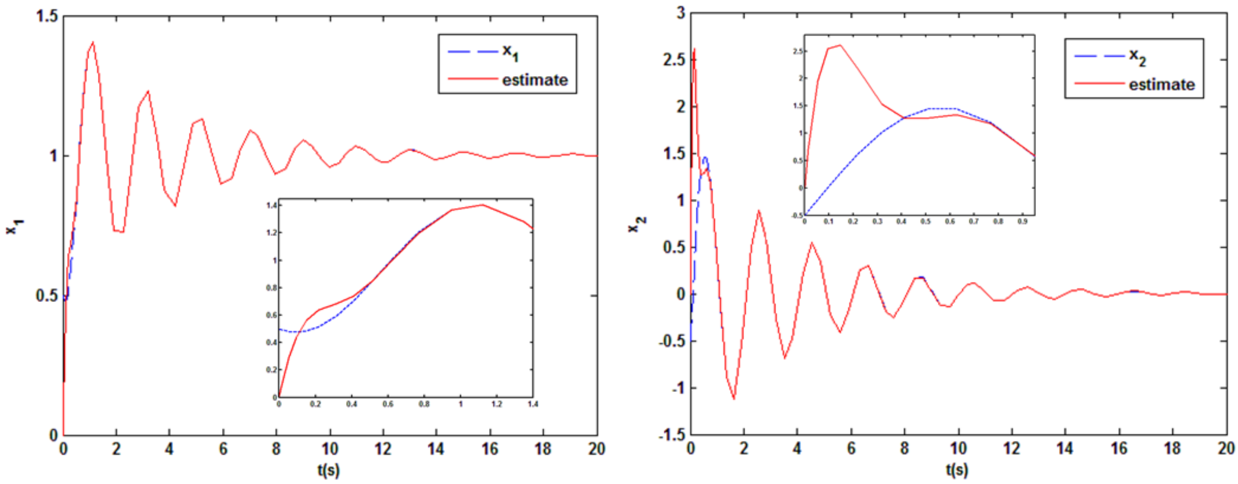
where, m is the pendulum's mass, l is its length, g is gravity constant, and b is friction coefficient at hinge. Letting $x_1 = \theta, x_2 = \dot{\theta}$, the state-space equation for small θ can be described in the canonical form by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{g}{l} & -\dfrac{b}{ml^2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_1 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_1 \end{bmatrix}$$

Initial conditions for this system and the optimal observer are considered as $x(0) = [0.5, -0.5]^T$ and $\hat{x}(0) = [0, 0]^T$, respectively. Also, its mechanical parameters in simulation are chosen as $l = 1, b = 1, m = 2,$ and $g = 10$, which are expressed in SI units.

Using the optimal observer in the canonical system's method (Section 2), its gain vector L is determined based on Algorithm 1 as $T_1 = 0, T_2 = 200\,ms$, and $\boldsymbol{L} = [-100, -13.35]^T$.



(a) (b)

Fig. 1. State trajectories $x, \hat{x}$ of the pendulum system, (a) state trajectories $x_1, \hat{x}_1$, (b) state trajectories $x_2, \hat{x}_2$

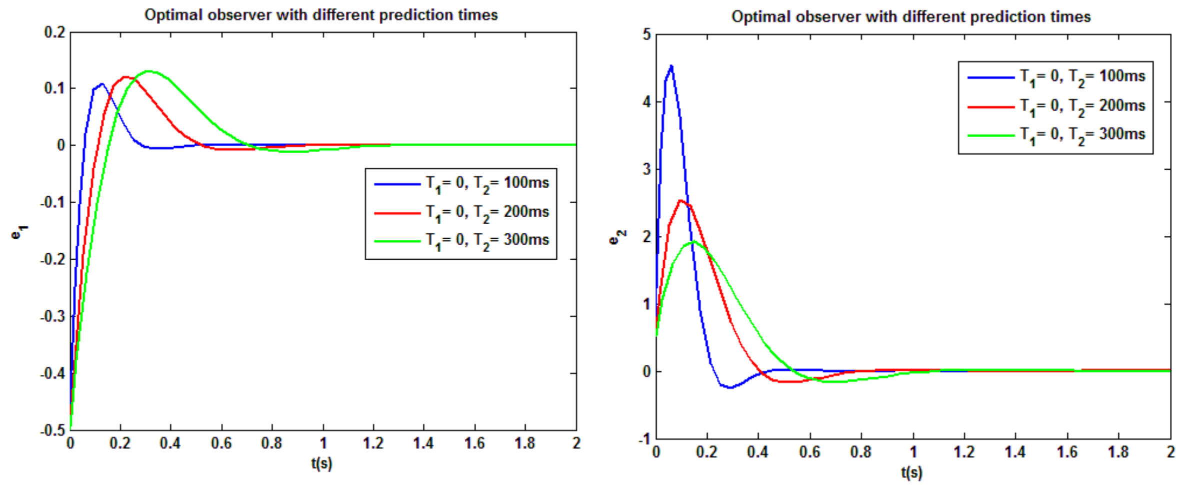
Simulation results of using the optimal observer are shown in Figure 1. As shown in Figure 1, the observer's estimations quickly converge to actual states of the system confirming performance of the optimal observer.

For evaluation of the optimal observer, let investigate the influence regarding choice of the prediction times. In this regard, various prediction times are considered and then, the related simulation results are shown in Figure 2.

For a more precise comparison, the integral time-square error (ITSE) is computed as a performance index for each case as follows:

$$\mathrm{ITSE} = \int_0^{t_f} te^2(t)\,dt$$

ITSE was chosen due to the importance of the magnitude of the transient error over the magnitude of the steady error in the performance of the observer. The numerical results are shown in Table 1. As can be observed, the performance index ITSE is enhanced with the increment of upper prediction time.



(a) (b)

Fig. 2. State observation errors for the pendulum system, (a) State error observation e_1, (b) state error observation e_2

Table 1. Measurement of ITSE as a performance index for the optimal observer in the pendulum system.

ITSE	Lower and upper prediction times		
	$T_1 = 0, T_2 = \ 00ms$	$T_1 = 0, T_2 = \ 00ms$	$T_1 = 0, T_2 = \ 00ms$
$\int_0^{t_f} te_1^2(t)dt$	$\int_0^{t_f} te_2^2(t)dt$	0.0008	0.0018
$\int_0^{t_f} te_2^2(t)dt$	0.1352	0.1443	0.1553

Example 2. Consider another canonical linear system that is described as follows (Na, et al. [29]):

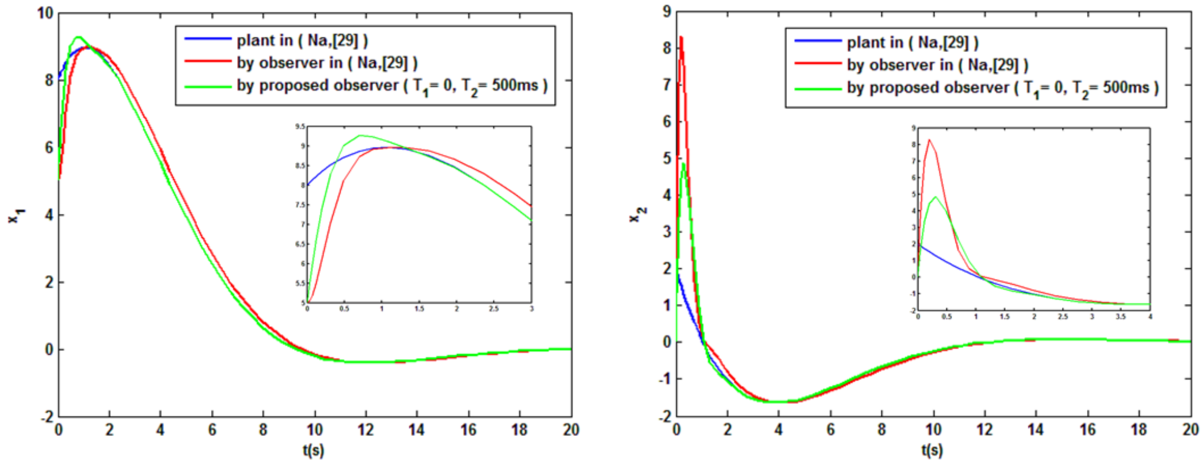
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.16 & -0.56 \end{bmatrix} \begin{bmatrix} x_1 \\ x_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_1 \end{bmatrix}$$

The initial conditions are $x(0) = [8,2]^T$ and $\hat{x}(0) = [5,0]^T$. Similar to Example 1, the gain vector L and prediction times are determined based on Algorithm1 as $T_1 = 0, T_2 = 500\mathrm{ms}$ and $L = [-5; -13.3333]^{\mathrm{T}}$. The optimal observer is simulated and results of this example are shown in Figure 3. Moreover, Table 2 shows the numerical results, which indicates the performance index, ITSE in is observer by [29] more than proposed observer. Comparing the estimation results of both observers shows that our proposed observer achieves a better performance than the observer in [29].

Table 2. Performance index ITSE for proposed observer and observer in Na, et al. [29]

ITSE	Observer in Na, et al. [29]	Proposed observer $(T_1 = 0, T_2 = \ 00ms)$
$\int_0^{t_f} te_1^2(t)dt$	3.549	0.1105
$\int_0^{t_f} te_2^2(t)dt$	4.577	2.194



(a) (b)

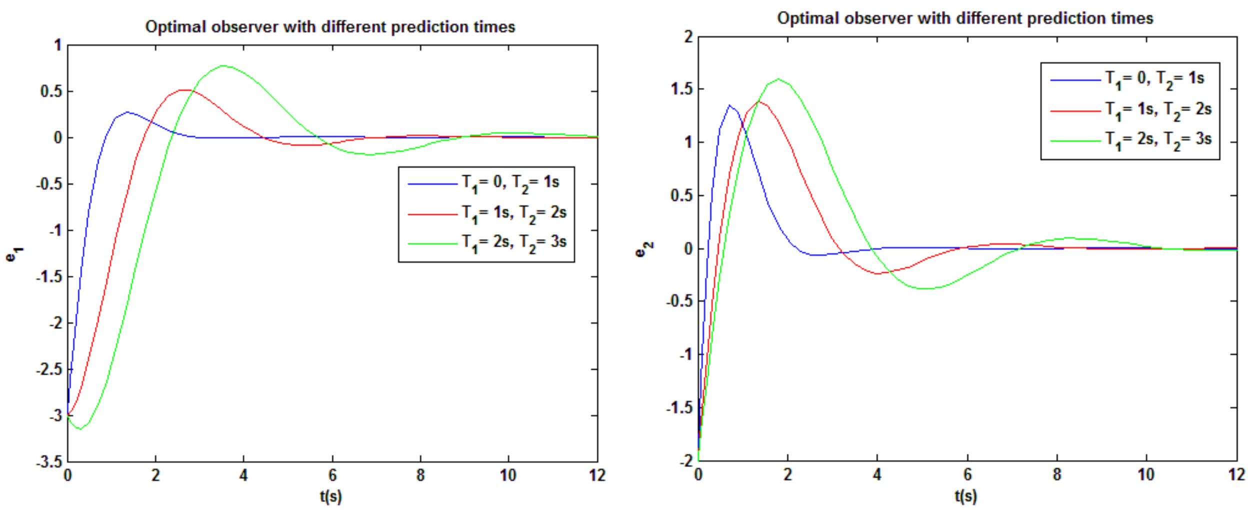
Fig. 3. State trajectories $x, \hat{x}$ of the linear system in the study by Na, [29], (a) state trajectories $x_1, \hat{x}_1$ (b) state trajectories $x_2, \hat{x}_2$

For investigating the influence regarding choice of the prediction times, various prediction times are considered but length of prediction horizon is kept the same, i.e., $T_2 - T_1 = 1$s. The results are shown in Table 3 and Figure 4. As can be seen, ITSE as a performance index is increased with the increment of upper and lower prediction times.

Considering results of these two examples in canonical form, it is clear that the estimations quickly converge to actual states of the system, confirming performance of the optimal observer. Moreover, transient responses of the proposed optimal observer are acceptable.

Table 3. Measurement of ITSE as a performance index for the optimal observer in the linear system (Na, [29])

ITSE	Lower and upper prediction times		
	$T_1 = 0, T_2 = \ s$	$T_1 = 1, T_2 = \ s$	$T_1 = 2, T_2 = \ s$
$\int_0^{t_f} te_1^2(t)dt$	0.401	3.513	11.380
$\int_0^{t_f} te_2^2(t)dt$	1.221	3.894	8.962



(a) State error observation e_1 (b) State error observation e_2

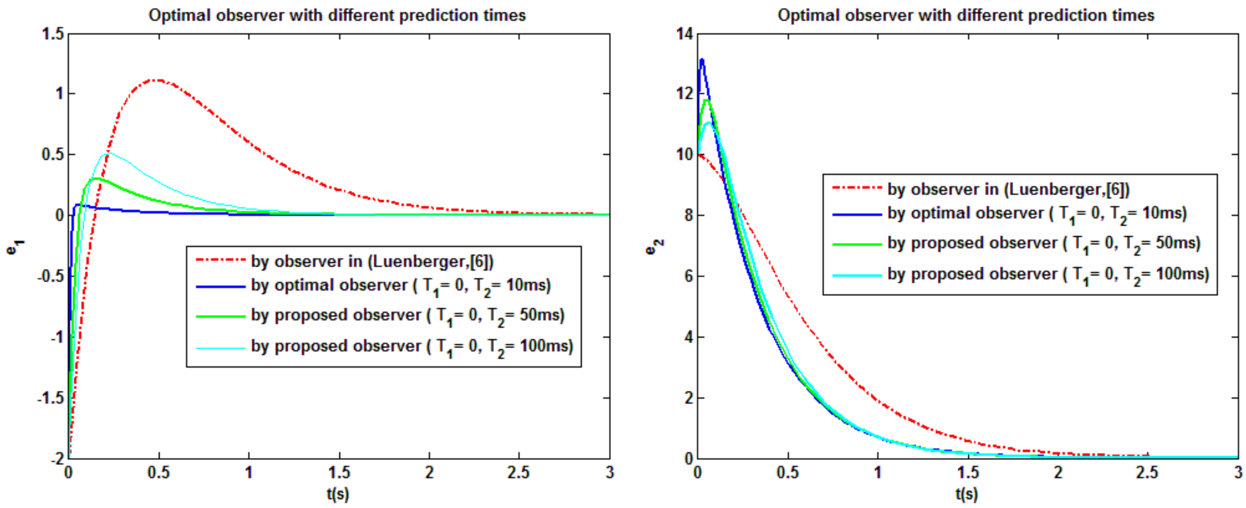
Fig. 4. State observation errors of the linear system in the study by Na, [29].

Example 3: Consider the following state -space of the system in complete form [6].

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_1 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_1 \end{bmatrix}$$

Initial conditions for this system and the observer states are $\mathrm{x}(0) = [2, -10]^{\mathrm{T}}$ and $\hat{\mathrm{x}}(0) = [0,0]^{\mathrm{T}}$, respectively. The optimal observer is designed in the complete form (Section 3) and its performance is compared with that of the Luenberger observer [6], using different prediction times, results of which are shown in Figure 5.



(a) (b)

Fig. 5. State observation errors for the linear system in the study by Luenberger, [6], (a) state error observation e_1 , (b) state error observation e_2

In this regard, based on the Luenberger observer design [6], the gain vector is chosen as $\mathrm{L} = [-3, -4]^{\mathrm{T}}$. For the optimal observer, the gain vector L and prediction times are determined based on Algorithm 2 as $\mathrm{T}_1 = 0, \mathrm{T}_2 = 10$ ms , $\mathrm{L} = [-149; -298]^{\mathrm{T}}, \mathrm{T}_1 = 0, \mathrm{T}_2 = 50$ ms , $\mathrm{L} = [-29; -58]^{\mathrm{T}}$ and $\mathrm{T}_1 = 0, \mathrm{T}_2 = 100$ ms , $\mathrm{L} = [-14; -28]^{\mathrm{T}}$.

Comparing the estimation results of both observers in Figure 5 and Table 4 reveals that our proposed observer achieves a better performance than the observer in [6].

Moreover, the results on the influence regarding choice of different prediction times in proposed observer are shown in Table 4. As can be seen, observation errors and ITSE are increased with the increment of upper prediction times.

Table 4. Measurement of ITSE as a performance index for the optimal observer and observer in Luenberger, [6]

ITSE	Observer in Leunberger [6]	Proposed observer Lower and upper prediction times		
		$T_1 = 0, T_2 = 10ms$	$T_1 = 0, T_2 = 50ms$	$T_1 = 0, T_2 = 100ms$
$\int_0^{t_f} te_1^2(t)dt$	0.0071	0.0003	0.0071	0.0284
$\int_0^{t_f} te_2^2(t)dt$	6.024	5.562	6.024	6.588

As can be seen from Tables 1-4, observation errors are decreased with the decrement of prediction times.

All the results obtained from assessing the examples showed that the proposed observer improves linear observers̓ performance while asymptotic convergence of the state estimation error to the zero is ensured.

5. Conclusions

In this study, a new optimal observer methodology was presented based on state prediction for linear systems. In this approach, observer parameters are determined such that the approximated receding horizon of the estimation error is minimized (prediction property) while asymptotic convergence of the state estimation error to zero is guaranteed. In this technique, the gain of the proposed observer is optimally chosen by a predictive approach among those satisfying the Luenberger observer condition. In addition, since there is no need for online optimization, the proposed optimal observer can be executed simply, especially for linear systems with fast dynamics. The interests of the proposed approach were also illustrated in the simulation results.

Developing a proposed optimal observer for time-varying linear systems and uncertain linear systems with bounded disturbances and noises can be considered a subject for further research. These cases can occur in many real-world applications, such as mechanical systems, robot dynamics, servo systems, and valve actuators, etc.

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