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Application of Erlang Formulae in Next Generation Networks

Erik Chromy, Tibor Misuth, Adam Weber
Institute of Telecommunications, Faculty of Electrical Engineering and Information Technology, Slovak University of Technology, Bratislava, Slovak Republic
Email: chromy@ut.fei.stuba.sk, misuth@ut.fei.stuba.sk, weber.ado@gmail.com

***Abstract*— The paper deals with the possibility of the Erlang B and Erlang C formula utilization in Next Generation Networks (NGN). Based on the common properties of synchronous and asynchronous networks it is possible the utilization of Erlang formulas also for asynchronous networks. It is possible to describe traffic in NGN networks by calculation of following parameters – loss, link utilization and bandwidth. The second part of this paper deals with the possibility of application of Erlang B formula in contact center. Contact center is one of the many examples of the NGN networks. By application of Erlang B formula in contact center environment it is possible to monitor following important parameters - number of agents, probability of call blocking, utilization of agents and mean number of calls in the system.**

***Index Terms*— Erlang B Formula, extended Erlang B Formula, Erlang C formula, Next Generation Networks, QoS, Contact Center**

1. Introduction

Much attention is focused on description of traffic in asynchronous packet networks (represented by ATM and IP networks) [1-5]. There are various methods for traffic description. Many of them are complex with high computational requirements. The description by Markov chains is one of them. Therefore the Erlang formulas [6-7] can be very efficient and simple way how we can describe traffic parameters in asynchronous networks.

Erlang formulas use traffic parameters such as loss, delay, bandwidth and link utilization. These parameters are especially important from the Quality of Service (QoS) [8-10] providing point of view. Hence the Erlang formulas seem to help us in the field of Quality of Service in Next Generation Networks (NGN) [11-13].

The paper proposes the idea of utilization of Erlang B and Erlang C formulas for description of traffic in NGN networks. The second part of the paper deals with the possibility of application of Erlang B formula in contact center environment.

Erlang theory is described in detail in [14].

This paper is organized as follows. In section 2, we discuss about asynchronous networks (ATM and IP). In Section 3, we analyze Erlang B and Erlang C formulas. Section 4 and 5 represent results obtained by calculations through Erlang B and Erlang C formulas. The last Section 6 deals with the possibility of application of Erlang B and extended Erlang B formulas in contact center environment.

2. Asynchronous Networks

Asynchronous Transfer Mode (ATM) was the emerging network technology in the beginning of 90-ties. The main reasons for ATM development were:

- Increasing demand for telecommunication and information technologies and services.
- Convergence of data and voice communication.

ATM technology combines fast packet transfer with synchronous transfer through virtual circuits and virtual paths. It is cell switching mode which offers low transfer latency with high level of Quality of Service and ensures the support for voice, data and video service with high transfer rates.

Networks based on Internet Protocol (IP) provide datagram service. IP transfer is non-connection oriented and its main feature is that it is best effort service. It means that the packet will be transferred in the best way, without forced delay and without unnecessary packet losses. The transfer of IP datagrams is done without guarantee of packet delivery.

Services such as voice and real-time video are very sensitive on particular traffic parameters. The main of these parameters are delay, loss and error rate. We have to note that various types of services have also different bandwidth requirements. Therefore there is need for guarantee of these parameters. This guarantee is called Quality of Service.

QoS requirements are: end-to-end delay, jitter, packet losses, bandwidth, link utilization.

3. Erlang Formulas

Two Erlang formulas (B and C) are used to describe the traffic in asynchronous networks. Calculations were performed in Matlab environment.

3.1 The Erlang B Formula

The Erlang B formula represents the ratio of lost calls. Therefore it is sometimes called “loss Erlang formula”. It is defined as follows:

$$B=\frac{\frac{A^N}{N!}}{\sum_{k=0}^{N}\frac{A^k}{k!}}=\frac{A^N}{N!}\cdot\frac{1}{1+A+\frac{A^2}{2!}+\frac{A^3}{3!}+\ldots+\frac{A^N}{N!}}.100[\%], \qquad (1)$$

where B is ratio of lost calls [%], A is total offered traffic [Erl] and N is number of channels (links).

The first Erlang formula can be written also in the following form:

$$B=\frac{1}{1+\sum_{k=1}^{m}\left(\frac{N}{A}\right)\cdot\left(\frac{N-1}{A}\right)\cdots\left(\frac{N-k+1}{A}\right)}. \qquad (2)$$

The use of (2) can significantly decrease the computational requirements and we can also calculate the load of the system with the higher values [6-7].

The following conditions must be met for the first Erlang formula:

- The flow of requests (calls) originates randomly with exponential distribution of incoming requests, which means the higher distance between requests, the lower number of these cases.
- Service time has similar distribution, i.e. exponential decreasing of requests with higher service time.
- The flow of requests is steady, as if it comes from infinite number of request sources.
- There is full availability of requests to all served links.
- Rejected requests do not return to incoming flow, therefore there are not repeated requests.
- No two requests will originate together [6].

IP traffic brings radical changes into telephone networks. The condition 2 is not fulfilled. And because the traffic from one source is considerable increasing, also the condition 5 can not be met.

3.2 The Erlang C Formula

The Erlang C formula also assumes infinite number of traffic sources. These sources generate the traffic A for N lines. The incoming request is inserted into waiting queue if all links N are occupied. Waiting queue can store infinite number of requests concurrently. This Erlang formula calculates probability of creation of waiting queue in the case of traffic A, and if it is assumed that the blocked calls will remain in the system until they are served [6].

$$C = \frac{\frac{A^N}{N!}\cdot\frac{N}{N-A}}{\sum_{k=0}^{N-1}\frac{A^k}{k!}+\frac{A^N}{N!}\cdot\frac{N}{N-A}}, \text{ where N>A} \qquad (3)$$

where A is total offered traffic [Erl], N is number of channels (links) and C is probability of waiting for service.

- Dividing numerator and denominator by $\sum_{k=0}^{N}\frac{A^k}{k!}$ and
- consecutive use of Erlang B formula (1).

$$C = \frac{N\cdot B}{N-A\cdot(1-B)}. \qquad (4)$$

3.3 Common Characters of Asynchronous and Synchronous Networks

Erlang formulas were primary intended for traffic description in synchronous networks. The idea is to use these formulas also for asynchronous networks.

Table 1：Synchronous and asynchronous networks

Synchronous networks		Asynchronous networks	
B	lost calls ratio	B	loss rate
C	probability of waiting for service	C	probability of delay
A [Erl]	total offered traffic	A [%]	link utilization
N	number of channels (links)	N [Mbit/s]	bandwidth

Probability of delay C for asynchronous networks represents the latency which occurs during transmission in the case of the heaviest traffic. This delay occurs in IP networks due to waiting queues in buffers in network nodes. Unfortunately, there is no similar parameter for jitter in synchronous networks, hence through Erlang formulas it can not be estimated.

4. Analysis of Erlang B Formula

This part represents results obtained by calculations through Erlang B formula. Input parameters were given as follows:

- One of parameters is constant.
- Other parameter was increased in given step sequence.

It is known that we can calculate the loss B through Erlang B formula if we have given link utilization A and bandwidth N. But it is also possible to calculate the link utilization A by method of bisection, if we know the bandwidth N and loss B [7].

4.1 Bandwidth and Loss in the Case of Constant Link Utilization

The tendencies are following:

- The loss B is decreasing if the bandwidth is increasing and link utilization is constant.
- The loss B is increasing if the link utilization A is increasing.

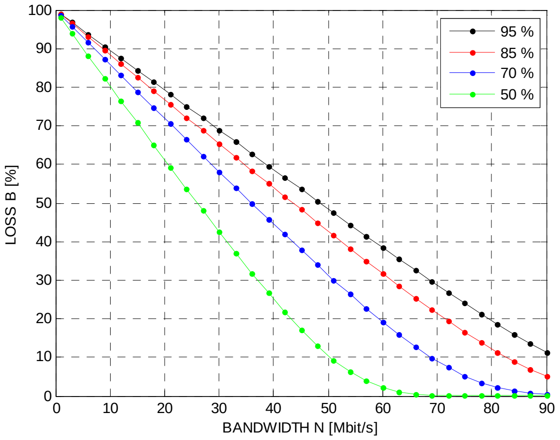

Fig 1: Dependency between loss and bandwidth in the cases of constant utilization

4.2 Loss and Link Utilization in the Case of Constant Bandwidth

The tendencies are following:

- By increasing link utilization *A* the loss *B* is also increasing if the bandwidth *N* is constant.
- By increasing bandwidth *N* the loss *B* is decreasing if the link utilization *A* is constant.

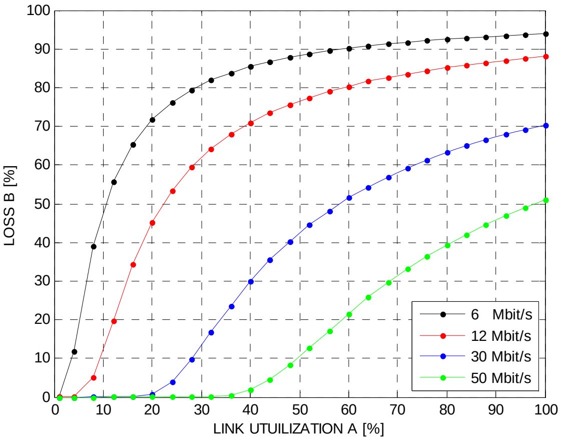

Fig 2: Dependency between loss and link utilization in the case of constant bandwidth

5. Analysis of Erlang C Formula

This part represents the results obtained by calculations through Erlang C formula (4). By this equation we can calculate the possibility of delay *C* and loss *B* if we know the two parameters – the link utilization *A* and bandwidth *N*. Also in this case the input parameters were given as follows:

- One of parameters is constant.
- Other parameter was increased in given step sequence.

5.1 Link Utilization and Probability of Delay in the Case of Constant Bandwidth

The tendencies are following:

- In the case of constant probability of delay *C* and increasing bandwidth *N* the link utilization *A* can be higher.
- In the case of constant link utilization *A* and increasing bandwidth *N* the probability of delay *C* is decreasing.
- In the case of increasing link utilization *A* and constant bandwidth *N* the probability of delay *C* is increasing.

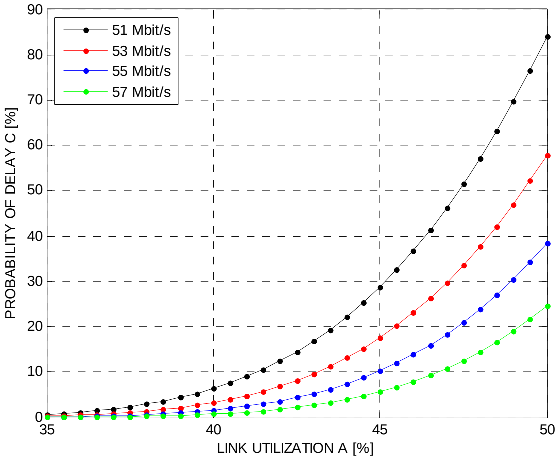

Fig 3: Probability of delay in the case of constant bandwidth and increasing link utilization

5.2 Link Utilization and Bandwidth in the Case of Constant Probability of Delay

By use of the Erlang B formula (4) and the method of bisection is possible to obtain the dependency of the link utilization *A* if the probability of delay *C* is constant and bandwidth *N* is increasing. The results are depicted in the Fig. 4.

The tendencies are following:

- In the case of constant link utilization *A* and increasing bandwidth *N* the probability of delay *C* is decreasing.
- In the case of constant bandwidth *N* and increasing probability of delay *C* the link utilization *A* is increasing.
- In the case of increasing bandwidth *N* and constant probability of delay *C* the link utilization *A* is increasing.

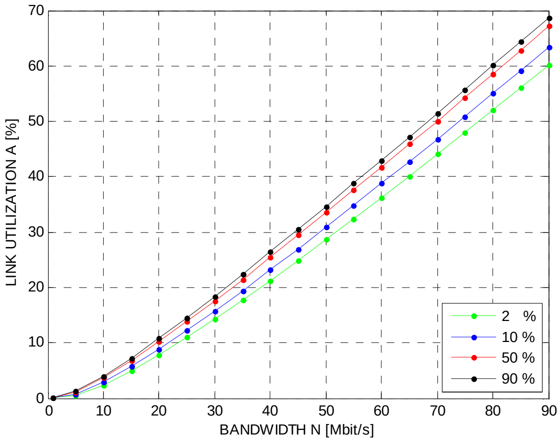

Fig 4: Link utilization in the case of constant probability of delay and increasing bandwidth

5.3 Probability of Delay and Loss if the Link Utilization and bandwidth are Changing

By use of the Erlang C formula (4) is possible to obtain dependencies of probability of delay C and loss B if the bandwidth N and link utilization A were changing. The obtained results are shown in the Fig. 5 and Fig. 6.

The tendencies are following:

- In the case of increasing link utilization A together with decreasing bandwidth N the loss B and probability of delay C are increasing.
- In the case of constant bandwidth N together with increasing link utilization A the loss B and probability of delay C are increasing.
- In the case of constant link utilization A together with increasing bandwidth N the loss B and probability of delay C are decreasing.

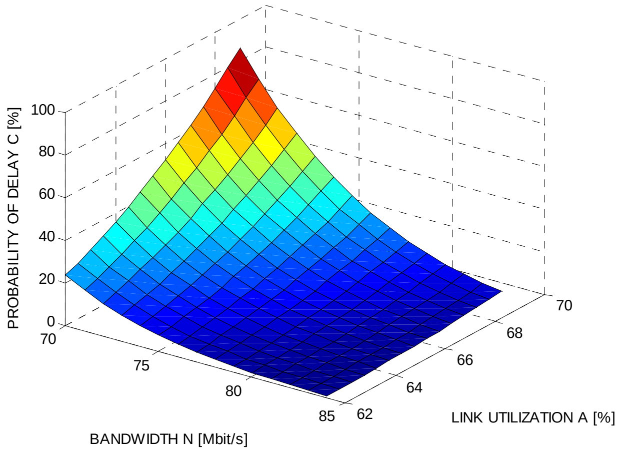

Fig 5: Dependency between probability of delay, bandwidth and link utilization

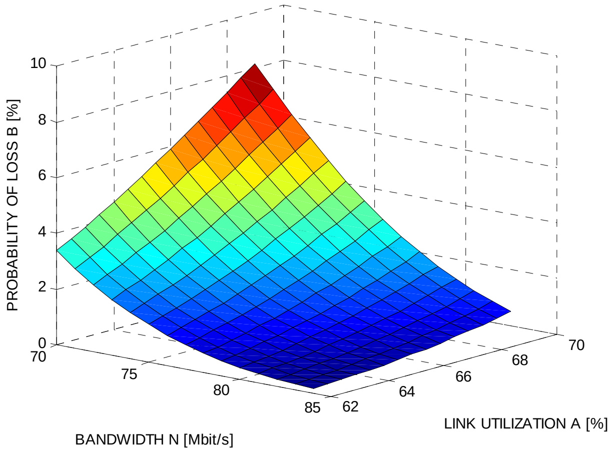

Fig 6: Dependency between probability of loss, bandwidth and link utilization

6. Application of Erlang B Formula in Contact Center Environment

Erlang B formula is basic mathematical model used for estimation of necessary access links capacity in contact centers. Furthermore, it enables modeling of the extreme and not usual case of contact center implementation without waiting queue. Incomming calls are either assigned to a free agent or are blocked and not served in case of no free agent is available. The most important output parameter of the model is the probability of call blocking P_B. To avoid problems and accuracy issues when computing results for large contact centers (many agents, therefore division of two large numbers), all calculations are done using slightly modified algorithm [15].

$$P_B(N,A)=\frac{1}{1+\sum_{i=1}^{N}\prod_{k=0}^{i-1}\frac{N-k}{A}}, \qquad (5)$$

where N is number of agents and A is traffic load.

For a system without waiting queue it is also important to determine the mean number of requests in the system K (i.e. mean number of occupied agents and links) and utilization of agents η [16]:

$$K=A(1-P_B), \qquad (6)$$

$$\eta = \frac{K}{N} = \frac{A(1-P_B)}{N}. \tag{7}$$

6.1 Input Parameters for Erlang B Formula Simulation

- incomming calls rate $\lambda = 667$ per hour,
- mean call service time $1/\mu = 150$ seconds,
- number of agents $N = 1\text{-}45$,
- simulation time $T_{SIM} = 30$ hours,
- steady state wait time = 1 hour,
- time step of simulation = 1 second.

The trends of important simulation outputs in dependence on number of agents are showed on Fig. 7.

From Fig. 7 is clear, that the trend of the probability of call blocking P_B has at the beginning linearly decreasing tendency, which is changing at the value of approx. $N = 25$ ($P_B = 20\%$) into exponential function (approaching to value 0). Utilization of agents η at the beginning is almost 100% to which occupation of the contact center K at the level of current agents count and linear increasing of this value is related as well. From value approx. $N = 25$ agent continual decreasing of agents utilization occurs. Mean number of requests K is changing to the constant value near 28. It is exactly the value of load A in Erlangs, for which the contact center is exposed by given input parameters.

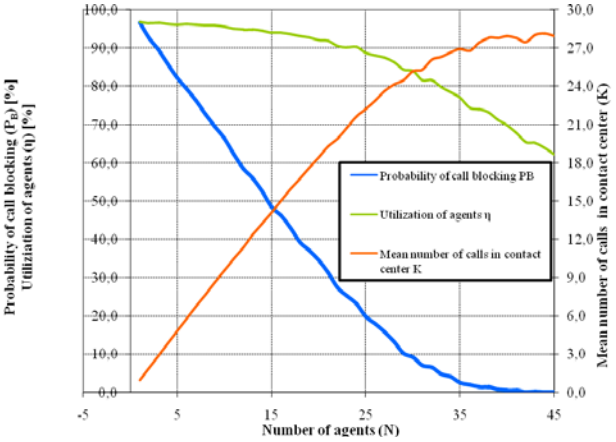

Fig 7: Simulation results for Erlang B formula

6.2 Extended Erlang B Formula

It is possible to modify the original Erlang B formula, so that we can consider also the traffic caused by repeated call attempts, in case of the unsuccessful previous attempt (thus, the call was rejected). This form of modified Erlang B formula is called extended Erlang B formula [15]. These repeated attempts to the contact center achieving can in specific cases significantly increase the overall traffic load. Therefore it is interesting to verify the behavior of the contact center also in this situation.

One new parameter is necessary – coefficient r (*recall factor*). Then the contact center traffic load consists of two components: incoming calls and repeated calls [17]:

$$A = A_0 + R = A_0 + A.P_B(A,N).r\,, \tag{8}$$

where A_0 represents the incomming calls (in Erlangs), R represents repeated calls a P_B is probability of call blocking according to the basic Erlang B formula. It is possible to modify (8) into form:

$$A(1 - P_B(A,N).r) = A_0\,. \tag{9}$$

6.3 Input Parameters of Extended Erlang B Formula Simulations

Input parameters of this simulation are the same than in the case of Erlang B formula simulation. Value of recall factor r is set to value 0.5. That means, that 50% of blocked calls are repeated and the caller is trying to make session once again. So the overall traffic load A is increasing.

The trends of important simulation outputs are showed on Fig. 8.

Fig. 8 includes also the trends of probability of call blocking P_B for basic Erlang B formula. It is evident, that by application of repeated calls mainly for lower number of agents higher call blocking occurs. With gradually increasing of the number of agents this difference decreases, because of the number of repeated calls decreases. The additional load generated by these calls is thus smaller. This phenomenon has the same effect on the two others monitored parameters (K a η).

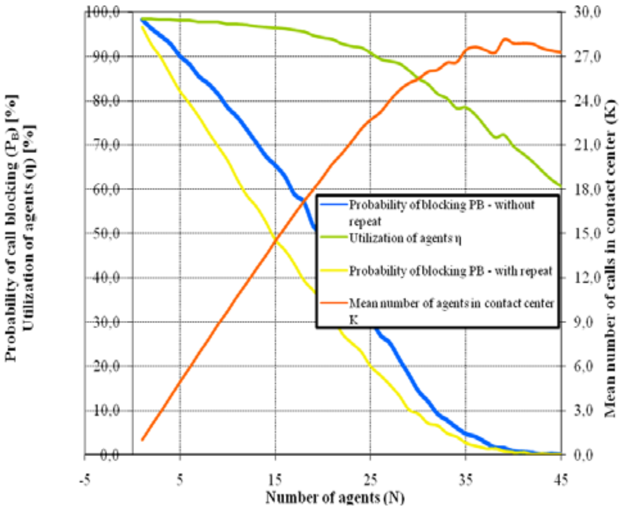

Fig 8: Simulation results for extended Erlang B formula

7. Conclusion

The paper proposes the idea of utilization of Erlang B and Erlang C formulas for description of traffic in NGN networks. All obtained tendencies through the calculations suggest the possibility of the Erlang formulas use in NGN networks (transport layer is represented through the ATM or IP network). Erlang B formula does not contain parameter for probability of delay, therefore this formula is more suitable for ATM network, because the probability of delay in this network is omissible. Through the Erlang C formula is possible to estimate the probability of delay, which usually occurs in IP networks. Description of traffic gives the opportunities to monitor the Quality of Service parameters in NGN networks. It seems that simplicity and unpretentiousness of Erlang formulas can be their strong advantage against other methods for traffic description in asynchronous networks, but the more future research is necessary in this field.

The paper deals also with the possibility of application of Erlang B formula in contact center environment. Through the simulation it is possible to monitor important QoS parameters - number of agents, probability of call blocking, utilization of agents and mean number of calls in the system. Great disadvantage of Erlang B formula is absence of waiting queue. Therefore, the better alternative is usage of the Erlang C formula.

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Erik Chromy: was born in Veľký Krtíš, Slovakia, in 1981. He received the Master degree in telecommunications in 2005 from Faculty of Electrical Engineering and Information Technology of Slovak University of Technology (FEI STU) Bratislava. In 2007 he submitted PhD work from the field of Observation of statistical properties of input flow of traffic sources on virtual paths dimensioning and his scientific research is focused on optimizing of processes in convergent networks. Nowadays he works as assistant professor at the Institute of Telecommunications of FEI STU Bratislava.

Tibor Misuth: was born in Žilina, Slovakia, in 1984. He is a student of PhD. study at Institute of Telecommunications, Faculty of Electrical Engineering and Information Technology of Slovak University of Technology Bratislava. He focuses on application of Erlangs' equations both in classic telecommunication networks and modern IP networks.

Adam Weber: was born in Kežmarok, Slovakia, in 1987. He received the Master degree in telecommunications in 2011 from Faculty of Electrical Engineering and Information Technology of Slovak University of Technology (FEI STU) Bratislava. He focuses on application of Erlangs' equations in telecommunication networks.