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Interval-Valued Fuzzy Soft Subhemiring of Hemiring and it's Application

V. Lambodharan
Assistant Professor in Mathematics, Department of Mathematics, Er. Permal Manakalei College of Engineering
E-mail: mathslambo@gmail.com
ORCID iD: https://orcid.org/0000-0002-3550-1871

N. Anitha
Assistant Professor, Department of Mathematics, Periyar University Post graduate and Research Studies, Dharmapuri-636701
E-mail: anithapupgec@periyaruniversity.ac.in
ORCID iD: https://orcid.org/0000-0003-0381-2437

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Abstract: In this paper, we initiated the idea of interval-valued fuzzy soft set (IVFSS) and a few results, Operations, definitions, and properties also a few properties and characteristics of IVFSSs and Prove some of theorem and we discuss a few examples uses of soft set in finding a selection taking problem. Also initiated the comparable measure of two IVFSSs and discuss with the Presentation of medical application problem.

Index Terms: Fuzzy set; soft set (FS); fuzzy soft set (FSS), interval-valued fuzzy soft set (IVFSS); soft sub hemiring of a hemiring.

1. Introduction

Nowadays [1] Molodtsov introduced a new concept using various mathematical tools finding with suspense which old ideas cannot switch. He applied many more problems of this area in evaluating many more problems in, business, Statistics, economic, etc. Right now the soft set idea is developing speedily. A further study [2, 5, 6] presented the soft set and apply to find a few selection taking problem and [3] open the notion of a FSS, an additional common idea of aFS and a FSS of the result. Zou [4] began the idea of the soft set lacking situation, [7] offered the idea of multiset, [8] he also stated the ideas of fuzzy dynamic IVFSS. [10] Present the idea of probability FSS and [9] presented the notion of expert set. [11] Presented the concept of collective soft set. [12] accessible the idea of IVFSS by combining the IVFSS [13, 14] soft set model. A second prior, the soft set thought has been broadly focused on a fundamental level and sharpness next Molotov's work. [9] at first introduced the thoughts of soft subset, void soft set and so on. Theyadditionally gave a couple of procedure on soft set and approved De Morgan's laws. Ali [15] corrected a couple of Mix-ups of previous investigations and characterized some new procedure on soft sets. From that point onward, [15] Comparatively concentrated on some basic homes identified with the pristine tasks and explored some arithmetical Frameworks of soft sets. Sezgin [16] delayed the notional piece of cycles on soft set. Soft mappings, soft uniformity and soft set connection mappings have been given in [17-27]. In this work, we give the idea of on IVFSS, a few hypotheses, and clinical analysis and mathematical model. Additionally, we build up some properties and uses of IVFSS.

2. Preliminary

In this part, we able to remember some basic idea of fuzzy set, fuzzy soft set on interval-valued fuzzy soft set required in this part.

Definition 2.1[15] An interval- valued fuzzy set $\overline{A}$ over U is a mapping such that

$$\overline{A}: U \to D[0,1], \tag{2.1}$$

Where $D([0,1])$ closed subintervals of [0,1], the set of all interval- valued fuzzy sets $(IVFS)$ on U is denoted by$\overline{P}(U)$.Suppose that $\overline{A} \in \overline{P}(U)$,$\forall\, a \in U, \chi_a(a) = [\chi_{a^-}(a), \chi_{a^+}(a)]$ is said to be the value of membership of a member a to A. $\chi_{a^-}(a)$ and $\chi_{a^+}(a)$ are called for the infimum and supremum degrees of membership of a to A. Where $0 \leq \chi_{a^-}(a) \leq \chi_{a^+}(a) \leq 1$.

Definition 2.2[14] The subset, union, intersection, and complement of the interval valued fuzzy sets are stated as follows. Let $\overline{A}, \overline{B} \in \overline{P}$(U), hence

(a) The complement of $\overline{A}$ is denoted by $\overline{A}^{\,c}$ where

$$\chi_{\overline{A}^{\,c}}(a) = 1 - \chi_{\overline{A}}(a) = [1 - \chi_{a^+}(a), 1 - \chi_{a^-}(a)],$$

(b) The $'' \sqcap ''$ of $\overline{A}$ and $\overline{B}$ is represented by $\overline{A} \sqcap \overline{B}$ then

$$\chi_{\overline{A}\sqcap\overline{B}}(a) = \inf\left[\chi_{\overline{A}}(a), \chi_{\overline{B}}(a)\right]$$
$$= [\inf\left(\chi_{\overline{A}^-}(a), \chi_{\overline{B}^-}(a)\right), \inf\left(\chi_{\overline{A}^+}(a), \chi_{\overline{B}^+}(a)\right)]$$

(c) The $'' \sqcup ''$ of $\overline{A}$ and $\overline{B}$ is represented by $\overline{A} \sqcap \overline{B}$ then

$$\chi_{\overline{A}\sqcup\overline{B}}(a) = \sup\left[\chi_{\overline{A}}(a), \chi_{\overline{B}}(a)\right]$$
$$= [\sup\left(\chi_{\overline{A}^-}(a), \chi_{\overline{B}^-}(a)\right), \sup\left(\chi_{\overline{A}^+}(a), \chi_{\overline{B}^+}(a)\right)]$$

(d) A is a subset of B denoted by $A \sqsubseteq B$ if $\chi_{\overline{A}^-}(a) \leq \chi_{\overline{B}^-}(a), \chi_{\overline{A}^+}(a) \leq \chi_{\overline{B}^+}(a)$

Definition 2.3 ([14]) Let two interval valued fuzzy soft sets X and Y the compability measure $\overline{\rho}(X, Y)$ is stated as follows

$$\overline{\rho}(X, Y) = [\rho^-(X, Y), \rho^+(X, Y)] \tag{2.2}$$

Such that

$$\rho^-(X, Y) = \inf[\rho_1(X, Y), \rho_2(X, Y)$$
$$\rho^+(X, Y) = \sup[\rho_1(X, Y), \rho_2(X, Y)$$

Where

$$\rho_1(X, Y) = \sup_{a\in A}\{\inf[\chi_{X^-}(a), \chi_{Y^-}(a)]\}/\sup_{a\in A}[\chi_{X^-}(a)]$$
$$\rho_1(X, Y) = \sup_{a\in A}\{\inf[\chi_{X^+}(a), \chi_{Y^+}(a)]\}/\sup_{a\in A}[\chi_{X^+}(a)]$$

Let U be the universal set and $\mathbb{P}$ be the parameters set. Let $P(U)$ be the power set of U , $X \sqsubseteq U$.

Definition 2.4 ([14]) A pair $(F, \mathbb{P})$ is said to be soft set over, where F is the mapping $F: \mathbb{P} \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universal set U.

Definition 2.5 Let U is a universal set and let $\mathbb{P}$ is parameters set. Let I_U be the subsets of U. Let $X \sqsubseteq \mathbb{P}$. A pair $(F, \mathbb{P})$ is said to be a soft fuzzy set over U where F is a function given by

$$F: X \to I_U.$$

Definition 2.6 ([12]) Let $U = \{a_1, a_2, \dots, a_{k-1}, a_k\}$be the universal set of elements and
$\mathbb{P} = \{p_1, p_2, \dots, p_{k-1}, p_k\}$ Be the universal set of parameters. The set $(U, \mathbb{P})$ be a soft universe.
Let $F: \mathbb{P} \to I_U$, where $I_U$the collection of all fuzzy subsets of U,and let χ is a fuzzy subset of $\mathbb{P}$.
Let $F_\chi: \mathbb{P} \to I_U \times I$ be a function stated as follows:

$$F_\chi(p) = \left(F(p), \chi(p)\right)$$

Then, F_χ is said to be interval valued fuzzy soft set over $(U, \mathbb{P})$.where, for every parameter p_k,
$F_\chi(p_k) = \left(F(p_k), \chi(p_k)\right)$ shows not only the degree of the belongingness of the element of U in $F(p_k)$.
Therefore $F_\chi(p_k)$ write as follows:

$$F_{\chi}(p_k) = \left(\{(a_1, F(p_k)(a_1)), (a_2, F(p_k)(a_2), \dots, (a_k, F(p_k)(a_k)\}, \chi(p_k)\right)$$

Where $F(p_k)(a_1), F(p_k)(a_2), \dots, F(p_k)(a_k)$ are the values of members and $\chi(p_k)$ is the value of such members.

Definition 2.7 ([12]) Let U be the universal and $\mathbb{P}$ a parameters set. $\overline{P}(U)$ Represents the set of all Interval- valued fuzzy sets of U.Let$X \sqsubseteq \mathbb{P}$. A pair $(\overline{F}, X)$ be an Interval- valued fuzzy soft set over U, here $\overline{F}$ is a function defined as $\overline{F}: X \to \overline{P}(U)$

3. On Interval- valued fuzzy soft set (IVFSS)

This section we give the idea of interval- valued fuzzy soft set. In our interval- valued fuzzy soft set, a value is involved with the parameter fuzzy sets despite the fact describing an nterval- valued fuzzy soft set.

Definition 3.1. Let $U = \{a_1, a_2, \dots, a_k\}$ be the common universal set of objects and $\mathbb{P} = \{p_1, p_2, \dots, p_k\}$ be the common universal set of parameters. The pair $(U, \mathbb{P})$ is said to be a soft universe. Let $\overline{F}: \mathbb{P} \to \overline{P}(U)$ and χ be a fuzzy set of $\mathbb{P}$, that is $\chi: \mathbb{P} \to [0,1]$, where $\overline{P}(U)$ be a set of all interval- valued fuzzy subsets on U. let $\overline{F}_{\chi}: \mathbb{P} \to \overline{P}(U) \times I$ be a mapping stated as follows:

$$\overline{F}_{\chi}(p) = \left(\overline{F}(p), \chi(p)\right).$$

Then, $\overline{F}_{\chi}$ is said to be an Interval- valued fuzzy soft set over $(U, \mathbb{P})$. For every parameters p_k ,

$\overline{F}_{\chi}(p_k) = (\overline{F}(p_k)(a), \chi(p_k)$ But also the degree of option of such objects which symbolized by $\chi(p_k)$. so we can write$\overline{F}_{\chi}(p_k)$. Therefore $\overline{F}_{\chi}(p_k)$ as follows:

$$\overline{F}_{\chi}(p_k) = (\left\{\left(a_1, \overline{F}(p_k)(a_1)\right), \left(a_2, \overline{F}(p_k)(a_2)\right), \dots, \left(a_k, \overline{F}(p_k)(a_k)\right)\right\}, \chi(p_k)\Big)$$

Example.1. Let $U = \{a_1, a_2, a_3\}$ is a universal set, and parameters set $\mathbb{P} = \{p_1, p_2, p_3\}$. And let $\chi: \mathbb{P} \to I$.Define the mapping $\overline{F}_{\chi}: \mathbb{P} \to \overline{P}(U) \times I$ as follows:

$$\overline{F}_{\chi}(p_1) = (\{(a_1, [0.4,0.7]), (a_2, [0.8,0,9]), (a_3, [0.6,0.9])\}, 0.7),$$
$$\overline{F}_{\chi}(p_2) = (\{(a_1, [0.2,0.5]), (a_2, [0.0,0,4]), (a_3, [0.2,0.6])\}, 0.6),$$
$$\overline{F}_{\chi}(p_3) = (\{(a_1, [0.7,0.8]), (a_2, [0.2,0,4]), (a_3, [0.0,0.5])\}, 0.2),$$

Then, $\overline{F}_{\chi}$is a interval- valued fuzzy soft set over $(U, \mathbb{P})$Also we can represent the matrix form

$$\overline{F}_{\chi} = \begin{bmatrix} [0.4,0.7] & [0.8,0.9] & [0.6,0.9] & 0.7 \\ [0.2,0.5] & [0.0,0.4] & [0.2,0.6] & 0.6 \\ [0.7,0.8] & [0.2,0.4] & [0.0,0.5] & 0.4 \end{bmatrix}.$$

Definition 3.2. Let $\overline{F}_{\chi}$ and $\overline{F}_{\phi}$be two interval- valued fuzzy soft set over$(U, \mathbb{P})$. $\overline{F}_{\chi}$ is said to be a interval- valued fuzzy soft subset of $\overline{\eta}_{\phi}$, and we write $\overline{F}_{\chi} \sqsubseteq \overline{\eta}_{\phi}$ if

(i) $\chi(p)$ is a fuzzy subset of $\phi(p)$, for every $\in \mathbb{P}$,
(ii) $\overline{F}(p)$ is an Interval- valued fuzzy subset of $\overline{\eta}(p)$, for every$\in \mathbb{P}$.

Example.2. Let $U = \{a_1, a_2, a_3\}$ is a universal set, and parameters set

$\mathbb{P} = \{p_1, p_2, p_3\}$.where $p_1 = low$, $p_2 = average$, $p_3 = green$.Let $\overline{F}_{\chi}$ be an Interval- valued fuzzy soft set over $(U, \mathbb{P})$ stated as follows:

$$\overline{F}_{\chi}(p_1) = (\{(a_1, [0.2,0.4]), (a_2, [0.6,0,8]), (a_3, [0.4,0.6])\}, 0.5),$$
$$\overline{F}_{\chi}(p_2) = (\{(a_1, [0.0,0.4]), (a_2, [0.0,0,3]), (a_3, [0.2,0.4])\}, 0.5),$$
$$\overline{F}_{\chi}(p_3) = (\{(a_1, [0.6,0.7]), (a_2, [0.2,0,2]), (a_3, [0.2,0.4])\}, 0.3),$$

Then, $\overline{F}_{\chi}$is an interval- valued fuzzy soft set over $(U, \mathbb{P})$.Let $\overline{\eta}_{\phi}$ another interval- valued fuzzy soft set over $(U, \mathbb{P})$ stated as follows

$$\overline{\eta}_{\phi}(p_1) = (\{(a_1, [0.4,0.7]), (a_2, [0.8,0,9]), (a_3, [0.6,0.8])\}, 0.7),$$
$$\overline{\eta}_{\phi}(p_2) = (\{(a_1, [0.3,0.5]), (a_2, [0.3,0,4]), (a_3, [0.4,0.6])\}, 0.6),$$
$$\overline{\eta}_{\phi}(p_3) = (\{(a_1, [0.8,0.9]), (a_2, [0.3,0,5]), (a_3, [0.3,0.6])\}, 0.4),$$

Note that $\overline{F}_{\chi}$ is an interval- valued fuzzy soft set subset of $\overline{\eta}_{\phi}$.

Definition 3.3. Let two interval- valued fuzzy soft sets $\overline{F}_{\chi}$ and $\overline{\eta}_{\phi}$over $(U, \mathbb{P})$ are called equal, and that can be written by $\overline{F}_{\chi} = \overline{\eta}_{\phi}$ if $\overline{F}_{\chi}$ is an interval- valued fuzzy soft subset of $\overline{\eta}_{\phi}$and $\overline{\eta}_{\phi}$is an interval- valued fuzzy soft subset of $\overline{F}_{\chi}$. In other words, $\overline{F}_{\chi} = \overline{\eta}_{\phi}$ is equal the below conditions are satisfied:

(i) $\chi(p) = \phi(p), \forall p \in \mathbb{P}$,
(ii) $\overline{F}(p) = \overline{\eta}(p), \forall p \in \mathbb{P}$.

Definition 3.4. Two Interval- valued fuzzy soft sets is said to be empty interval- valued fuzzy Soft set, it is symbolized by $\overline{\emptyset}_{\chi}$ if $\overline{\emptyset}_{\chi}: \mathbb{P} \to \overline{P}(U) \times I$ such that

$$\overline{\emptyset}_{\chi}(p) = \left(\overline{F}(p)(a), \chi(p)\right)$$

Where $\overline{F}(p) = [0.0,0.0] = [0.0]$ and $\chi(p) = 0 \; \forall \; p \in \mathbb{P}$.

Definition 3.5 An interval- valued fuzzy soft set is said to be complete interval- valued fuzzy soft set it is symbolized by $\overline{\Omega}_{\chi}$ if $\overline{\Omega}_{\chi}: \mathbb{P} \to \overline{P}(U) \times I$ such that

$$\overline{\Omega}_{\chi} = \left(\overline{F}(p)(a), \chi(p)\right),$$

Where $\overline{F}(p) = [1,1] = [1]$,and $\chi(p) = 1, \forall \; p \in \mathbb{P}$.

4. On Interval Valued Fuzzy Soft Set Operation

In this part we initiateinterval valued fuzzy soft sets some basic operation that is to say complement, $'' \sqcup ''$ and $'' \sqcap ''$ a few result of this operation.

Definition 4.1 Let $\overline{F}_{\chi}$ is an interval valued fuzzy soft sets over $(U, \mathbb{P})$, hence the complement operation of $\overline{F}_{\chi}$, represented by ${\overline{F}_{\chi}}^{c}$ and is stated by ${\overline{F}_{\chi}}^{c} = \overline{\eta}_{\phi}$, therefore $\phi(p) = c(\chi(p))$ and $\overline{\eta}(p) = \overline{c}\left(\overline{F}(p)\right), \forall p \in \mathbb{P}$, and $\overline{c}$ is a interval valued fuzzy soft sets complement.

Example.3.Think through interval valued fuzzy soft sets $\overline{F}_{\chi}$ over $(U, \mathbb{P})$ in the example.1

$$\overline{F}_{\chi} = \begin{bmatrix} [0.4,0.7] & [0.8,0.9] & [0.6,0.9] & 0.7 \\ [0.2,0.5] & [0.0,0.4] & [0.2,0.6] & 0.6 \\ [0.8,0.9] & [0.2,0.3] & [0.0,0.5] & 0.4 \end{bmatrix}$$

By using the direct fuzzy complement for $\chi(p)$ and Interval- valued complement for$\overline{F}(p)$, we have ${\overline{F}_{\chi}}^{c} = \overline{\eta}_{\phi}$ where

$$\overline{\eta}_{\phi} = \begin{bmatrix} [0.5,0.8] & [0.3,0.4] & [0.3,0.6] & 0.3 \\ [0.7,0.9] & [0.8,1.0] & [0.6,0.9] & 0.4 \\ [0.3,0.4] & [0.9,0.8] & [0.6,1.0] & 0.6 \end{bmatrix}$$

Theorem 4.2. Let $\overline{F}_{\chi}$ be an interval valued fuzzy soft sets over$(U, \mathbb{P})$.Then, the following are true

$$({\overline{F}_{\chi}}^{c})^{c} = \overline{F}_{\chi}$$

Proof: Since ${\overline{F}_{\chi}}^{c} = \overline{\eta}_{\phi}$, then $({\overline{F}_{\chi}}^{c})^{c} = \overline{\eta}^{c}{}_{\phi}$

By definition 4.1,$\overline{\eta}_{\phi} = \left(\overline{c}\left(\left(\overline{F}(p)\right)\right), c(\chi(p))\right)$

$$\overline{\eta}_{\phi}{}^{c} = \left(\overline{c}\left(\overline{c}\left(\overline{F}(p)\right)\right), c\left(\chi(p)\right)\right)$$
$$= \left(\overline{F}(p), \chi(p)\right)$$
$$= \overline{F}_{\chi} //$$

Definition 4.3. Let tow interval valued fuzzy soft sets $(\overline{F}_{\chi}, M)$ and $(\overline{\eta}_{\phi}, N)$ the union of interval- valued fuzzy soft set $\overline{F}_{\chi} \sqcup \overline{\eta}_{\phi}$, is a interval- valued fuzzy soft set $(\overline{\Upsilon}_{\tau}, L)$ where $L = M \sqcup N$ and $\overline{\Upsilon}_{\tau}: \mathbb{P} - \overline{P}(U) \times I$ is stated by $\overline{\Upsilon}_{\tau}(p) = \left(\overline{\Upsilon}(p), \tau(p)\right)$

Such that $\overline{\Upsilon}(p) = \overline{F}(p) \sqcup \overline{\eta}(p)$ and $\tau(p) = \kappa\left(\chi(p), \phi(p)\right)$, where τ is an $s-$ norm and $\overline{\Upsilon}(p) = [\sup\left(\chi_{\overline{F}(p)}{}^{-}, \chi_{\overline{\eta}(p)}{}^{-}\right) \ \sup\left(\chi_{\overline{F}(p)}{}^{+}, \chi_{\overline{\eta}(p)}{}^{+}\right)]$.

Example.4. Consider interval- valued fuzzy soft set and $\overline{\eta}_{\phi}$by example 2.By using the Interval- valued fuzzy union and basic fuzzy union is represented by $\overline{F}_{\chi} \sqcup \overline{\eta}_{\phi} = \overline{\Upsilon}_{\tau}$where

$$\overline{\Upsilon}_{\tau}(p_1) = (\{a_1, [\inf(0.2,0.4), \inf(0.4,0.7)], (a_2, [\inf(0.6,0.8), \inf(0.8,0.9)]),$$
$$(a_3, [\inf(0.4,0.6), \inf(0.6,0.9)])\}, \sup(0.5,0.7))$$
$$= (\{(a_1, [0.5,0.8], (a_2, [0.8,0.9]), (a_3, [0.6,0.9])\}, 0.7)$$

In the same way we get

$$\overline{\Upsilon}_{\tau}(p_2) = (\{(a_1, [0.3,0.5], (a_2, [0.3,0.4]), (a_3, [0.4,0.6])\}, 0.6),$$
$$\overline{\Upsilon}_{\tau}(p_3) = (\{(a_1, [0.8,0.9], (a_2, [0.3,0.5]), (a_3, [0.3,0.6])\}, 0.4)$$

In the matrix form we write

$$\overline{\Upsilon}_{\tau}(p) = \begin{bmatrix} [0.4,0.5] & [0.8,0.9] & [0.6,0.9] & 0.7 \\ [0.3,0.5] & [0.3,0.5] & [0.4,0.6] & 0.6 \\ [0.8,0.9] & [0.3,0.5] & [0.3,0.6] & 0.4 \end{bmatrix}.$$

Theorem 4.4. Let $\overline{F}_{\chi}$,$\overline{\eta}_{\phi}$and $\overline{\Upsilon}_{\tau}$ be any three interval- valued fuzzy soft sets then, the below results are equivalent.

(i) $\overline{F}_{\chi} = \overline{\eta}_{\phi} = \overline{\eta}_{\phi} \sqcup \overline{F}_{\chi}$.
(ii) $\overline{F}_{\chi} \sqcup \left(\overline{\eta}_{\tau} \sqcup \overline{\Upsilon}_{\tau}\right) = (\overline{F}_{\chi} \sqcup \overline{\eta}_{\phi}) \sqcup \overline{\Upsilon}_{\tau}$
(iii) $\overline{F}_{\chi} \sqcup \overline{F}_{\chi} \sqsubseteq \overline{F}_{\chi}$
(iv) $\overline{F}_{\chi} \sqcup \overline{X}_{\chi} = \overline{X}_{\chi}$
(v) $\overline{F}_{\chi} \sqcup \overline{\emptyset}_{\chi} = \overline{F}_{\chi}$

Proof: (i) $\overline{F}_{\chi} \sqcup \overline{\eta}_{\phi} = \overline{\Upsilon}_{\tau}$ by the **definition 4.4**, we have $\overline{\Upsilon}_{\tau}(p) = \left(\overline{\Upsilon}_{\tau}(p)(a), \tau(p)\right)$ such that $\overline{\Upsilon}(p) = \overline{F}(p) \sqcup \overline{\eta}(p)$ and $\tau(p) = s\left(\chi(p), \phi(p)\right)$. But $\overline{\Upsilon}(p) = \overline{F}(p) \sqcup \overline{\eta}(p) = \overline{\eta}(p) \sqcup \overline{F}(p)$ since union is commutative and $\tau(p) = s\left(\chi(p), \phi(p)\right) = s\left(\eta(p), \chi(p)\right)$ also s norm is commutative therefore$\overline{\eta}_{\phi} \sqcup \overline{F}_{\chi} = \overline{\Upsilon}_{\tau}$. Hence the other results (ii), (iii), (iv) and (v) proof is straightforward of **definition 4.4**.

Definition 4.5. Let two Interval- valued fuzzy soft sets $(\overline{F}_{\chi}, M)$ and $(\overline{\eta}_{\phi}, N)$ the intersection of Interval- valued fuzzy soft set $\overline{F}_{\chi} \sqcap \overline{\eta}_{\phi}$, is a interval- valued fuzzy soft set $(\overline{\Upsilon}_{\tau}, L)$ where $L = M \sqcup N$ and $\overline{\Upsilon}_{\tau}: \mathbb{P} \to \overline{P}(C) \times I$ is stated

$$\overline{\Upsilon}_{\tau}(p) = \left(\overline{\Upsilon}(p), \tau(p)\right)$$

Such that $\overline{\Upsilon}(p) = \overline{F}(p) \sqcap \overline{\eta}(p)$ and $\tau(p) = \kappa\left(\chi(p), \phi(p)\right)$, where τ is an $s-$norm and

$$\overline{\Upsilon}(p) = [\max\left(\chi_{\overline{F}(p)}{}^{-}, \chi_{\overline{\eta}(p)}{}^{-}\right) \ \max\left(\chi_{\overline{F}(p)}{}^{+}, \chi_{\overline{\eta}(p)}{}^{+}\right)].$$

Example.5. Consider interval- valued fuzzy soft sets $\overline{F}_{\chi}$ and $\overline{\eta}_{\phi}$ by **example 2**.using interval valued fuzzy intersection and is symbolized by $\overline{F}_{\chi} \sqcap \overline{\eta}_{\phi} = \overline{\Upsilon}_{\tau}$where

$$\overline{\Upsilon}_{\tau}(p_1) = (\{a_1, [\inf(0.2,0.4), \inf(0.4,0.7)], (a_2, [\inf(0.6,0.8), \inf(0.8,0.9)]), (a_3, [\inf(0.4,0.6), \inf(0.6,0.9)])\}, \inf(0.5,0.7)) = (\{(a_1, [0.2,0.4], (a_2, [0.4,0.7]), (a_3, [0.4,0.6])\}, 0.5)$$

In the same way we get

$$\overline{\Upsilon}_{\tau}(p_2) = (\{(a_1, [0.0,0.4], (a_2, [0.0,0.3]), (a_3, [0.2,0.4])\}, 0.5),$$
$$\overline{\Upsilon}_{\tau}(p_3) = (\{(a_1, [0.6,0.7], (a_2, [0.2,0.2]), (a_3, [0.2,0.4])\}, 0.2)$$

In the matrix form we write

$$\overline{\Upsilon}_{\tau}(p) = \begin{bmatrix} [0.2,0.4] & [0.4,0.7] & [0.4,0.6] & 0.5 \\ [0.0,0.4] & [0.0,0.3] & [0.2,0.4] & 0.5 \\ [0.6,0.7] & [0.2,0.2] & [0.2,0.4] & 0.2 \end{bmatrix}.$$

Theorem 4.6. Let $\overline{F}_{\chi}$,$\overline{\eta}_{\phi}$and $\overline{\Upsilon}_{\tau}$ be any three interval- valued fuzzy soft sets then, the below results are equivalent.

(i) $\overline{F}_{\chi} = \overline{\eta}_{\phi} = \overline{\eta}_{\phi} \sqcap \overline{F}_{\chi}$.
(ii) $\overline{F}_{\chi} \sqcap (\overline{\eta}_{\tau} \sqcap \overline{\Upsilon}_{\tau}) = (\overline{F}_{\chi} \sqcap \overline{\eta}_{\phi}) \sqcap \overline{\Upsilon}_{\tau}$
(iii) $\overline{F}_{\chi} \sqcap \overline{F}_{\chi} \sqsubseteq \overline{F}_{\chi}$
(iv) $\overline{F}_{\chi} \sqcap \overline{X}_{\chi} = \overline{X}_{\chi}$
(v) $\overline{F}_{\chi} \sqcap \overline{\emptyset}_{\chi} = \overline{F}_{\chi}$

Proof: (i) $\overline{F}_{\chi} \sqcap \overline{\eta}_{\phi} = \overline{\Upsilon}_{\tau}$ by the **definition 4.4,** we have $\overline{\Upsilon}_{\tau}(p) = \left(\overline{\Upsilon}_{\tau}(p)(a), \tau(p)\right)$ such that $\overline{\Upsilon}(p) = \overline{F}(p) \sqcap \overline{\eta}(p)$ and $\tau(p) = s(\chi(p), \phi(p))$. But $\overline{\Upsilon}(p) = \overline{F}(p) \sqcup \overline{\eta}(p) = \overline{\eta}(p) \sqcup \overline{F}(p)$ since union is commutative and $\tau(p) = s(\chi(p), \phi(p)) = s(\eta(p), \chi(p))$ also s norm is commutative hence $\overline{\eta}_{\phi} \sqcap \overline{F}_{\chi} = \overline{\Upsilon}_{\tau}$. And so the other results (ii), (iii), (iv) and (v) proof is straightforward of **definition 4.7.**

Theorem.4.7. Let $\overline{F}_{\chi}$ and $\overline{\eta}_{\phi}$ be any two interval- valued fuzzy soft sets. Then the De Morgan's results are

(i) $(\overline{F}_{\chi} \sqcup \overline{\eta}_{\phi})^c = \overline{F}_{\chi}^{\ c} \sqcap \overline{\eta}_{\phi}^{\ c}$
(ii) $(\overline{F}_{\chi} \sqcap \overline{\eta}_{\phi})^c = \overline{F}_{\chi}^{\ c} \sqcup \overline{\eta}_{\phi}^{\ c}$

Proof: Consider

$$\overline{F}_{\chi}^{\ c} \sqcap \overline{\eta}_{\phi}^{\ c} = (((\overline{c}(\overline{F}(p), c(\chi(p)))) \sqcap \left(\overline{c}\left(\overline{\eta}(p), c(\phi(p))\right)\right)$$
$$= ((\overline{c}(\overline{F}(p) \sqcup \overline{c}(\overline{\eta}(p))^{c} (\chi(p) \sqcup \phi(p))^{\ c})$$
$$= (\overline{F}_{\chi} \sqcup \overline{\eta}_{\phi})^c$$
$$\overline{F}_{\chi}^{\ c} \sqcup \overline{\eta}_{\phi}^{\ c} = (((\overline{c}(\overline{F}(p), c(\chi(p)))) \sqcup \left(\overline{c}\left(\overline{\eta}(p), c(\phi(p))\right)\right)$$
$$= ((\overline{c}(\overline{F}(p) \sqcap \overline{c}(\overline{\eta}(p))^{c} (\chi(p) \sqcap \phi(p))^{\ c})$$
$$= (\overline{F}_{\chi} \sqcap \overline{\eta}_{\phi})^c$$

Theorem 4.8. Let $\overline{F}_{\chi}$,$\overline{\eta}_{\phi}$and $\overline{\Upsilon}_{\tau}$ be any three interval- valued fuzzy soft sets then, the below results are equivalent.

(i) $\overline{F}_{\chi} \sqcup \left(\overline{\eta}_{\phi} \sqcap \overline{\Upsilon}_{\tau}\right) = \left(\overline{F}_{\chi} \sqcup \overline{\eta}_{\phi}\right) \sqcap \left(\overline{F}_{\chi} \sqcup \overline{\Upsilon}_{\tau}\right)$.
(ii) $\overline{F}_{\chi} \sqcap \left(\overline{\eta}_{\phi} \sqcup \overline{\Upsilon}_{\tau}\right) = \left(\overline{F}_{\chi} \sqcap \overline{\eta}_{\phi}\right) \sqcup \left(\overline{F}_{\chi} \sqcap \overline{\Upsilon}_{\tau}\right)$.

Proof (i) for all $p \in \mathbb{P}$,

$$\lambda_{\overline{F}(a) \sqcup \left(\overline{\eta}(a) \sqcap \overline{\Upsilon}(a)\right)}(a) = \left[\max\left(\lambda_{\overline{F}(a)}{}^{-}(a), \lambda_{\overline{\eta}(a) \sqcap \overline{Y}(a)}{}^{-}(a)\right), \max\left(\lambda_{\overline{F}(a)}{}^{+}(a), \lambda_{\overline{\eta}(a) \sqcap \overline{Y}(a)}{}^{+}(a)\right)\right]$$
$$= [\max(\lambda_{\overline{F}(a)}{}^{-}(a), \min\left(\lambda_{\overline{\eta}(a)}{}^{-}(a), \lambda_{\overline{\Upsilon}(a)}{}^{-}(a)\right),$$

$$(\max\left(\lambda_{\overline{F}(a)}{}^{+}(a), min\left(\lambda_{\overline{\eta}(a)}{}^{+}(a), \lambda_{\overline{Y}(a)}{}^{+}(a)\right)\right]$$
$$= [\min\left(max\left(\lambda_{\overline{F}(a)}{}^{-}(a), \lambda_{\overline{\eta}(a)}{}^{-}(a)\right), max\left(\lambda_{\overline{F}(a)}{}^{-}(a), \lambda_{\overline{Y}(a)}{}^{-}(a)\right)\right),$$
$$\min\left(max\left(\lambda_{\overline{F}(a)}{}^{+}(a), \lambda_{\overline{\eta}(a)}{}^{+}(a)\right), max\left(\lambda_{\overline{F}(a)}{}^{+}(a), \lambda_{\overline{Y}(a)}{}^{+}(a)\right)\right)]$$
$$= \lambda_{(\overline{F}(a)\sqcup\overline{\eta}(a)\sqcap(\overline{F}(a)\sqcup\overline{\Upsilon}(a)}(a)$$
$$\lambda_{\chi(a)\sqcup(\phi(a)\sqcap\tau(a))}(a) = \sup\{\lambda_{\chi(a)}(a), \lambda_{\phi(a)\sqcap\tau(a)}(a)\}$$
$$= \sup\left\{\lambda_{\chi(a)}(a), inf\left(\lambda_{\phi(a)}(a), \lambda_{\tau(a)}(a)\right)\right\}$$
$$= \inf\{\sup\left(\lambda_{\chi(a)}(a), \lambda_{\phi(a)}(a)\right), \sup\left(\lambda_{\chi(a)}(a), \lambda_{\tau(a)}(a)\right)\}$$
$$= \inf\{\lambda_{\chi(a)\sqcup\phi(a)}(a), \lambda_{\chi(a)\sqcup\tau(a)}(a)\}$$
$$= \lambda_{(\chi(a)\sqcup\phi(a))\sqcap(\chi(a)\sqcup\tau(a)}(a).$$

In same way to prove (ii).

5. On Interval- valued fuzzy soft set ($\wedge$(**AND**) and $\vee$(**OR**)) operation

In this section we state interval- valued fuzzy soft set $\wedge(AND)$ and $\vee(OR)$ operation and with application of selection taking problem with example interval- valued fuzzy soft set.

Definition 5.1 $(\overline{F}_{\chi}, M)$ and $(\overline{\eta}_{\phi}, L)$ be an two interval- valued fuzzy soft sets, then $(\overline{F}_{\chi}, M)AND(\overline{\eta}_{\phi}, L)$ symbolized by $(\overline{F}_{\chi}, M)\wedge(\overline{\eta}_{\phi}, L)$ is stated by

$$\left(\overline{F}_{\chi}, M\right)\wedge\left(\overline{\eta}_{\phi}, L\right) = \left(\overline{\Upsilon}_{\tau}, M\otimes N\right),$$

Where $\overline{\Upsilon}_{\tau}(\delta,\zeta) = \left(\Upsilon(\delta,\zeta), \lambda(\delta,\zeta)\right), \forall\ (\delta,\zeta) \in M\otimes N$, such that $\overline{\Upsilon}(\delta,\zeta) = \overline{F}(\delta) \sqcap \overline{\eta}(\zeta)$ and $\lambda(\delta,\zeta) = t\left(\chi(\delta), \eta(\zeta)\right), \forall(\delta,\zeta) \in M\otimes N)$.

Example 5 Consider the three objects a_1, a_2, a_3, that is $U = \{a_1, a_2, a_3\}$, also parameters be $\mathbb{P} = \{p_1, p_2, p_3\}$ which refer to some acts giving some different target. Suppose a company wants to purchase one such engine subject upon two parameters only. Take two observations $\overline{F}_{\chi}$ and $\overline{\eta}_{\phi}$ by two specialists M and N, respectively, stated below,

$$\overline{F}_{\chi}(p_1) = (\{(a_1, [0.2,0.4]), (a_2, [0.6,0.8]), (a_3, [0.4,0.6])\}, 0.5),$$
$$\overline{F}_{\chi}(p_2) = (\{(a_1, [0.0,0.4]), (a_2, [0.0,0.3]), (a_3, [0.2,0.4])\}, 0.5),$$
$$\overline{F}_{\chi}(p_3) = (\{(a_1, [0.6,0.7]), (a_2, [0.2.0.2]), (a_3, [0.2,0.4])\}, 0.2),$$
$$\overline{\eta}_{\phi}(p_1) = (\{(a_1, [0.4,0.6]), (a_2, [0.3,0.7]), (a_3, [0.5,0.6])\}, 0.4),$$
$$\overline{\eta}_{\phi}(p_2) = (\{(a_1, [0.4,0.6]), (a_2, [0.5,0.7]), (a_3, [0.0,0.6])\}, 0.2),$$
$$\overline{\eta}_{\phi}(p_1) = (\{(a_1, [0.2,0.7]), (a_2, [0.5,0.8]), (a_3, [0.3,0.4])\}, 0.3),$$

Then find $\wedge$ between two Interval- valued fuzzy soft sets, hence we have

$$(\overline{F}_{\chi}, M)\wedge(\ \overline{\eta}_{\phi}, N) = \left(\overline{\Upsilon}_{\tau}, M\otimes N\right),$$
$$\overline{\Upsilon}_{\tau}(p_1, p_1) = (\{(a_1, [0.2,0.4]), (a_2, [0.3,0.7]), (a_3, [0.4,0.6])\}, 0.4),$$
$$\overline{\Upsilon}_{\tau}(p_1, p_1) = (\{(a_1, [0.2,0.4]), (a_2, [0.5,0.7]), (a_3, [0.0,0.4])\}, 0.2),$$
$$\overline{\Upsilon}_{\tau}(p_1, p_3) = (\{(a_1, [0.2,0.4]), (a_2, [0.5,0.8]), (a_3, [0.3,0.4])\}, 0.3),$$
$$\overline{\Upsilon}_{\tau}(p_2, p_1) = (\{(a_1, [0.0,0.4]), (a_2, [0.0,0.3]), (a_3, [0.2,0.4])\}, 0.4)\ ,$$
$$\overline{\Upsilon}_{\tau}(p_2, p_2) = (\{(a_1, [0.0,0.4]), (a_2, [0.0,0.3]), (a_3, [0.0,0.4])\}, 0.2),$$
$$\overline{\Upsilon}_{\tau}(p_2, p_3) = (\{(a_1, [0.0,0.4]), (a_2, [0.0,0.3]), (a_3, [0.2,0.4])\}, 0.3),$$
$$\overline{\Upsilon}_{\tau}(p_3, p_1) = (\{(a_1, [0.4,0.6]), (a_2, [0.2,0.2]), (a_3, [0.0,0.4])\}, 0.2),$$
$$\overline{\Upsilon}_{\tau}(p_3, p_2) = (\{(a_1, [0.4,0.4]), (a_2, [0.2,0.2]), (a_3, [0.0,0.4])\}, 0.2),$$
$$\overline{\Upsilon}_{\tau}(p_3, p_3) = (\{(a_1, [0.2,0.7]), (a_2, [0.2,0.2]), (a_3, [0.2,0.4])\}, 0.2).$$

Table 1.

	a_1	a_2	a_3	χ
(p_1, p_1)	[0.2,0.4]	[0.3,0.7]	[0.4,0.6]	0.3
(p_1, p_2)	[0.2,0.4]	[0.5,0.7]	[0.0,0.4]	0.2
(p_1, p_3)	[0.2,0.4]	[0.3,0.8]	[0.3,0.4]	0.3
(p_2, p_1)	[0.0,0.4]	[0.0,0.3]	[0.2,0.4]	0.4
(p_2, p_2)	[0.0,0.4]	[0.0,0.3]	[0.0,0.4]	0.2
(p_2, p_3)	[0.0,0.4]	[0.0,0.3]	[0.2,0.4]	0.3
(p_3, p_1)	[0.4,0.6]	[0.2,0.2]	[0.2,0.4]	0.2
(p_3, p_2)	[0.4,0.6]	[0.2,0.2]	[0.0,0.4]	0.2
(p_3, p_3)	[0.2,0.7]	[0.2,0.2]	[0.2,0.4]	0.2

Table 2. Assign grade value $g_{e\in P}(a_k)$

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
a_1	-0.9	-0.6	-0.9	0.0	0.0	0.0	1.0*	1.1*	0.9*
a_2	0.5*	1.4*	1.4*	-0.4*	-0.3*	-0.3*	-0.9*	-0.8*	-0.8*
a_3	0.5*	-0.9*	-0.6*	0.4*	0.2*	0.4*	-0.3*	-0.4*	-0.2*
χ	0.4	0.2	0.3	0.4	0.2	0.3	0.2	0.2	0.2

Here and now, to evaluate the correct product, we initially solve the statistical grade $g_{e\in P}(a_k)$ for every $e \in$ P such that

$$g_{e\in P}(a_k) = \sum_{a\in C}\{(r_k^{-} - \chi_{\Upsilon(e_k)}{}^{-}(a))\oplus((r_k^{+} - \chi_{\Upsilon(e_k)}{}^{+}(a))\}$$

This value shown in table 1 and 2. Let $P = \{e_1 = (p_1, p_1), e_2 = (p_1, p_2), \ldots, e_9 = (p_3, p_3)\}$

Now we assign the large value (Marked * in table 2)in every row not including lost row which is the grade of such product of engine compared to every pair of parameters (look table 3). Now , the mark of every such a engine is evaluated by taking the add and multiple of these score with the equivalent value of χ. The engine with the largest value is the looked-for engine. We donot consider the grade of the engine against the pair $(p_k, p_k), k = 1,2,3,$ as both the parameters are the equal.Now evaluate largest value.

$$\text{Value}(a_1) = (1\otimes 0.2) + (1.1\otimes 0.2) = 0.24$$
$$\text{Value}(a_2) = (1.4\otimes 0.2) + (1.4\otimes 0.3) = 0.70$$
$$\text{Value}(a_3) = (0.4\otimes 0.4) + (0.4\otimes 0.3) = 0.28$$

The company will chose the engine with the largest value . Hence they will purchase engine a_2.

Table 3. Grade Value

	e_1	e_1	e_1	e_1	e_1	e_1	e_1	e_1	e_1
a_i	a_2, a_3	a_2	a_2	a_3	a_3	a_3	a_1	a_1	a_1
χ	0.4	0.2	0.3	0.4	0.2	0.3	0.2	0.2	0.2

Definition 5.2. If $(\overline{F}_{\chi}, M)$ and $(\overline{\eta}_{\phi}, N)$ are two interval- valued fuzzy soft set, then

$$((\overline{F}_{\chi}, M)\vee\left(\overline{\eta}_{\phi}, N\right) = \left(\overline{\Upsilon}_{\tau}\ M\otimes N\right),$$

Where $\overline{\Upsilon}_{\tau}(\delta,\zeta) = \left(\Upsilon(\delta,\zeta), \lambda(\delta,\zeta)\right), \forall\ (\delta,\zeta) \in M\otimes N,$ such that $\overline{\Upsilon}(\delta,\zeta) = \overline{F}(\delta) \sqcup \overline{\eta}(\zeta)$ and $\lambda(\delta,\zeta) = t\left(\chi(\delta), \eta(\zeta)\right), \forall(\delta,\zeta) \in M\otimes N)$.

Consider the **example 5** then find $\wedge(AND)$ between two interval- valued fuzzy soft set sets, hence we have $(\overline{F}_{\chi}, M)\vee(\overline{\eta}_{\phi}, N) = \left(\overline{\Upsilon}_{\tau}, M\otimes N\right),$

$$\overline{\Upsilon}_{\tau}(p_1,p_1) = (\{(a_1,[0.2,0.4]),(a_2,[0.3,0.7]),(a_3,[0.4,0.6])\},0.5),$$
$$\overline{\Upsilon}_{\tau}(p_1,p_1) = (\{(a_1,[0.4,0.6]),(a_2,[0.6,0.8]),(a_3,[0.5,0.6])\},0.5),$$
$$\overline{\Upsilon}_{\tau}(p_1,p_3) = (\{(a_1,[0.2,0.7]),(a_2,[0.6,0.8]),(a_3,[0.4,0.6])\},0.5),$$
$$\overline{\Upsilon}_{\tau}(p_2,p_1) = (\{(a_1,[0.4,0.6]),(a_2,[0.0,0.7]),(a_3,[0.5,0.6])\},0.5)\ ,$$
$$\overline{\Upsilon}_{\tau}(p_2,p_2) = (\{(a_1,[0.4,0.6]),(a_2,[0.5,0.7]),(a_3,[0.2,0.4])\},0.5),$$
$$\overline{\Upsilon}_{\tau}(p_2,p_3) = (\{(a_1,[0.2,0.7]),(a_2,[0.5,0.8]),(a_3,[0.3,0.4])\},0.5),$$
$$\overline{\Upsilon}_{\tau}(p_3,p_1) = (\{(a_1,[0.6,0.7]),(a_2,[0.3,0.7]),(a_3,[0.5,0.6])\},0.4),$$

$\overline{\Upsilon}_\tau(p_3, p_2) = (\{(a_1, [0.6,0.7]), (a_2, [0.5,0.7]), (a_3, [0.2,0.4])\}, 0.2),$
$\overline{\Upsilon}_\tau(p_3, p_3) = (\{(a_1, [0.6,0.7]), (a_2, [0.5,0.8]), (a_3, [0.3,0.4])\}, 0.3),$

Remark 1. We apply the same procedure in **example 5** for $''\vee''$ operation if the company wants to purchase engine depending on any one of the parameters only.

6. Comparable measure of two Interval- valued fuzzy soft set

Definition 6.1. Comparable between two interval- valued fuzzy soft sets. $\overline{F}_\chi$and $\overline{\eta}_\phi$, represented by $E(\overline{F}_\chi, \overline{\eta}_\phi)$ is stated by

$$E\left(\overline{F}_\chi, \overline{\eta}_\phi\right) = [\theta^-(\overline{F}, \overline{\eta}) \odot w(\chi, \phi), \theta^+(\overline{F}, \overline{\eta}) \odot w(\chi, \phi)], \text{ Such that}$$
$$\theta^-(\overline{F}, \overline{\eta}) = \inf\left(\theta_1(\overline{F}, \overline{\eta}), \theta_2(\overline{F}, \overline{\eta})\right),$$
$$\theta^+(\overline{F}, \overline{\eta}) = \sup\left(\theta_1(\overline{F}, \overline{\eta}), \theta_2(\overline{F}, \overline{\eta})\right)$$

Where

$$\theta_1(\overline{F}, \overline{\eta}) = 0 \text{ If } \overline{\chi}_{\overline{F}_k}{}^-(a) = 0, \forall\, k,$$
$$\theta_1(\overline{F}, \overline{\eta}) = \textstyle\sum_i^k \sup_{a\in A}\left\{\inf\left(\overline{\chi}_{\overline{F}_k}{}^-(a), \overline{\chi}_{\overline{\chi}_k}{}^-(a)\right)\right\} / \textstyle\sum_i^k \sup_{a\in A}\left[\overline{\chi}_{\overline{F}_k}{}^-(a)\right], \text{ other values.}$$
$$\theta_2(\overline{F}, \overline{\eta}) = \sum_i^k \sup_{a\in A}\left\{\inf\left(\overline{\chi}_{\overline{F}_k}{}^+(a), \overline{\chi}_{\overline{\chi}_k}{}^+(a)\right)\right\} \Big/ \sum_i^k \sup_{a\in A}[\overline{\chi}_{\overline{F}_k}{}^-(a)]$$
$$w(\chi(p), \phi(p)) = 1 - (\sum |\chi(p) - \phi(p)| / (\sum |\chi(p) + \phi(p)|)$$

Definition 6.2 Let $\overline{F}_\chi$ and $\overline{\eta}_\phi$ be an tow interval- valued fuzzy soft sets over universal$(U, \mathbb{P})$, we said to be two interval- valued fuzzy soft sets are similar if $\theta^-(\overline{F}_\chi, \overline{\eta}_\phi) \geq 0.5$.

Theorem 6.3. Let $\overline{F}_\chi$ and $\overline{\eta}_\phi$ be a two interval- valued fuzzy soft sets over universal $(U, \mathbb{P})$,then the below results are true:

(i) In general $w(\overline{F}_\chi, \overline{\eta}_\phi)$ is not equal to $w\left(\overline{\eta}_\phi, \overline{F}_\chi\right)$,
(ii) $0 \leq \theta^-\left(\overline{F}_\chi, \overline{\eta}_\phi\right)$ and $\theta^+(\overline{F}_\chi, \overline{\eta}_\phi) \leq 1$,
(iii) $\overline{F}_\chi = \overline{\eta}_\phi$ which implies $w\left(\overline{F}_\chi, \overline{\eta}_\phi\right) = 1$,
(iv) $\overline{F}_\chi \sqsubseteq \overline{\eta}_\phi \sqsubseteq \overline{\Upsilon}_\tau$ which implies $w(\overline{F}_\chi, \overline{\Upsilon}_\tau) \leq w\left(\overline{\eta}_\phi, \overline{\Upsilon}_\tau\right)$,
(v) $\overline{F}_\chi \sqcap \overline{\eta}_\phi = \emptyset$ if and only if $w(\overline{F}_\chi, \eta_\phi) = 0$.

Proof: (i) The proof using **definition 6.1** (ii) by the **definition 6.1** straightforward of this proof.

$$\theta_1(\overline{F}, \overline{\eta}) = 0 \text{ if } \overline{\chi}_{\overline{F}_k}{}^-(a) = 0,$$
$$\theta_1(\overline{F}, \overline{\eta}) = \textstyle\sum_i^k \sup_{a\in A}\left\{\inf\left(\overline{\chi}_{\overline{F}_k}{}^-(a), \overline{\chi}_{\overline{\chi}_k}{}^-(a)\right)\right\} / \textstyle\sum_i^k \sup_{a\in A}\left[\overline{\chi}_{\overline{F}_k}{}^-(a)\right], \text{ Otherwise}$$

If $\overline{\chi}_{\overline{F}_k}{}^-(a) = 0, \forall\, k$, then $\theta^-(\overline{F}, \overline{\eta}\,) = 0$, and, if $\overline{\chi}_{\overline{F}_k}{}^-(a) \neq 0$, for some k , then if it evidently $\theta^-(\overline{F}, \overline{\eta}\,) \geq 0$,After $\theta^+(\overline{F}, \overline{\eta}\,) = \sup\left(\theta_1(\overline{F}, \overline{\eta}\,), \theta_2(\overline{F}, \overline{\eta}\,)\right)$, suppose that $\theta_1(\overline{F}, \overline{\eta}\,) = 1$ and $\theta_2(\overline{F}, \overline{\eta}\,) = 1$, then $\theta^+(\overline{F}, \overline{\eta}) = 1$, this means , if $\theta_1(\overline{F}, \overline{\eta}\,) < 1$ and $\theta_2(\overline{F}, \overline{\eta}\,) < ,1$ then $\theta^+(\overline{F}, \overline{\eta}) \leq 1$. (iii),(iv),(v) proof is straight forward by the definition 6.1. //

Example 6.Let $\overline{F}_\chi$ be an Interval- valued fuzzy soft set over $(U, \mathbb{P})$ stated as follows

$\overline{F}_\chi(p_1) = (\{(a_1, [0.4,0.8]), (a_2, [0.5,0.9]), (a_3, [0.2,0.4])\}, 0.5),$
$\overline{F}_\chi(p_2) = (\{(a_1, [0.5,0.7]), (a_2, [0.2,0.4]), (a_3, [0.0,0.5])\}, 0.7),$
$\overline{F}_\chi(p_3) = (\{(a_1, [0.8,0.9]), (a_2, [0.2,0.6]), (a_3, [0.9,1.0])\}, 0.9).$

Let $\overline{\eta}_\phi$ be the interval- valued fuzzy soft set over $(U, \mathbb{P})$ is stated as follows:

$$\overline{\eta}_\phi(p_1) = (\{(a_1, [0.2,0.5]), (a_2, [0.6,0.8]), (a_3, [0.3,0.4])\}, 0.4),$$
$$\overline{\eta}_\phi(p_2) = (\{(a_1, [0.7,0.9]), (a_2, [0.6,0.7]), (a_3, [0.5,0.9])\}, 0.8),$$
$$\overline{\eta}_\phi(p_3) = (\{(a_1, [0.5,0.8]), (a_2, [0.4,0.6]), (a_3, [0.6,0.8])\}, 0.7),$$

Here

$$w(\chi(p), \phi(p)) = 1 - (\sum|\chi(p) - \phi(p)|/(\sum|\chi(p) + \phi(p)|)$$
$$= 1 - (|(0.5 - 0.4| + |(0.7 - 0.8| + |(0.9 - 0.7|/|(0.5 + 0.4| + |(0.7 + 0.8| + |(0.9 + 0.7|$$

This approximately value is 0.88,
Using **(39), (40) in (38)** after calculating, we get

$$\theta_1(\overline{F}, \overline{\eta}) = 0.81$$
$$\theta_2(\overline{F}, \overline{\eta}) = 0.82$$

Then

$$\theta^-(\overline{F}, \overline{\eta}) = \inf\left(\theta_1(\overline{F}, \overline{\eta}), \theta_2(\overline{F}, \overline{\eta})\right) = \inf(0.81,0.82) = 0.81$$
$$\theta^+(\overline{F}, \overline{\eta}) = \sup\left(\theta_1(\overline{F}, \overline{\eta}), \theta_2(\overline{F}, \overline{\eta})\right) = \sup(0.81,0.82) = 0.82$$

Hence, the similar value of two interval- valued fuzzy soft sets $\overline{F}_\chi$ and $\overline{\eta}_\phi$ will be

$$E\left(\overline{F}_\chi, \overline{\eta}_\phi\right) = [\theta^-(\overline{F}, \overline{\eta}) \odot w(\chi, \phi), \theta^+(\overline{F}, \overline{\eta}) \odot w(\chi, \phi)]$$
$$= [(0.81) \odot (0.88), (0.82 \odot 0.88)]$$
$$= [0.71,0.72].$$

Hence the similar $\overline{F}_\chi$ and$\overline{\eta}_\phi$.

7. Comparable Measure of Medical Application

In this part, we are able to attempt to evaluation the probability that an ill man or woman having certain warning signs is stricken by a viral infection. Hence, we first bring together an interval valued fuzzy soft sets type for viral infection and the interval valued fuzzy soft sets of warning symbol for the ill man or woman. After, we discover the comparison measure of those sets. If they are significantly same, then we finish that the one is maybe tormented by a viral infection. Let our common universal set holds the parameters ''sure'' and ''not at all' 'therefore $U = \{S, N\}$. Here the parameters set $\mathbb{P}$ is set of certain observable warning mark. Let $\mathbb{P} = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7\}$,where $''p_1 = Headache''$, $''p_2 = stomachpain''$, $''p_3 = cough''$, $''p_4 = loosemotion''$, $''p_5 = eye\ pain''$, $''p_6 = high\ sugure''$,$''p_7 = blood\ presure''$,our model Interval- valued fuzzy soft set viral fever H_χ is in **table-4** after can be assigned with a doctor Now , after speaking to the ill person , we can build interval- valued fuzzy soft set η_ϕ as in **table -5** Here and now we evaluated the measure of comparison of these 2 sets by applying the similar technique as in example 6, after the evaluation ,we come to be $\theta^-(\overline{\mathrm{H}}, \overline{\eta}) \odot h(\chi, \phi)$approximately 0.23<0.5.Hence the two interval valued fuzzy soft set**s** are not comparable. Hence we determine that the man or woman is not ill from the viral infection.

Table 4. Model interval valued fuzzy soft sets for viral temperature

H_χ	p_1	p_2	p_3	p_4	p_5	p_6	p_7
S	1.0	0.0	0.0	1.0	1.0	1.0	1.0
N	0.0	1.0	1.0	0.0	0.0	0.0	0.0
χ	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Table 5. Interval valued fuzzy soft sets ill man or women

F_a	p_1	p_2	p_3	p_4	p_5	p_6	p_7
S	[0.4,0.5]	[0.3,0.6]	[0.0,0.3]	1.0	[0.5,0.7]	0.0	[0.4,0.5]
N	[0.7,0.9]	[0.6,0.8]	[0.7,0.9]	[0.4,0.6]	1.0	[0.5,0.7]	[0.4,0.6]
χ	0.4	0.6	0.5	0.7	0.2	0.6	0.3

8. Conclusion

In this paper, we have introduced the concept of interval-valued fuzzy soft set and studied some of its properties. The complement, union, intersection, "AND," and "OR" operations have been defined on the interval-valued fuzzy soft sets. An application of this theory is given in solving a decision making problem. Similarity measure of two interval-valued fuzzy soft sets are discussed and its application to medical diagnosis.

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Authors' Profiles

Mr. V. Lambodharan, M.Sc, M.Phil, B.Ed, Research scholar Periyar university PG Extension centre, Dharmapuri, Published various international journal and paper presented national international conferences, and also participated in various national international conferences seminar workshop ect.

Dr. N. Anitha, M.Sc, M.Phil, Ph.D, Assistant Professor, Department of Mathematics, Periyar university PG Extension centre, Dharmapuri, Published various international journal and paper presented national international conferences, and also participated in various national international conferences seminar workshop ect. She has seventeen years of teaching experiences and thirteen years of research experiences she also guide three research scholars and awarded one Ph.D scholar

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