I.J. Wireless and Microwave Technologies, **2011, 5, 67-74**
Published Online October 2011 in MECS (http://www.mecs-press.net)
DOI: 10.5815/ijwmt.2011.05.10

Available online at http://www.mecs-press.net/ijwmt

Cross Layer Design and Performance Analysis of HARQ Schemes in Multi-Relay Networks

Cui-Qin Dai, Li Jiang, Qian-Bin Chen, Guang-Yu Wang

Chongqing Key Lab of Mobile Communication Technology, Chongqing University of Posts and Telecommunications, Chongqing China

Abstract

We propose a cross-layer design combining adaptive modulation and coding (AMC) at the physical layer with a hybrid automatic repeat request (HARQ) protocol at the data link layer in multi-relay networks with imperfect channel state information (CSI). Based on a simple and distributed relay selection strategy, we derive the closed-form expressions for performance metrics such as average packet loss rate (PER) and spectral efficiency; and further formulate an optimization model to maximize the spectral efficiency under the target PER constraint. The numerical results show that the average PER is lower and the spectral efficiency is higher with the number of the relay increasing.

Index Terms: Relay selection; hybrid automatic repeat request (HARQ); adaptive modulation and coding (AMC); imperfect channel state information (CSI)

1. Introduction

The cross-layer design proposed in [1], allows the physical layer to interact and share information with higher layers (e.g., the data link and the network layer) to achieve significant performance gains. Adaptive modulation and coding (AMC) at the physical layer and hybrid automatic repeat request (HARQ) at the data link layer are both extensively studied to match the transmission rates to time-varying conditions. The performances of type-I, type-II and type-III HARQ protocols in cross-layer design have been researched in [2]. As a result, type-II HARQ protocol has a compromise performance between type-I and type-III HARQ protocol. However, the mentioned cross-layer designs are only considered in the point-to-point wireless communication link. The authors proposed a simple and distributed relay selection strategy for multi-relay HARQ channels, and derived the outage probability of the system in the l -th HARQ round [3]. But it has been assumed that the channel state information (CSI) can be perfectly estimated at the receiver without any errors, which unfortunately is over-

* Corresponding author.
E-mail address: daicq@cqupt.edu.cn, lili-lijiang1985@163.com, chenqb@cqupt.edu.cn, Guangyu.Wang@infineon.com

optimistic in practice. In this paper, we fill the gap between [2]-[3] by jointly designing the type-II HARQ protocol and the AMC schemes in multi-relay networks with imperfect CSI.

2. System Model

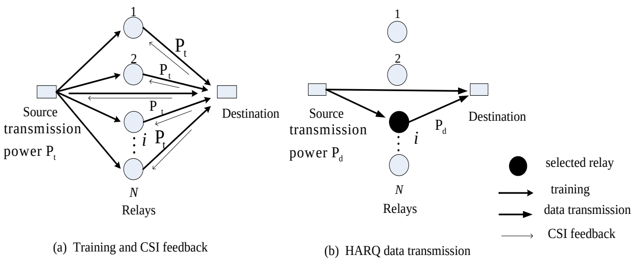

Figure 1. system model

The system model, described in Fig.1, consists of a source denoted as S, one or more relays denoted as R_i, $i=1,2,\cdots N$ and one destination denoted as D. Suppose each link is independent of the others. The channel coefficient h_{SD}, h_{SR_i} and h_{R_iD} of $S-D$, $S-R_i$ and R_i-D links are i.i.d. complex Gaussian random variables denoted as $h_{SD} \sim CN(0,\sigma_{SD}^2)$, $h_{SR_i} \sim CN(0,\sigma_{SR_i}^2)$ and $h_{R_iD} \sim CN(0,\sigma_{R_iD}^2)$. The power gains are denoted as $g_{SD}=|h_{SD}|^2$, $g_{SR_i}=|h_{SR_i}|^2$ and $g_{R_iD}=|h_{R_iD}|^2$ with mean σ_{SD}^2, $\sigma_{SR_i}^2$ and $\sigma_{R_iD}^2$.

Assume the source and the relay use the same transmit power, the transmit powers for the training sequence and the data packet are P_t and P_d respectively, and both the relay and the destination use the minimum mean square error estimation (MMSE) [4]. The signal received by node j is $y_{ij}=h_{ij}x_i+n_j$, where x_i is the signal transmitted by node i, n_j is the independent AWGN noise with variances N_0, where $i\in\{S,R\}$ and $j\in\{R,D\}$. The channel coefficient of imperfect CSI can be written as

$$h_{ij}=\hat{h}_{ij}+\tilde{h}_{ij} \tag{1}$$

where $\hat{h}_{ij}$ is the estimated channel coefficient, $\tilde{h}_{ij}$ is the estimated error. $\hat{h}_{ij}=\frac{\sigma_{ij}^2\sqrt{P_t}}{\sigma_{ij}^2P_t+N_0}y_{ij}$, $y_{ij}\sim CN(0,\sigma_{ij}^2P_t+N_0)$ and $\tilde{h}_{ij}\sim CN(0,\frac{\sigma_{ij}^2N_0}{\sigma_{ij}^2P_t+N_0})$.

Inspired by the relay selection scheme [3], which selects the relay with the highest power gain, we have the following selection procedure:

Step1 During the first symbol period, the source broadcasts a training sequence, with which the N available relays and the destination estimate channel coefficient $\hat{h}_{SR_i}$ and $\hat{h}_{SD}$ of $S-R_i$ and $S-D$ links, where $i=1,2,\cdots N$.

Step2 During the second symbol period, the i-th relay which has finished estimating $S-R_i$ link transmits a training sequence to the destination. The destination estimates the channel coefficient $\hat{h}_{R_iD}$ of R_i-D link.

Step3 The i -th relay has a timer T_i, proportionally to the inverse of $\min\{\hat{g}_{SR_i}, \hat{g}_{R_iD}\}$, where $\hat{g}_{SR_i}$ and $\hat{g}_{R_iD}$ denote the estimated power gain of $S-R_i$ and R_i-D links.

Step4 Whenever the first relay finished its timer, it broadcasts a flag packet toward the other relays to make them silent and announce itself as the selected relay.

After the best relay R_{sel} is selected, during the first HARQ round, the selected relay and the destination listen to the source transmit data packet. At the end of the transmission, the destination sends both the source and the selected relay a one-bit ACK or NACK, respectively. As long as NACK is received after each HARQ round and the maximum number of HARQ rounds is not reached, the source successively transmits the same packet. Suppose the selected relay decodes the message after HARQ round k, while the destination has not yet decoded the message correctly. For all HARQ rounds $k<l$, the selected relay helps the source transmit data packet.

3. Performance Analysis

A. *Average Packet Loss Rate* $PER(l)$

Assume the maximum HARQ round is L, let $PER(l)$ denote the average PER after l HARQ rounds, and k denote the earliest HARQ round after which the selected relay stops listening to the current message. For $k<l$, the selected relay listens for k HARQ rounds and helps the source by simultaneous transmission for the remaining ($l-k$) HARQ rounds; for $k \ge l$, the selected relay does not help the source during l HARQ rounds, $PER(l)$ can be calculated as

$$\begin{aligned} PER(l) &= \sum_{k=1}^{l-1} \overline{PER}_{SD,t=l}\, \overline{PER}_{R_{sel}D,t=l-k}\, (\overline{p}_{out,t=k-1})^{k-1}(1-\overline{p}_{out,t=k}) \\ &+ \sum_{k=l}^{L} \overline{PER}_{SD,t=l}\, (\overline{p}_{out,t=k-1})^{k-1}(1-\overline{p}_{out,t=k}) \end{aligned} \tag{2}$$

where $\overline{PER}_{ij,t}$ denotes the average PER for $i-j$ link after t HARQ rounds; $\overline{p}_{out,t}$ denotes the average outage probability for the selected relay after t HARQ rounds.

To compute $\overline{p}_{out,t}$, the mutual information between source and the selected relay for each HARQ round is given by

$$I = \frac{1}{2}\log_2(1+\rho \hat{g}_{SR_{sel}}) \tag{3}$$

where $\rho = \frac{P_d}{N_0}$, the outage probability for the selected relay for each transmission mode can be calculated as

$$P_{out,SR_{sel},n} = P\{I < R_n\} = P\left\{\hat{g}_{SR_{sel}} < \frac{2^{2R_n}-1}{\rho}\right\} \tag{4}$$

here $u_n = \frac{2^{2R_n}-1}{\rho}$, R_n denotes the coding rate for each transmission mode.

Refer to [3], the approximate PDF of $\hat{g}_{SR_{sel}}$ required in (4) can be calculated as

$$p_{\hat{g}_{SR_{sel}}}(\hat{g}) = p_{\hat{g}_{R_{sel}D}}(\hat{g}) \approx -\frac{1}{2}\frac{1}{\sqrt{1-\prod_{i=1}^{N}\left(1-\exp(-\hat{g}\left(\frac{1}{\hat{g}_{SR_i,av}}+\frac{1}{\hat{g}_{R_iD,av}}\right))\right)}}$$
$$\times \sum_{i=1}^{N}\exp(-\hat{g}\left(\frac{1}{\hat{g}_{SR_i,av}}+\frac{1}{\hat{g}_{R_iD,av}}\right))\times(-\left(\frac{1}{\hat{g}_{SR_i,av}}+\frac{1}{\hat{g}_{R_iD,av}}\right))$$
$$\prod_{\substack{j=1\\ j\neq i}}^{N}\left(1-\exp(-\hat{g}\left(\frac{1}{\hat{g}_{SR_j,av}}+\frac{1}{\hat{g}_{R_jD,av}}\right))\right) \quad (5)$$

By substituting (5) into (4), $P_{out,SR_{sel},n}$ is obtained as

$$P_{out,SR_{sel},n} = 1-\sqrt{1-\prod_{i=1}^{N}\left(1-\exp(-u_n\left(\frac{1}{\hat{g}_{SR_i,av}}+\frac{1}{\hat{g}_{R_iD,av}}\right))\right)} \quad (6)$$

Then, $\overline{p}_{out,t}$ in (2) is achieved as

$$\overline{p}_{out,t} = \sum_{n=1}^{5} P_{\gamma}(n) P_{out,SR_{sel},n} \quad (7)$$

where $P_{\gamma}(n)$ is the probability for choosing mode n.

Next, the average PER for $i-j$ link after t HARQ rounds $\overline{PER_{ij,t}}$ will be calculated.
The instantaneous PER expression of imperfect CSI at the t-th HARQ round can be written as

$$PER_{t,n}(\hat{\gamma}) = \begin{cases} 1, & 0<\hat{\gamma}<\gamma_{pn} \\ a_{t,n}\exp(\frac{-b_{t,n}P_d\,\hat{\gamma}}{\sigma^2_{\tilde{h}_{ij}}p_d+N_0}) & ,\hat{\gamma}>\gamma_{pn} \end{cases} \quad (8)$$

where n is the mode index; $\hat{\gamma}$ is the received instantaneous SNR; $a_{t,n}$, $b_{t,n}$ and γ_{pn} in table I are used to fit the parameter of PER curve. Suppose the instantaneous PER is guaranteed to be no bigger than p_0 for each chosen AMC mode at the physical layer, then the PER at the data link layer is no bigger than P_{loss}, we have $p_0{}^L \le P_{loss}$, and thus, $p_0 \le P_{loss}^{1/L} = p_{t\arg et}$.

Inverting the PER expression in (8), the intervals of AMC modes can be calculated as

$$\hat{\gamma}_n = \frac{1}{d_{t,n}}\ln(\frac{\overline{a}_{t,n}}{P_{t\arg et}}), n=1,2,\ldots N$$
$$\hat{\gamma}_{N+1} = +\infty \quad (9)$$

where $d_{t,n} = \dfrac{\bar{b}_{t,n} P_d}{\sigma^2_{\tilde{h}_{ij}} P_d + N_0}$, $\bar{a}_{t,n}(t) = \sqrt[t]{a_{1,n} \ldots a_{t,n}}$ and $\bar{b}_{t,n}(t) = \dfrac{b_{1,n} + \ldots + b_{t,n}}{t}$.

TABLE I. AMC TRANSMISSION MODE FOR TYPE-II HARQ

II-HARQ	Mode1	Mode2	Mode3	Mode4	Mode5
Modulation	BPSK	QPSK	QPSK	16-QAM	16-QAM
R_n (bits/sym.)	0.50	1.00	1.50	3.00	3.00
$a_{1,n}$	1525.9	424.06	27.429	126.88	133.27
$b_{1,n}$	6.0354	2.6532	0.8483	0.4446	0.243
$a_{2,n}$	0.2608	163.34	1940	99.5411	359.118
$b_{2,n}$	3.5497	1.5291	2.4865	0.4986	0.4812
$a_{3,n}$	13.134	401.49	149.355	178.966	224.796
$b_{3,n}$	7.8735	3.1401	2.4442	0.5944	0.5547
$a_{4,n}$	1.0888	1807.2	46.2931	207.57	1064
$b_{4,n}$	8.7707	3.9224	2.3718	0.6092	0.7082

Refer to [5], the PDF of the received SNR $\hat{\gamma}$ of $S-D$ link is given by

$$p_{\hat{\gamma}}(\hat{\gamma}) = \frac{1}{\hat{\gamma}_{SD,av}} \exp(-\frac{\hat{\gamma}}{\hat{\gamma}_{SD,av}}) \quad (10)$$

Hence, the probability of choosing mode n for $S-D$ link is calculated as

$$P_{\hat{\gamma}}(n) = \int_{\hat{\gamma}_n}^{\hat{\gamma}_{n+1}} p_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} = \exp(-\frac{\hat{\gamma}_n}{\hat{\gamma}_{SD,av}}) - \exp(-\frac{\hat{\gamma}_{n+1}}{\hat{\gamma}_{SD,av}}) \quad (11)$$

Using (5), the probability of choosing mode n for $R_{sel}-D$ link is calculated as

$$\begin{aligned} P_{\hat{\gamma}}(n) &= \int_{\hat{\gamma}_n}^{\hat{\gamma}_{n+1}} p_{\hat{\gamma}_{R_{sel}D}}(\hat{\gamma}) d\hat{\gamma} \\ &= \int_{\hat{\gamma}_n}^{\hat{\gamma}_{n+1}} -\frac{1}{2} \frac{1}{\sqrt{1-\prod_{i=1}^{N}\left(1-\exp(-\hat{\gamma}\left(\frac{1}{\hat{\gamma}_{SR_i,av}}+\frac{1}{\hat{\gamma}_{R_iD,av}}\right))\right)}} \times \\ & \sum_{i=1}^{N} \exp(-\hat{\gamma}\left(\frac{1}{\hat{\gamma}_{SR_i,av}}+\frac{1}{\hat{\gamma}_{R_iD,av}}\right)) \times (-\left(\frac{1}{\hat{\gamma}_{SR_i,av}}+\frac{1}{\hat{\gamma}_{R_iD,av}}\right)) \\ & \prod_{\substack{j=1 \\ j\neq i}}^{N}\left(1-\exp(-\hat{\gamma}\left(\frac{1}{\hat{\gamma}_{SR_j,av}}+\frac{1}{\hat{\gamma}_{R_jD,av}}\right))\right) d\hat{\gamma} \end{aligned} \quad (12)$$

where $\hat{\gamma}_{SR_i,av}$ and $\hat{\gamma}_{R_iD,av}$ are the average SNR.

According to [2], the average PER of $i-j$ link, at each transmission mode, is given by

$$\overline{PER}_n = \frac{1}{P_{\hat{\gamma}}(n)} \int_{\hat{\gamma}_n}^{\hat{\gamma}_{n+1}} p_{\hat{\gamma}}(\hat{\gamma}) PER_{t,n}(\hat{\gamma}) d\hat{\gamma} \tag{13}$$

Using (11), (12) and (13), $\overline{PER}_{ij,t}$ in (2) is calculated as

$$\overline{PER}_{ij,t} = \sum_{n=1}^{N} P_{\hat{\gamma}}(n) \overline{PER}_n \tag{14}$$

B. Spectral Efficiency and Optimization Model

The average number of the transmission is

$$\bar{N}(l, \bar{P}(\hat{\gamma})) = 1 + \bar{P} + \bar{P}^2 + \cdots + \bar{P}^{(l-1)} = \frac{1-\bar{P}^l}{1-\bar{P}} \tag{15}$$

where $\bar{P}(\hat{\gamma})$ is equal to $PER(l)$.

The spectral efficiency $\bar{S}_e(\hat{\gamma})$ after l HARQ rounds can be calculated as

$$\begin{aligned} \bar{S}_e(\hat{\gamma}) &= \frac{1}{\bar{N}(l, \ \bar{P}(\hat{\gamma}))} \sum_{n=1}^{N} R_n P_{\hat{\gamma}}(n) \\ &= \frac{1-\bar{P}(\hat{\gamma})}{1-\bar{P}(\hat{\gamma})^l} \sum_{n=1}^{N} R_n P_{\hat{\gamma}}(n) \end{aligned} \tag{16}$$

Hence, we formulate the optimization model under the target PER constraint as follows

$$\begin{aligned} &\max_{\hat{\gamma}} \quad \bar{S}_e(\hat{\gamma}) \\ &s.t. \quad \bar{P}(\hat{\gamma}) \le P_{target} \end{aligned} \tag{17}$$

Iteration algorithm can be used to solve (17).

4. Simulation Results

In this section, the performance of the proposed cross-layer is verified by numerical method. According to [2], we set $P_{loss} = 0.01$, $L = 4$ and $l = 2$.

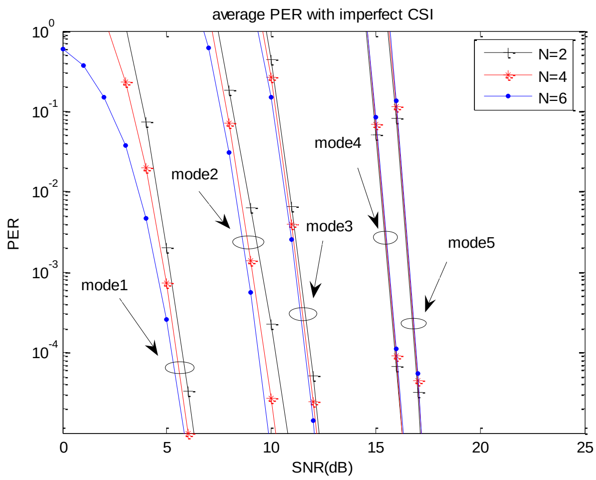

Figure 2. Average PER of type-II HARQ with imperfect CSI

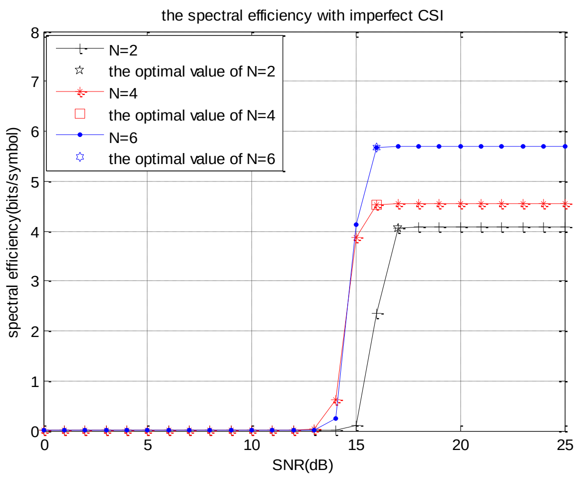

Figure 3. Spectral Efficiency of type-II HARQ with imperfect CSI

Fig.2 shows the average PER of the multi-relay system with imperfect CSI. We can find out that with the number of the relay increasing, the average PER of the system becomes smaller. As the transmission mode increases, the difference between the average PER becomes small, in mode 4 and mode 5, the average PER of $N=(2,4,6)$ becomes the same line. One reason is that as the number of the relay increases, the PDF of the power gain for the $S-R_{sel}$ and the $R_{sel}-D$ link in (5) decreases, which leads the outage probability $\overline{p}_{out,t}$ in (7) and the average PER in (14) to become small; the other reason is that as the transmission mode increases, the coding rate R_n increases, therefore the outage probability $\overline{p}_{out,t}$ in (7) increases.

Fig.3 shows the spectral efficiency of the system with imperfect CSI. We can see that when $N=6$, the spectral efficiency is the highest; when $N=2$, the spectral efficiency is the lowest. From Fig.2, we know that when $N=2,4,6$, the average PER become smaller at the same SNR. So when the HARQ round l and

$P_{target} = P_{loss}^{(1/l)}$ are fixed, the average PER of $N = (2,4,6)$ required to satisfy P_{target} becomes orderly smaller. So at the same SNR, when $N = 6$, the system can use higher modulation. As we know, the higher modulation has higher spectral efficiency. So when $N = 6$, the system has the highest spectral efficiency. We also see that as the SNR increases, the spectral efficiency cannot increase unlimitedly. Under the constraint of the system QoS, when the spectral efficiency increases to an optimal value, it will be a constant.

5. Conclusion

In this paper, we proposed a cross-layer design in multi-relay networks with imperfect CSI. Using the derived expressions of the metrics, an optimization problem is formulated to maximize the system spectral efficiency under the target PER constraint. Numerical results show that the average PER is lower and the spectral efficiency is higher with the number of the relay increasing.

Acknowledgment

This work was supported by the National Key Technology R&D Program (2008BAH30B10), the Natural Science Foundation of China (60972070), the Natural Science Foundation of Chongqing, China (CSTC2009BA2090) and the Foundation of Chongqing Educational Committee, China (KJ100514).

References

[1] Srivastava, V. Motani, M., "Cross-layer design:A survey and the road ahead", Communication Magazine, IEEE, vol.43, 2005, pp.112-119.

[2] Changchun Li, LinglingXu, Dongfeng Yuan and Feijin Shi, "Performance analysis and comparison of HARQ schemes in cross layer design", ChinaCOM, Xian, 2009, pp. 1-5

[3] Mahan, B., Hjorungnes, A. and Debbah, M., "Outage Probability Analysis of Multi-Relay Delay-Limited Hybrid-ARQ Channels", Vehicular Technology Conf. (VTC 2010-Fall), 2010, pp.1-5

[4] Shaolei Ren, Letaief,K.B. and de Marca, J.R.B., "Outage reduction in cooperative networks with limited feedback", IEEE Trans., vol.58, no.3, March 2010, pp.748-752

[5] John G. Proakis and Masoud Salehi. , "Digital Communications" New York, McGrawHill, 1989.