Robust 4-D Hyperchaotic DNA Framework for Medical Image Encryption

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Abstract: With the integration of cloud computing approaches in the healthcare systems, medical images are now processed and stored remotely on third-party servers. For such digital medical image data, privacy, protection, and security must be maintained by using image encryption methods. The aim of this paper is to design and apply a robust medical encryption framework to enhance the protection of medical image transformation and the patient information confidentiality. The proposed Framework encrypt the digital medical images using DNA computation and hyperchaotic RKF-45 random sequence approach. For which, the DNA computation is enhanced by applying hyperchaoticRKF-45 random key to the different Framework phases. The simulation results on different medical images were measured with various security analyses to prove the proposed framework randomness and coherent. Simulation results showed the ability of the hyperchaotic DNA encryption framework to withstand multiple electronic attacks with high performance compared to its counterparts of encryption algorithms. Finally, simulation and comparative studies have shown that, the proposed cryptography framework reported UACI and NPCR values 33.327 and 99.603 respectively, which are near to the theoretical optimal value.

Index Terms: Image Encryption, Medical Image, DNA Computing, Hyperchaotic System.

1. Introduction

In the contemporary internet world, e-Health services plays a vital role [1, 2]. With the assistance of cloud computing and telecommunication model, the medical data are transmitted and stored from one location to another via different network media [3]. The e-Health services necessitate the medical image enabled health information exchange to provide doctors for faster access, interpretation and clinical diagnosis. Therefore, providing patient’s medical image confidentiality is an essential security aspect of eHealth services.

For medical diagnosis purpose, the sent medical image should be received in perfect condition without any loss [4-6]. Cryptography is a science of applying logic and complex mathematics to design strong encryption methods to secure data and to retrieve the original data back by applying decryption [7-10].

Chaos-based ciphers can be used to employ secured and robust cryptosystem, due to the sensitivity to the deterministic pseudorandom behavior and initial conditions [11-13]. Hence, ciphers Chaos-based have been adapted for medical applications security [14-16]. However, most of the chaos encryption systems have showed severe security weaknesses that leave them open to classical attacks [17-19].

Due to the DNA molecule special characteristics, large information density and vast parallelism; DNA coding is applied to cryptography for improving the security and efficiency of encryption schemes [20-24].

Inspired by the DNA computing and chaos advantages, in this paper, a secure robust image encryption framework is proposed. The proposed cryptography Framework aims to enhance the confidentiality of patient information by protection of the digital medical images transformation. The proposed cryptography Framework encrypt the digital medical images using DNA computation and 4D chaotic system by utilizing hyperchaotic RKF-45 random key to the different Framework phases. Experimental and simulation results reveal that the proposed medical image cryptography framework exhibited high confidentiality and security in e-health care systems.
The rest of the manuscript is organized as follows: Section 2 presents the existing work related to image encryption. In section 3 and 4, a brief description of DNA Computing and Runge-Kutta-Fehlberg Method (RKF45) is presented. In section 5, the proposed medical image encryption HC RK45-DNA Framework is elaborated. In section 6 and 7, the simulation and security analysis of the proposed HC RK45-DNA encryption framework on different medical images is investigated and discussed. Finally, in section 8 the main findings of this work are discussed.

2. Related Work

Hu et al. [25] introduced a DNA-based image cryptography, for which, they overcome the DNA complementary operation limitations by including the DNA operation into the diffusion process. In [24] Shyam et al. presented a novel image DNA computing encryption scheme, where they utilized the DNA sequences to encode the information and the XOR operation to encrypted the image.

Meanwhile, Chen et al. [26] proposed an encryption approach that classified an excellent performance due the features of 3D chaos maps such as periodicity. Li [27] has studied the performance of the Hierarchical Chaotic Image Encryption (HCIE) algorithm. HCIE is a two-stage permutation-based image encryption technique. The first stage permutes the image by dividing it into different blocks. While, the second stage permutes each image block pixel values. The author has remarked that the HCIS algorithm is less secure compared to the non-hierarchical structures permutation algorithms.

Zhao et al. [28] have introduced an image encryption chaos based system. The proposed system adapts the Arnold chaotic maps, where it is based on the permutation and substitution stages. For which, the pseudo random vectors for permutation are generated using Arnold map. Authors reported that the algorithm is secure in nature and has shown effective results. In [29] the authors introduced an image encryption model by generating a DNA masks using 2D logistic map; which was then used to produce the DNA matrix.

3. DNA Computing

DNA computing was proposed by L. Adleman [30] to solve the complex computational problem. In his research, Adleman discovered that DNA contains high computational and storage capability. He used DNA computation to solve the directed Hamiltonian route problem including seven vertices. For which, the molecules assumed as vertices and are encoded in a molecule sequence, then applied computations by chemical operations in lab.

DNA is a storing information particle in the living organisms. The DNA sequence contains four various bases; Adenine (A), Thymine (T), Cytosine (C) and Guanine (G) [31-33]. Therefore, in DNA computing; data is interpreted by four genetic character $P=\begin{pmatrix}A, T, C, G\end{pmatrix}$ as an alternative of the binary character $P=(0, 1)$. By using DNA four genetic bases to encode 11, 10, 01 and 00, there are 24 coding schematics. However, only eight coding schematics content the Watson–Crick complement rules, given in Table 1. [31]. Moreover, in DNA computing [34, 35], the subtraction and addition executions are performed based on the classical rules of binary subtraction and addition rules.

<table>
<thead>
<tr>
<th>Rule</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>00</td>
<td>00</td>
<td>01</td>
<td>01</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>T</td>
<td>11</td>
<td>11</td>
<td>10</td>
<td>10</td>
<td>01</td>
<td>01</td>
<td>00</td>
<td>00</td>
</tr>
<tr>
<td>G</td>
<td>01</td>
<td>10</td>
<td>00</td>
<td>11</td>
<td>00</td>
<td>11</td>
<td>01</td>
<td>10</td>
</tr>
<tr>
<td>C</td>
<td>10</td>
<td>01</td>
<td>11</td>
<td>00</td>
<td>11</td>
<td>00</td>
<td>10</td>
<td>01</td>
</tr>
</tbody>
</table>

4. Runge-Kutta-Fehlberg Method (RKF45)

Runge-Kutta method is emerged for the numerical solving of initial value problems (IVP) with oscillating solution. It can be adapted to developed accurate high order numerical solution by finding the value of the functions’ self without needing to calculate its derivatives [36]. To ensure the accuracy of the IVP solution, the problem need to be solved using two step sizes $h=2$ and $h$; then comparing the solutions at the mesh points correlating to the greater step size. However, this method necessitates a large computation for the smaller step size and have to be replicated if the agreement is insufficient. To try to resolve the above problem, the Runge-Kutta-Fehlberg (RKF45) method is developed [37]. RKF45 regulates if the proper step size ($h$) is being used. For which, two distinct approximations of the solution at each step are developed and compared. If the solutions are close in agreement to a certain accuracy, the approximation solution is accepted, otherwise the step size is minimized. While, the step size is maximized, if the two answers agree to a more significant than required. Each step requires the computation of the following:

$$k1 = hf(t_k, y)$$  \hspace{1cm} (1)
\[ k2 = hf(t_k + \frac{1}{4} h, y_k + \frac{1}{4} k1) \] (2)

\[ k3 = hf(t_k + \frac{3}{8} h, y_k + \frac{1}{32} k1 + \frac{9}{32} k2) \] (3)

\[ k4 = hf(t_k + \frac{12}{13} h, y_k + \frac{1932}{2197} k1 - \frac{7200}{2197} k2 + \frac{7296}{2197} k3) \] (4)

\[ k5 = hf(t_k + h, y_k + \frac{439}{216} k1 - 8 k2 + \frac{443}{110} k3 - \frac{443}{389} k2) \] (5)

\[ k6 = hf(t_k + \frac{1}{} h, y_k - \frac{8}{27} k1 + 2 k2 - \frac{3544}{216} k3 + \frac{1859}{4104} k4 - \frac{11}{40} k5) \] (6)

Then, the IVP approximation solution is determined using the following equation:

\[ y_{k+1} = y_k + \frac{16}{135} k1 + \frac{6656}{12825} k3 + \frac{28561}{56430} k4 - \frac{9}{50} k5 + \frac{2}{55} k6 \] (7)

5. The Proposed Medical Image Encryption Framework

The proposed framework hybrid the hyperchaotic RKF-45 random sequence and DNA computation for medical image encryption. The proposed schematic framework is shown in Fig.1.

A 4D hyperchaotic system is adapted to generate a random sequence. The obtained hyperchaotic sequence key is incorporated to the different Framework phases. The dynamics of the adapted 4D Hyperchaotic system has been presented by the following non-linear equations [38]:

\[
\begin{align*}
    x' &= a(x_2 - x_1) + \lambda_1 x_4 \\
    \dot{x_2} &= c x_1 - x_1 x_3 + \lambda_2 x_4 \\
    \dot{x_3} &= b x_3 + x_1 x_2 + \lambda_3 x_4 \\
    \dot{x_4} &= -dx_1
\end{align*}
\] (8)

The 4D Hyperchaotic system contains two non-linear product terms. For which, \(a, b, c, d, \lambda_1, \lambda_2\) and \(\lambda_3\) are the system’s control parameters. For parameter values, \(a = 35, b = 3, c = 35, d = 5, \lambda_1 = 1, \lambda_2 = 0.2, \lambda_3 = 0.3\), the 4D Hyperchaotic system’s Lyapunov exponent is \(0.5, 0.2117, 0, -38.7068\); for which the system presents a super hyperchaotic behavior.
5.1. Hyperchaotic RKF-45 Random Sequence Generation Algorithm

The Hyperchaotic RKF-45 random sequence key generation methodology is as follows:

**Step 1:** Iterate the 4D-hyperchaotic system equation 8 with \((h, x_n)\) for \(N\) iteration to eliminate the harmful transitional procedure effect and to increase the security.

**Step 2:** For \((n \times m)\) times iterate equation 8; using the RKF45 equation 7 to calculate the values of \(x1, x2, x3\) and \(x4\).

**Step 3:** Each \(x_i\) value generate two different key elements \(c_i^1\) and \(c_i^2\) where \(i = 1, 2, 3; 4\) using the following equation:

\[
c_i^1 = floor((|x_i| - floor(|x_i|)) \times 10^{15}) / 10^8 \mod 256
\]

\[
c_i^2 = floor((|x_i| - floor(|x_i|)) \times 10^{15}) \mod 10^8 \mod 256
\]

**Step 4:** Concatenate \(c_i^1\) and \(c_i^2\) elements to obtain the random sequence key \(k\).

5.2. Medical Image Encryption Algorithm

The proposed image encryption framework for medical images is given in detail as follows:

**Step 1:** Import the DICOM image \(P\) and get \((m \times n)\) the size of \(P\).

**Step 2:** Generate the Hyperchaotic RKF-45 random sequence \(k\), as discussed above.

**Step 3:** Perform image bit scrambling, where, the intensity values of \(P\) are represented by the binary values \(b\) that correspond to them. Then, divide image \(P\) intensity values into 8 bit planes.

**Step 4:** Fetch and sort different values of the hyperchaotic RKF45 sequence key \(k\) is, to obtain the index sequence \(k^f\). Then, based on the index sequence moves \(b\) to \(b^f_k\).

**Step 5:** Choose randomly a DNA encoding rule to encode \(b^f_k\) into a DNA sequence \(d1\).

**Step 6:** apply DNA addition operation to \(d1\) elements to obtain \(d2\).

**Step 7:** Fetch a sequence \(k_{mn}\) from \(k\), and transform it to a binary sequence \(k_b\).

**Step 8:** Encode \(k_b\) with the same encoding rule (from step 5) to obtain \(d3\).

**Step 9:** Based on the DNA XOR operation, perform: \(b2 = k_b\ XOR\ d3\). Then, convert the binary sequence \(b2\) to a cipher image \(Pc\).

6. Simulation and Security Analysis

With the aim of investigating the efficiency and capabilities of the proposed HC RK45-DNA encryption framework; Matlab R2015b was used for the implementation purposes. All of the study experiments were carried out on Intel(R), Core i7-4910 MQ CPU 2.90GHz and 16GB RAM.

The proposed encryption framework was applied on six medical images. The initial parameters of the proposed HC RK45-DNA frame work are set as following “\(x1(0), x2(0), x3(0), x4(0) = 0.12, 0.23, 0.34, 0.45\)”. All the medical images and their counterparts’ encrypted/decrypted images are depict in Fig.2. Fig 2 demonstrates that; no effective information can be obtained from the encrypted images and are in a random noise style.

6.1. Key sensitivity analysis

Key sensitivity is a crucial part to assess the strength of the encryption method. Whereby, a little change on the encryption/decryption key value leads to a completely different encrypted/decrypted image. To experiment the proposed HCRK45-DNA key sensitivity, the value of the initial parameter \(x_0\) is changed from 0.12 to 0.12e-10; from the original encryption key. The related decrypted images after the key alteration are shown in Fig.3. Fig 3 demonstrates that, applying decryption with a slightly differing key will not give the original image; which proves that the proposed HC RK45-DNA has adequate capability of resisting exhaustive attack.

6.2. Histogram Analysis

The histogram of the image illustrates the image pixel value distribution. The encrypted medical images histograms are shown in Fig.4. From Fig.4 it’s clear that; the encrypted medical images histograms are very stale and smooth. This uniform encrypted image distribution will not provide any meaningful information to an attacker; therefor, the proposed HC RK45-DNA resists statistical attacks effectively.
Fig. 2. Medical original and encrypted test images

Fig. 3. Decrypted medical images with correct key $x_0 = 0.12$ and incorrect key $x_0 = 0.12e - 10$
6.3. Correlation Coefficient Analysis

Correlation coefficient is a crucial metric to assess the correlation between the image adjacent pixels. To verify the proposed HC RK45-DNA encryption security, the correlation coefficient is analyzed in the vertical, horizontal and diagonal direction. The formula of the correlation coefficient is given by:

\[
\rho_{xy} = \frac{\text{cov}(x,y)}{\sqrt{\text{d}(x)} \sqrt{\text{d}(y)}}
\]  

(11)

\[
\text{d}(x) = \frac{1}{W \times H} \sum_{p=1}^{W} \sum_{q=1}^{H} [x(p,q) - E(x)]^2
\]  

(12)

\[
\text{cov}(x,y) = \frac{1}{W \times H} \sum_{p=1}^{W} \sum_{q=1}^{H} [x(p,q) - E(x)] [y(p,q) - E(y)]
\]  

(13)

\[
E(x) = \frac{1}{W \times H} \sum_{p=1}^{W} \sum_{q=1}^{H} x(p,q)
\]  

(14)

The correlation coefficient values of the original medical images and the encrypted images that correspond to them are reported in Table 2. From Table 2, the horizontal, vertical, and diagonal correlation coefficient values are close to "0" for encrypted images. Thereby indicates that; the randomness of the encrypted image pixels reached maximum. Fig. 5 shows the adjacent pixels correlation in original and encrypted image.

Table 2. The Correlation Coefficient of Original Test and Encrypted Images

<table>
<thead>
<tr>
<th>Medical Image</th>
<th>Vertical correlation $\rho_{xy}$</th>
<th>Horizontal correlation $\rho_{xy}$</th>
<th>Diagonal correlation $\rho_{xy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Original Image</td>
<td>Encrypted Image</td>
<td>Original Image</td>
</tr>
<tr>
<td>IMG01</td>
<td>0.9970</td>
<td>0.00021</td>
<td>0.99729</td>
</tr>
<tr>
<td>IMG02</td>
<td>0.9962</td>
<td>-0.00109</td>
<td>0.9977</td>
</tr>
<tr>
<td>IMG03</td>
<td>0.9989</td>
<td>0.00048</td>
<td>0.9985</td>
</tr>
<tr>
<td>IMG04</td>
<td>0.9983</td>
<td>0.00015</td>
<td>0.9973</td>
</tr>
</tbody>
</table>

Fig. 5 reveals that; the high correlation existence in the original image is scattered in its resulting encrypted images; which verifies the good encryption security the proposed HCRK45-DNA algorithm provides.
6.4. Information Entropy Analysis

The entropy of the image measures the amount of information redundancy in an event. Bigger information entropy indicates more randomness and higher security the encryption algorithm provides. Since each byte has 8 bits, the maximum information amount present in a byte of an image should be 8. The information entropy of an image is defined as:

\[ H(Q_i) = -\sum_{i=0}^{255} p(Q_i) \log_2 p(Q_i) \]

Where \( p(Q_i) \) is the occurring probability of symbol \( Q_i \). The information entropy values of the medical original and encrypted images are given in Table 3. As shown in Table 3, the encrypted images entropy values are near to the ideal value “8”. Hence, the encrypted image pixels are independent of each other statistically, which indicates the difficulty to decrypt the HC RK45-DNA encrypted images.

<table>
<thead>
<tr>
<th>Medical Image</th>
<th>Original Image Entropy</th>
<th>Encrypted Image Entropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMG01</td>
<td>6.94104236604948</td>
<td>7.99649256554477</td>
</tr>
<tr>
<td>IMG02</td>
<td>5.29059671063522</td>
<td>7.98910516850633</td>
</tr>
<tr>
<td>IMG03</td>
<td>6.39696019981591</td>
<td>7.99873639562868</td>
</tr>
<tr>
<td>IMG04</td>
<td>6.08646923173147</td>
<td>7.9989390680445</td>
</tr>
</tbody>
</table>

6.5. Differential Attack Analysis

To assess the ability of the proposed HC RK45-DNA system to resist differential attack; the unified average changing intensity (UACI) and the number of pixel change rate (NPCR) are measured. If a small variation in the original medical image can effectuate to a great variation in the encrypted image, then the algorithm can subsequently be able to effectively defend against differential attacks. The UACI and NPCR are calculated by the following equations:

\[ \text{NPCR} = \frac{1}{w \times h} \sum_{i=1}^{w} \sum_{j=1}^{h} D_{ij} \times 100 \]

\[ D_{ij} = \begin{cases} 0, & Q1(i,j) = Q2(i,j) \\ 1, & Q1(i,j) \neq Q2(i,j) \end{cases} \]

\[ \text{UACI} = \frac{1}{w \times h} \sum_{i=1}^{w} \sum_{j=1}^{h} D_{ij} \frac{(Q1(i,j) - Q2(i,j))^2}{255} \times 100 \]

Theoretically the optimal values of UACI and NPCR are 33.4635 and 99.6094, respectively. A pixel is selected randomly from the original medical image and is changed for 10 times; the average values of UACI and NPCR are calculated.

The calculated UACI and NPCR results are summarized in Table 4. As reported in Tables 4 the NPCR values are nearing to 99.6 which illustrate that, at most all the pixels between encrypted images \( Q_1 \) and \( Q_2 \) have been changed.
entirely. Furthermore, the reported UACI values are nearing the theoretical optimal value. Therefore, the results indicate that the proposed HC RK45-DNA satisfy the expected secure performance against differential attacks.

Tables 4. The UACI and NPCR Values of the Medical Test Images

<table>
<thead>
<tr>
<th>Medical Image</th>
<th>NPCR</th>
<th>UACI</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMG01</td>
<td>99.105679</td>
<td>32.762518</td>
</tr>
<tr>
<td>IMG02</td>
<td>99.565625</td>
<td>33.327747</td>
</tr>
<tr>
<td>IMG03</td>
<td>99.603908</td>
<td>33.014542</td>
</tr>
<tr>
<td>IMG04</td>
<td>99.603908</td>
<td>33.012166</td>
</tr>
</tbody>
</table>

7. Discussion

In this study, a 4D hyperchaotic framework based on RKF45 and DNA to be utilized in the e-healthcare systems is presented. The 4D hyperchaotic system has ten terms, two of which are nonlinear. The maximum Lyapunov exponent of the 4D hyperchaotic system is 0.5. The bigger the Lyapunov exponent is, the faster the trajectories separate the chaotic region is. Thus, the dynamic behavior of the system is better than one-dimensional chaotic system. For solving the 4D hyperchaotic system and generating a random sequence key, RKF-45 algorithm has been chosen. Which leads to enhance the uncertainty and randomness in the hyperchaotic system. Hence, to adequate capability of resisting exhaustive attack by increasing the cryptography key sensitivity. Moreover, the framework is conceptually based on the DNA biological operation with conjunction with XOR and computational operations, which increases the complexity of the framework. Therefore, as shown by the experimental studies the proposed 4D hyperchaotic RK45-DNA framework effectively satisfy the secure performance against resists statistical attacks and differential attacks.

8. Conclusion

In order to improve the security of e-health care systems, medical image encryption is the best method to conserve the patient’s information confidentiality. This study aims to propose a robust and efficient medical image encryption framework based on 4-D hyperchaotic RKF45-DNA. The proposed framework chaotic sequences are produced by the RKF45 method. Then, the sequence are integrated to 4-D hyperchaotic to generate the random key sequence. Moreover, DNA addition and subtraction operations were adopted to increase the proposed framework cipher unpredictability and efficiency. For the proposed encryption framework the DICOM image is used as the input image. The security analysis, including key sensitive, correlation, histogram and information entropy, verify that the proposed encryption framework has good complexity and security in medical image. The key space is sufficiently large to withstand the brute-force attack. Moreover, NPCR and UACI analysis are conducted, results show that the proposed HC-RK45-DNA framework is robust against differential attacks. In future work, the proposed framework will be improved by studying different 4-D Hyperchaotic system. In addition, an enhanced version of the proposed framework will be studied by proposing and examining higher Hyperchaotic system dimensions.

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