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Results on Coregular Perfect Domination of Line Graph and Relation with Different Dominations of Graph

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Abstract: For any graph G = (V, E),the line graph $L(G)$of a graph G is a graph whose set of vertices is the union of set of edges of G in which two vertices of $L(G)$ are adjacent if and only if the corresponding edges of G are adjacent. A dominating set $D_1 \subseteq V[L(G)]$ is called coregular perfect dominating set, if the induced subgraph $< V[L(G)] - D_1 >$ is regular. The minimum cardinality of vertices in such a set is called coregular perfect domination number in $L(G)$ and is represented by $\gamma_{cop}[L(G)]$.

In this Article, we study the graph theoretic properties of $\gamma_{cop}[L(G)]$ and many bounds were obtained in terms of elements of G and its relationship with other domination parameters were found .Our investigation on this work is to establish the application oriented standard results in the field of domination theory for several kinds of new concepts which are playing an important role of application.

Index Terms: Graph, Line graph, Co- regular perfect dominating set , Co-regular perfect domination number.

1. Introduction

The theory of graph is an extremely useful tool for solving combinatorial problems in different areas such as geometry,algebra, number theory,topology, operations research ,and computer science. Graphs serve as mathematical models to analyze many real world problems successfully. Some problems in physics, chemistry, communication science, computer technology, genetics , psychology , and sociology can be formulated as problems in Graph theory.

In this Article the graphs considered here are finite and simple. In general we follow the notations of Harary [7].

Let G = (V,E) be a graph of order $|V| = p$ and $|E| = $q. For any vertex v ∈ V, the open neighborhood of v is the set N(v) $= \{u \in V/uv \in E\}$ and the closed neighborhood is the set $N[v] = N(v) \cup \{v\}$. In general we use $\langle X \rangle$ to denote the subgraph induced by the set of vertices X.The notations $\alpha_0(G)$ $[\alpha_1(G)]$ is the minimum number of vertices (edges) in a vertex (edge) cover of G. Also $\beta_0(G)$ $[\beta_1(G)]$ is the maximum number of vertices (edges) in a maximal independent set of vertex (edge) of G. The maximum degree of a vertex v is denoted by $\Delta(G)$. The minimum distance between any two farthest vertices of a connected graph G is called diameter of G, and is denoted by $diam(G)$. If S is a subset of $V(G)$, then we denote by $\langle S \rangle$, the subgraph induced by S.

A line graph $L(G)$ is the graph whose vertices corresponds to edge of G and two vertices in $L(G)$ are adjacent if and only if the corresponding edges in G are adjacent.

We will start some basic definitions from domination theory.

A dominating set S of G is total dominating set if the induced sub graph $\langle S \rangle$ has no isolated vertices. The total domination number $\gamma_t(G)$of G must have a minimum cardinality of a total dominating set of G see [5].

In [8,15], a connected dominating set S is a dominating set whose induced sub graph $< S >$ is connected. The connected domination number of a graph G is represented by $\gamma_c(G)$, is the cardinality at a minimum of a connected dominating set.

A dominating set S_1 of a graph G is perfect dominating set if each vertex of V-S_1 is adjacent to exactly one vertex of S_1. The minimum cardinality of a perfect dominating set of G is a perfect domination number and is denoted by $\gamma_p(G)$, see [6].

A dominating set S of a graph G is a co-regular perfect dominating set if the induced subgraph $\langle V-S\rangle$ is regular. The co-regular perfect domination number $\gamma_{cop}(G)$ of G is the minimum cardinality of a co-regular perfect dominating.

This research study focuses on a a new concept in domination theory.

A dominating set S of $L(G)$ is a co-regular perfect dominating set if the induced subgraph $\langle V[L(G)]-S\rangle$ is a regular. The co-regular perfect domination number $\gamma_{cop}L(G)$ is the minimum cardinality of a co-regular perfect dominating set.

For the more reference see [2,4,7,9-15].

2. Results

We begin our investigation in to co-regular perfect domination in line graph.

Theorem 1:

1. For any Cycle C_p, with $p \geq 3$ vertices,

$$\gamma_{cop}L(C_p) = p - 2\left\lfloor\frac{p}{3}\right\rfloor$$

2. For any star $K_{1,p}$, with $p \geq 3$ vertices,

$L(K_{1,p}) = 1$.

Remark1. For any Wheel W_p with $p \geq 4$ vertices $\gamma_{cop}\, L(W_p)$ does not exists.

Theorem 2 : For any connected (p,q) graph G with $p \geq 3$ vertices , then

$\gamma_{cop}L(G) + diam\,(G) \leq$ p + $\gamma_{split}(G) + \Delta(G)$.

proof : Let J = { $e_1, e_2, \dots\dots\dots, e_n$}$\subseteq$ E(G) be the minimal set of edges which constitutes the largest path between any two distinct vertices $u, v \in V(G)$, as a result dist (u, v) = diam (G).

Now assume there exists atleast one vertex $v \in$ V(G) with $\Delta(G) = \deg(v)$. Let D = $\{v_1, v_2, \dots\dots, v_m\} \subseteq V(G)$ and $\forall\ v_k \in D$,$1 \leq k \leq m$, is adjacent to atleast one vertex of V (G) – D and if $\langle V(G)-D\rangle$ has more than one component, then D is a γ_{split} - set of G. Further let A = $\{v_1, v_2, \dots\dots, v_n\} = V[L(G)]$. Select a set K ⊆ A such that $\forall\ v_i \in A-K$ is adjacent to exactly one vertex of K and N[K] = V[L(G)]. Then K is the minimal perfect dominating set of L(G). If $\langle A-K\rangle$ is not regular, then attach a set of vertices $\{v_j\} \in \{A-K\}$ which make $\langle A-K \cup \{v_j\}\rangle$ regular. Hence $|K \cup \{v_j\}|$ + dist $(u,v) \leq |V(G)| + |D| + \Delta(G)$ attains $\gamma_{cop}L(G) + diam\,(G) \leq$ p + $\gamma_{split}(G) + \Delta(G)$.

The relationship between $\gamma_c(G)$, $\gamma_t(G)$ with co-regular perfect domination of a line graph $L(G)$ is established in the following theorem .

Theorem 3: For any non trivial (p, q) graph G, $\gamma_{cop}L(G) \leq \gamma_t + \gamma_c$ +1 and G $\neq W_p$.

Proof: Suppose G = W_p .Then by Remark 1 $\gamma_{cop}L(G)$ does not exists.

Let S = $\{v_1, v_2, \dots\dots, v_n\} \subseteq V(G)$ be the minimum set of vertices which covers all the vertices in G. In the subgraph $\langle S\rangle$, if deg(v_i) ≥ 1, $\forall\ v_k \in S$,$1 \leq i \leq n$, then S forms a γ_t set of G.Otherwise if deg(v_i)< 1 for some $v_i \in S$,then add the minimum number of vertices $\{v_j\} \in N(v_i)$ to the vertices of S . Then S ∪ $\{v_j\}$ forms a minimal total dominating set of G.

Let $S_1 = \{v_1, v_2, \dots\dots, v_k\}$ be the collection of all end vertices in G. Suppose $S_2 \subset \{V(G) - S_1\}$ be the minimal set of vertices such that N$[v_i] =$ V(G) $\forall\ v_i \in S_2$. Then S_2 forms a minimal dominating set of G.

Further if S_2 has exactly one component, then S_2 itself is a connected dominating set of G.Suppose S_2 has more than one component then attach the minimum set of vertices $\{v_k\}$, such that S_3 = $S_2 \cup \{v_k\}$ which are in u-v path , ∀ u , v ∈ $\{V(G) - S_2\}$. Hence S_3 is a minimal connected dominating set of G.

Let E_1 = $\{e_1, e_2, \dots\dots\dots, e_n\} \subseteq$ E(G); E_2 = $\{e_1, e_2, \dots\dots\dots, e_m\} \subseteq$ E(G). Then $\forall\ e_i \in E_1$ are incident with $\forall\ v_i \in \gamma_t$ set of G and $\forall\ e_j \in E_2$ are incident with $v_j \in \gamma_c$ set of G. Let E_3 = $\{e_1, e_2, \dots\dots\dots, e_k\}$ = E(G) . Then $\{v_1, v_2, \dots\dots\dots, v_k\}$ = V$[L(G)]$ corresponding to the elements of E_3 . Also H_1 = $\{v_1, v_2, \dots\dots\dots, v_n\} \subset V[L(G)]$ corresponding to the elements of E_2. Suppose K $\subset V[L(G)]$ and every vertex of $V[L(G)] - K$ is adjacent to exactly one element of K such that N [K] = $V[L(G)]$. Then $|H_1| < |K|$ and also $|H_2| < |K|$. But it is easy to verify that $|\{H_1\} \cup \{H_2\}| + 1 \geq |K|$ and if $\langle V[L(G)] - \{H_1\} \cup \{H_2\}\rangle$ is regular, then $\{H_1\} \cup \{H_2\}$ is a γ_{cop} set of $L(G)$. Hence

$|\{H_1\} \cup \{H_2\}| \geq$ S ∪ $\{v_j\} + |S_3| + 1$ gives the required result.

Theorem 4: For any non trivial $(p\,, q\,)$ graph G, with $p \geq 3$ vertices then ,

$\gamma_{cop}L(G) + \beta_1(G) \leq$ p + diam (G) .

Proof: Let V = { $v_1, v_2, \dots\dots\dots\dots, v_n$} = V (G) with $|V|$ = p. Let E_1 = $\{e_1, e_2, \dots\dots, e_n\} \subseteq$ E(G) be the edge set in G, which constitute the diametral path in G. Clearly $|E_1|$ = diam (G).

Let J = $\{e_1, e_2, \dots\dots, e_i\} \subseteq$ E(G),$1 \leq i \leq n$ is a set of maximum edges in G, such that for any $e_i, e_j \in J$, N(e_i)∩ N(e_j) = e and e ∈ E(G) – J. Clearly J forms maximal edge independent set of G, with $|J| = \beta_1(G)$.

Suppose M $\subset \beta_1$ and H $\subset E(G) - \beta_1$. Then in L (G), $\{M\} \cup \{\beta_1\} \subset$ V [L(G)]. Now assume $\forall\ v_i \in [\{M\} \cup \{\beta_1\}]$ is adjacent to exactly one vertex of V [L(G)]- $[\{M\} \cup \{\beta_1\}]$ such that N$[\{M\} \cup \{\beta_1\}] = V[L(G)]$ .Then $\{M\} \cup \{\beta_1\}$ is a

γ_p set of $L(G)$. For coregularity,if $\langle$V [L(G)] $-$ $[\{M\} \cup \{\beta_1\}]\rangle$ is regular, then $\{M\} \cup \{\beta_1\}$ is a γ_{cop} set of $L(G)$.If it is not the case, then consider a set $\{v_i\} \in$ V[L(G)]- $[\{M\} \cup \{\beta_1\}]$ which gives is regular. Clearly $\left|[\{M\} \cup \{\beta_1\}] \cup v_j\right|$ + $|J|$ $\leq |V| + |E_1|$ gives $\gamma_{cop}L(G) + \beta_1(G) \leq$ p + diam (G).

Theorem 5 : For any connected graph G ,with $p \geq 3$ vertices,then

$\gamma_{cop}L(G) + \gamma^1(G) \leq$ p - α_0 (G) + 1.

Proof : Let E_1 = { $e_{1,}e_2, \dots \dots \dots . \dots, e_n\} \subseteq$ E(G) be the set of all non end edges in G. Suppose there exists a minimal set of edges E_2 = { $e_{1,}e_2, \dots \dots \dots . \dots, e_m\} \subseteq E_1$ such that N[e_i] = E(G) , $\forall\, e_i \in E_2$, $1 \leq i \leq m$. Then E_2 forms a minimal edge dominating set of G.

Let B = $\{v_{1,}v_2, \dots \dots, v_i$ }$\subseteq$ V(G) be the minimal number of vertices which covers all the edges of G , such that $|B| = \alpha_0(G)$. Further if H = $\{v_{1,}v_2, \dots \dots \dots, v_m$ } be the vertex set of L(G). Suppose there exists K $\subseteq$ H such that N (v_i) $\cap$ N (v_j) = $\emptyset$ $\forall$ i,j $\in$ K , then K is the minimal perfect dominating set of L(G).

If $\langle H - K\rangle$ is regular, then K is γ_{cop} set of L(G). If not add the set of vertices $\{v_r$ }from{H - K} such that $\langle$ $H - K \cup \{v_r\}$ $\rangle$ is regular. Since V(G) = p, then $|K \cup \{v_r\}|$ - $|E_2| \leq$ $|V(G)|$ + $|B|$ + 1, which gives $\gamma_{cop}L(G) + \gamma^1(G) \leq$ p - α_0 (G) + 1.

The independent domination number i(G) of G is the minimum cardinality of a dominating set which are also independent, see [1]

A dominating set F $\subseteq V(G)$ is a cototal dominating set if the induced subgraph $\langle V - F\rangle$ has no isolated vertices. The cototal domination number $\gamma_{cot}(G)$ of G is the minimum cardinality of cototal domination set of G . see [10]

Theorem 6: For any non trivial graph G with $p \geq 3$ vertices, then

$\gamma_{cop}L(G) - i(G) \leq 2\beta_0(G)$ - γ_{cot} (G).

Proof : Let V_1 = $\{v_{1,}v_2, \dots \dots . \dots \dots, v_p$ } $\subseteq$ V(G), such that $\forall\, v_i \in V_1$,deg (v_i) = 0 in $\langle V_1\rangle$. Then V_1 forms independent dominating set of G .Now consider $L = \{v_{1,}v_2, \dots \dots \dots \dots, v_m$ }$\subseteq$ V(G) be the maximum set of vertices with dist (u,v) $\geq$ 2 and N (u) $\cap$ N (v) = x , $\forall$ u,v $\in L$ and $x \in$ V(G) – L . Clearly $|L| = \beta_0(G)$.

Now let F be the set of vertices with deg (v) = 1,$\forall\, v \in F$, and I = $\{v_{1,}v_2, \dots \dots \dots, v_k$ } $\subseteq$ V(G) be the set of vertices such that diam (a,b) $\geq$ 2 where a $\in F, b \in I$. Suppose B = F $\cup\, I$ covers all the vertices of G. Then B forms γ_{cot} set of G.

Further let K = $\{v_{1,}v_2, \dots \dots, v_r$ } be the vertex set of L (G) . Then there exists H $\subseteq$ K such that N (v_i) $\cap$ N (v_j) = $\emptyset$, $\forall\, i , j \in H$ then H is minimal perfect dominating set of L (G). If $\langle K - H\rangle$ is regular ,then H is a γ_{cop} set of L (G). If not , select the set of vertices $\{v_i\}$ from $\{K - H\}$ such that $\langle$ $K - H \cup \{v_i\}$ $\rangle$ is regular. Hence $|H \cup$ $\{v_i\}|$ - $|V_1| \leq 2$ $|L| - |B|$ gives $\gamma_{cop}L(G) - i(G) \leq 2\beta_0(G)$ - γ_{cot} (G).

Theorem 7: For any connected graph G with $p \geq 3$ vertices, then $\gamma_{cop}L(G) \leq 2\alpha_1(G)$ - 1.

Proof: Let S = { $e_{1,}e_2, \dots \dots \dots ., e_i\} \subseteq E(G)$ be the set of end edges in G .Then S $\cup\, J$ where $J \subseteq E(G) - S$ be the minimal set of edges which covers all the vertices of G.Such that $|S \cup J| = \alpha_1(G)$.

Further let V = $\{v_{1,}v_2, \dots . . \dots \dots, v_m$ } be the vertices of L (G) . Suppose there exists K $\subseteq$ V such that $\forall$ $v_r \in V - K$, N (v_i) $\cap$ N (v_j) = $\emptyset$, $\forall\, i , j \in V - K$. Then K is minimal perfect dominating set of L (G). If $\langle V - K\rangle$ is regular, then K is a γ_{cop} of L (G). If not select the set of vertices $\{v_k\}$ from $\{V - K\}$ such that $\langle$ $V - K \cup \{v_k\}$ $\rangle$ makes regular.

Hence $|K \cup$ $\{v_k\}|$ ≤ 2 $|$ $S \cup J| - 1$ gives $\gamma_{cop}L(G) \leq 2\alpha_1(G)$ - 1.

Theorem 8: For any connected graph (p, q) $graph$ G , $\gamma_{cop}L(G) + 3 \geq p$ - $\gamma_t(G)$.

Proof ; Let V = $\{v_{1,}v_2, \dots . . \dots \dots, v_n$ } be the vertex set of G, with $|V| = p$. Suppose D be the minimal dominating set of G such that consider $H \subset V$ where H $\in N(D)$ and $\exists\, H_1 \subset H$ such that $\langle H_1 \cup D\rangle$ has no isolates. Clearly $\{H_1 \cup D\}$ is a total dominating set of G. Now let $V_1 = \{v_{1,}v_2, \dots . . \dots \dots, v_m$ }be the vertices of L(G), then there exists K $\subseteq$ V_1 such that $\forall\, v_i \in V_1 - K$ is adjacent to exactly one vertex of K and N[K] = V[L(G)]. Thus K be the minimal perfect dominating set of L (G). If $\langle V_1 \cup K\rangle$ is regular , then K is a γ_{cop} of L(G) . If not select the set of vertices $\{v_i\}$ from $\{V_1 - K\}$ which makes $\langle V_1 - K \cup \{v_i\}\rangle$ as regular. Hence

$|K \cup$ $\{v_i\}|$ + 3 $\geq$ $|V|$ - $|H_1 \cup D|$ gives $\gamma_{cop}L(G) + 3 \geq p$ - $\gamma_t(G)$.

Example:

G L(G)

Fig. 1. G is a Graph and L(G) is line graph of a graph G

In fig.1 , G is a Graph and L(G) is line graph of a graph G. we can see
p(G) = 9 and Total domination number of a graph G = $\gamma_t(G)$ = { v_2 , v_4, v_5, v_6 } = 4
Coregular Perfect domination number in line graph L[G] is
$\gamma_{cop}[L(G)]$ = { e_1 , e_2, e_3, e_4,e_5, e_6} = 6.
Hence $\gamma_{cop}L(G) + 3 \geq p$ - $\gamma_t(G)$

Theorem 9: For any nontrivial graph G, $\gamma_{cop}L(G) \leq \gamma_{cop}(G) + \gamma^1(G) + 1$.

Proof : Let H ={ $e_{1,}e_2, \dots \dots, e_m$} $\subseteq E(G)$ such that for each $e_i \in H$, i = 1, 2, 3---..---m , N(e_i) ∩ H = φ .Then $|H| = \gamma^1(G)$.

Let V = {$v_{1,}v_2, \dots \dots, v_n$ } be the vertices of a graph G. Suppose there exists A ⊆ V(G) such that $\forall v_i \in V - A$ is adjacent to exactly one vertex of A . Then A is perfect dominating set of G. If $\langle V - A\rangle$ is regular, then A itself is a coregular perfect domination set of G with $|A| = \gamma_{cop}(G)$.

Suppose there exists $H_1 \subseteq H$ and $H_2 \subseteq E(G) - H$. Then in L (G), {$H_1 \cup H_2$} ⊂ V [L(G)] . Now assume every vertex v_i ∈ V [L(G)] − {$H_1 \cup H_2$} is adjacent to exactly one vertex of {$H_1 \cup H_2$} and N[{H_1} ∪ {H_2}] = V[LG] . Hence {$H_1 \cup H_2$} is a perfect dominating set . If ⟨V [L(G)] − {$H_1 \cup H_2$} ⟩ is regular. Clearly {$H_1 \cup H_2$} is a γ_{cop} set of L(G).

Then $|\{H_1 \cup H_2\}| \leq |A| + |H| + 1$ gives $\gamma_{cop}L(G) \leq \gamma_{cop}(G) + \gamma^1(G) + 1$.

Theorem 10: For any connected graph G, then $\gamma_{cop}L(G) + 3 \geq$ q -Δ(G) and G ≠ P_p with p ≥ 11.

Proof: Suppose $G = P_p$ with $p \geq 11$. Then $\gamma_{cop}L(G) + 3 \leq q$ - $\Delta(G)$.

Let E = { $e_{1,}e_2, \dots \dots, e_n$} be the edge set of G with $|E| = q$.Let F = { $u_{1,}u, \dots \dots, u_i$ } be the set of all non end vertices in G , then there exists at least one vertex $u_k \in F$ of maximum degree $\Delta(G)$.

Further let V_1 = { $v_{1,}v_2, \dots \dots, v_n$ } be the vertex set of L(G). Suppose there exists K ⊂ V_1 such that ∀ $v_r \in V_1 - K$ is adjacent to exactly one vertex of K and N[K] = $V_1[L(G)]$. Then K is the minimal perfect dominating set of $L(G)$. If $\langle V_1 - K\rangle$ is regular, then K is γ_{cop} set of L(G). If not select the set of vertices $\{v_j\} \in \{V_1 - K\}$,which makes $\langle V_1 - K \cup \{v_j\}\rangle$ regular.

Clearly $|K \cup \{v_j\}| + 3 \geq |E| - \Delta(G)$. It follows that $\gamma_{cop}L(G) + 3 \geq q$ - $\Delta(G)$.

Theorem 11: For any connected (p, q)graph G with $p \geq 3$ vertices, then $\gamma_{cop}L(G) \leq 3q$ - $2p$.

Proof: Let E={$e_{1,}e_2, \dots \dots e_n$} be the edge set of G with$|E|$= q. And let V={ $v_{1,}, v_2 \dots \dots, v_k$} be the set of vertices with $|V|$ = p .

Consider V_1 = { $v_{1,}, v_2 \dots \dots, v_k$} be the vertex set of L(G) .Suppose there exists K ⊂ V_1 such that ∀ $v_r \in V_1 - K$ is adjacent to exactly one vertex of K .Then K is the minimal perfect dominating set of L(G). If $\langle V_1 - K\rangle$ is regular, then K is itself γ_{cop} of L (G). Hence $|K| \leq 3|E| - 2|V|$ gives $\gamma_{cop}L(G) \leq 3q$ - $2p$.

Theorem 12: For any connected (p, q) graph G with $p \geq 3$ vertices , then

$$\gamma_{cop}L(G) \leq \left\lceil \frac{q+m}{2} \right\rceil + \Delta^1(G) .$$

Proof: Let E ={$e_{1,}e_2, \dots, e_m$} be the edge set of G, with $|E| = q$.Suppose F = {$v_{1,}v_2, \dots, v_m$ }⊆ $V(G)$ be the set of all end vertices in G with $|F|$ = m . Now suppose E_1 = { $e_{1,}e_2, \dots \dots, e_i$} be the set of all non end vertices of G.Then there exist atleast one edge of maximum degree $\Delta^1(G)$ in E_1. Further let V_1 = { $v_{1,}, v_2 \dots \dots, v_m$} be the vertices of L(G) corresponding to the elements of E. Suppose there exists K ⊆ V_1 such that ∀ $v_r \in V_1 - K$ is adjacent to exactly one vertex of K and $N(v_i) \cap N(v_j) = \varphi \ \forall \, i, j \in V - K$. Then K is the minimal perfect dominating set of L(G). If $\langle V_1 - K \rangle$ is regular then, K is γ_{cop} set of L(G). If not add the set of vertices $\{v_k\} \in \{V_1 - K\}$ such that $\langle V_1 - K \cup \{v_k\} \rangle$ is regular.

Hence $|K \cup \{v_k\}| \leq \frac{|E| + |F|}{2} + \Delta^1(G)$, which gives $\gamma_{cop}L(G) \leq \left\lceil \frac{q+m}{2} \right\rceil + \Delta^1(G)$.

Theorem 13: For any nontrivial graph G, then $\gamma_{cop}L(G) + \Delta(G) \geq \gamma_p(G) + C$, where C be the number of cutvertices in G and $G \neq P_p$.

Proof: Let $V_1 = \{ v_{1,}, v_2 \dots \dots, v_p \} \subset V(G)$ such that V(G)$-\{v_i\}$ has more than one component , $\forall \, v_i \in V_1$ such that $|V_1|$= C.Now assume there exists atleast one vertex $v \in V(G)$ with $\Delta(G) = \deg(v)$.

Let $S = \{ v_{1,}, v_2 \dots \dots \dots . \dots, v_m \} \subseteq V(G)$ be the minimum set of vertices which covers all the vertices in G. Suppose $\forall \, v_r \in V - S$ is adjacent to exactly one vertex of S. Then S itself is a γ_p set of G.

Let $H = \{ v_{1,}, v_2 \dots \dots \dots . \dots, v_m \}$ be the vertex set of L (G). If $H_1 \subseteq H$ be the cutvertices of L(G) and $H_2 \subseteq H - H_1$. Assume every vertex $v_i \in$ V [L(G)] $- \{H_1 \cup H_2\}$ is adjacent to exactly one vertex of $\{H_1 \cup H_2\}$ and N $[\{H_1\} \cup \{H_2\}]$ = V[LG] . Hence $\{H_1 \cup H_2\}$ is a perfect dominating set of L(G). If $\langle$ V [L(G)] $- \{H_1 \cup H_2\} \rangle$ is regular. Clearly $\{H_1 \cup H_2\}$ is a γ_{cop} set of L(G) .Then $|\{H_1 \cup H_2\}| + \Delta(G) \geq |S| + |V_1|$ gives $\gamma_{cop}L(G) + \Delta(G) \geq \gamma_p(G) + C$.

A edge dominating set of K of a line graph L(G) is a co-regular edge dominating set if the induced subgraph $\langle E[L(G)] - K \rangle$ is regular . The co-regular edge domination number is the minimum cardinality of a co-regular edge dominating set of L(G) and is denoted by $\gamma^1_{cr} L(G)$.

Theorem 14: For any non trivial connected graph G, with $p \geq 3$ vertices , then

$\gamma_{cop}L(G) + \gamma(G) \geq \gamma^1_{cr} L(G)$ with $G \neq K_{1,p}$ if $p \geq 6$.

Proof: Suppose G = $K_{1,p}$, p ≥ 6 . Then $\gamma_{cop}L\{K_{1,p}\} = 1$

Hence $\gamma_{cop}L(G) + \gamma(G) < \gamma^1_{cr} L(G)$, a contradiction.

Let $S = \{v_{1,} v_2, \dots \dots \dots, v_n\} \subseteq (G)$ be the set of vertices with, deg $(v_i) \geq 2 \ \forall \, v_i \in S$, $1 \leq i \leq n$. Suppose there exists a set $S_1 \subseteq S$ with diam $(u, v) \geq 2$, $\forall \, u, v \in S_1$, which covers all the vertices in G. Clearly S_1 forms a dominating set of G.

Now consider $V_1 = \{ v_{1,} v_2, \dots \dots \dots, v_m \}$ be the vertex set of L(G).Suppose there exists $K \subseteq V_1$,every vertex of $V_1 - K$ is adjacent to exactly one vertex of K and N[K] = $V[L(G)]$. Then K is the minimal perfect dominating set of L(G). If $\langle V_1 - K \rangle$ is regular, then K is a γ_{cop} set of L (G). Otherwise select the set of vertices $\{v_k\} \in V_1 - K$ such that $\langle V_1 - K \cup \{v_k\} \rangle$ is regular.

Further let K = $\{ e_{1,} e_2, \dots . \dots, e_m \}$ be the set of edges of L(G). There exist $H \subseteq K[L(G)]$ such that $\forall \, e_i \in K - H$ is adjacent to atleast one edge of H. Then H is minimal edge dominating set of L(G) . If $\langle K - H \rangle$ is regular , then H is γ^1_{cr} set of $L(G)$. Hence $|K \cup \{v_k\}| \cup |S_1| \geq |H|$ gives $\gamma_{cop}L(G) + \gamma(G) \geq \gamma^1_{cr} L(G)$.

Theorem 15: For any connected graph G , $\gamma_{cop}L(G) \leq \gamma_{ss}(G) + \gamma_c(G)$.

Proof: Since $\gamma_t(G) \leq \gamma_{ss}(G)$ and by the Theorem 3, the result follows .

3. Conclusion

In this paper we derived selected results on Coregular Perfect domination in line graph .These results establish key relationship between the Coregular Perfect domination in line graph and other parameters, including the domination number, the edge domination number, split domination number and entire domination number of a simple, and undirected graph.

The concept of Coregular Perfect domination in line graph gives the regularity of the vertices of V[L(G)] - D. Where D is the perfect domination number of L(G). Here we have derived some general results on the concept of Coregular Perfect domination. Further its relationships with other different domination parameters were obtained.

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