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A Mathematical Model for Capturing Cholera Spread and Containment Options

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Abstract

The explosive nature of cholera epidemic over the years in different parts of the world has been a subject of interest to scientists in proffering interventions towards controlling its spread. Over the years many models has been created by the following people Capaso and Pavari – Fontana (1973), Codeco (2001), Hartley, Tien (2009), Mukandivare (2009) etc. In the present study, we modify the Cholera model proposed by Mukandivare incorporating three (3) containment options such as vaccination, Therapeutic treatment and water treatment and solved the system analytically using Homotopy Perturbation Method. The results shows that with improved use of vaccination, therapy and proper sanitation we have a more healthy population. This research is therefore recommended to modelers who desire to know how homotopy perturbation methods works. The computations were done and further analyzed mathematically using a computer symbolic package MAPLE 13.

Index Terms: Homotopy Perturbation Method, SIR model, Equilibrium, Stability Analysis, Reproduction number.

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1. Introduction

Cholera is a severe intestinal infection caused by the ingesting contaminated food substances and water with v. cholerae bacterium. From the 200 Vibrio Choleraesero groups, only vibrio cholera 01 and 0139 that as the primary cause of the cholera disease and having survived the gastric acid barrier of the stomach, it penetrates the mucus lining of the intestinal epithelial. It then colonise the small intestine to produce enterotoxin which results to watery diarrhoea and if left untreated, results to death in few hours. Recently, the infection dose was estimated to be 102 - 103 cells. Cholera is seen in its severe form, by the onset of serious watery diarrhoea which leading to severe dehydration and possible death. This etiological agent can remain in faeces without losing its infectious ability up to 7-14 days before shedding back to the environment.

Several mathematical models of cholera transmission dynamics have been formulated, studied and reported including Akinwande, N. I., Abubakar, S. (2013) Akinwande, N.I. (2006), Jiya, M. (2010), Ochoche Jeffrey M. A (2013), Benyah, F (2007). P.T. Tian, S. Liao, J. Wang (2010), WHO Weekly Epidemiological Reports from 2000 – 2013, Yibeltal Negussie Bayleyegn (2009), R.T. Ashleigh, J. Tien, M. Eisenberg, J.D. David, M.Junling, N.F. David (2011) ,Jin Wang and Chairat Modnak (2011). Notably among these is Capaso and Pavari – Fontana (1973) who pains takingly observed the spread of cholera epidemics in the Mediterranean using a deterministic model. In 2001, Code φ proposed a cholera model which explicitly accounts for the environmental factor i.e. the *v.cholerae* concentration in the water supply denoted by B, into a regular SIR system.

Hartley, Morris *et al*, in extension of Codeco's work proposed general model which took into consideration the different infective states of *v.cholerae*, consisting of five (5) equations describing the relationships between the susceptible, infectious and removed classes, the dynamics of the hyper infective state and lower infective states of the population of vibrio cholera assuming the total population *N*, is constant. In 2009, Joh, Wang et al, proposed a modification to the existing cholera model considering the density of the pathogen for the infected class in the population taking his bearing from the relationship between the humans, environment and the population of the vibrios in water reservoir. Mukandavire (2009), in furtherance of the Code φ Cholera model proposed a model to study the cholera outbreak which occurred in Zimbabwe in 2008/2009 incorporating the human – to human into the model, by which the basic reproduction number (R_o) was derived, and this work was insightful enough to providing containment options measures on controlling cholera outbreak globally.

The aim of this paper is to solve Cholera epidemiological model using Homotopy Perturbation Method that will help us to project forward in our quest to salvage the lives of people within the population under consideration. The objective of this study is: To study the mathematical equation for the spread of disease in a single host species, To solve the model equation analytically using Homotopy perturbation method, To apply the .ghmodel equation to solving the equilibrium and the stability point, To examine the proportion of the susceptible, Infected, Recovered and the over a period of time with and without the use of the three (3) containment options, To determine the Reproduction Number which will help us estimate the rate of spread with time.

The work employes the use of Homotopy perturbation Method to solve Mukandivare's Cholera Model incorporating three containment options viz: Vaccination, Therapeutic Treatment and Water Sanitation.The Homotopy Perturbation Method is an analytical method for solving linear /nonlinear differential equations. This series expansion method uses power series which transforms every non-linearities to series of linear equations. It uses the idea of the homotopy from topology to create a convergent series solution using a homotopy-Maclaurin series transform the nonlinearities in the system of differential equation. When two (2) continuous functions moves from one topological space to another and one deforms into another it is said to be homotopic.

2. Model Variables and Parameters

Table 1. Showing the Model Variable

The variables and parameters of the *Mukandivare's* model are defined below: *S(t)* = Number of susceptible individuals at time t *I(t)*= Number of infected individuals at time t *B(t)* = Concentration of v.cholerae in water at time t R*(t)* = Recovered individual at time t *H* = Initial human population

Table 2. Showing the Model Parameters

ρ= Human Birth rate/Recruitment rate ψ= Human death rate σ= Rate at which people recover from cholera ϕ = *Natural death rate of V.Cholerae* κ = Concentration of Vibrio Cholerae in food and water that yield 50% chance of catching cholera disease. ε = The rate of shedding of V.Cholerae by humans through untreated wastes α = Rate of injesting V.Cholerae from contaminated sources β = Rate of contacting V.cholerae human contact δ = Death rate due to the disease $\lambda = \frac{B}{K+B}$ = The incidence which determines the rate of new infection or the chance of catching cholera as a result of contact with contaminated food and water. The total population is given as H = S + I + R

2.1. Model Formulation

$$\frac{dS}{dt} = \rho H - \alpha \frac{B}{K+B} S - \beta SI - \psi S \tag{1}$$

$$\frac{dI}{dt} = \alpha \frac{B}{K+B} S + \beta SI - (\sigma + \delta + \psi) I \tag{2}$$

$$\frac{dB}{dt} = \varepsilon I - \phi B \tag{3}$$

$$\frac{dR}{dt} = \sigma I - \psi R \tag{4}$$

2.2. Model Modification

We modify the Mukandivare's Cholera Model by adding the following containment options:

1. Vaccination of the susceptible individuals
2. Therapeutic Treatment (use of Oral rehydration therapy, Antibodies, electrolyte e.t.c.)
3. Water Sanitation/Purification

From which we came up with the following assumptions which help further inclusions of these strategies into the Codeco Model.

(a) Introducing vaccination to the susceptible at the rate of q so that $q(t)$S individual per time are removed from the susceptible class to the recovered class.
(b) Applying therapeutic treatment to the infected at the rate of u, so that $u(t)I$ individual per time are removed from the infected class and added to the recovered class.
(c) Water sanitation leading to the death of vibrios (V.Cholerae) at the rate of z.

Assuming a closed population where H =1, and $\lambda = \frac{B}{K+B}$, for a given period of time. We have,

The methodology employed in this work is using Homotopy perturbation Method to solve Mukandivare's Cholera Model incorporating three containment options viz: Vaccination, Therapeutic Treatment and Water Sanitation.

The Homotopy Perturbation Method is an analytical method for solving linear /nonlinear differential equations. This series expansion method uses power series which transforms every nonlinearities to series of linear equations. It uses the idea of the homotopy from topology to create a convergent series solution using a homotopy-Maclaurin series transform the nonlinearities in the system of differential equation. When two (2) continuous functions moves from one topological space to another and one deforms into another it is said to be homotopic.

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The total population is given as H = S + I + R

3.1. Model Formulation

$$\frac{dS}{dt} = \rho H - \alpha \frac{B}{K+B} S - \beta SI - \psi S \tag{5}$$

$$\frac{dI}{dt} = \alpha \frac{B}{K+B} S + \beta SI - (\sigma + \delta + \psi) I \tag{6}$$

$$\frac{dB}{dt} = \varepsilon I - \phi B \tag{7}$$

$$\frac{dR}{dt} = \sigma I - \psi R \tag{8}$$

3.2. Model Modification

We modify the Mukandivare's Cholera Model by adding the following containment options:

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(b) Applying therapeutic treatment to the infected at the rate of *u*, so that *u(t)I* individual per time are removed from the infected class and added to the recovered class.
(c) Water sanitation leading to the death of vibrios (V.Cholerae) at the rate of *z*.

Assuming a closed population where H =1, and $\lambda = \frac{B}{K+B}$, for a given period of time. We have,

$$\frac{dS}{dt} = \rho - \alpha\lambda S - \beta SI - \psi S - q(t)S \tag{9}$$

$$\frac{dI}{dt} = \alpha\lambda S + \beta SI - (\sigma + \delta + \psi) I - u(t)I \tag{10}$$

$$\frac{dB}{dt} = \varepsilon I - \phi B - zB \tag{11}$$

$$\frac{dR}{dt} = \sigma I - \psi R + uI + qS$$

With the initial conditions S(0)=0, I(0) = 0, B(0) = 0 and R(0) = 0. Let

$S = x_0 + px_1 + p^2x_2 + \ldots$ …… (12)

$I = y_0 + py_1 + p^2y_2 + \ldots$ …… (13)

$R = z_0 + pz_1 + p^2z_2 + \ldots$ …… (14)

$B = q_0 + pq_1 + p^2q_2 + \ldots$ …… (15)

Applying Homotopy perturbation to (a) we have

$S = x_0 + px_1 + p^2x_2 + p^3x_3 + \ldots\ldots\ldots$ (16)

So that,

$S' = \frac{dS}{dt} = x_0' + px_1' + p^2x_2' + p^3x_3' + \ldots$ …… (17)

Applying Homotopy perturbation to (a) and multiplying each contributory rate with p

$\frac{dS}{dt} - p\frac{dS}{dt} - p[\rho + \beta SI + (\alpha\lambda + \psi + q)S] = 0$…… (18)

$(1-p)\frac{dS}{dt} + p[\frac{dS}{dt} - \rho + \beta SI + (\alpha\lambda + \psi + q)S] = 0$…… (19)

Substituting S and $\frac{dS}{dt}$ *below*

$(1-p)(x_0' + px_1' + p^2x_2' + \ldots) + p[(x_0' + px_1' + p^2x_2' + \ldots) - \rho + \beta(x_0 + px_1 + p^2x_2 + \ldots)(y_0 + py_1 + p^2y_2 + \ldots) + (\alpha\lambda + \psi + q)(x_0 + px_1 + p^2x_2 + \ldots) - \psi] = 0$…… (20)

$$x_0' + px_1' + p^2x_2' - px_0' - p^2x_1' - p^3x_2' + px_0' + p^2x_1' + p^3x_2' + p\beta\,(x_0y_0 + p\,x_0y_1 + p^2\,x_0y_2 + px_1y_0 + p^2\,x_1y_1 + p^3\,x_1y_2 + p^2\,x_2y_0 + p^3\,x_2y_1 + p^4\,x_2y_2 + \ldots.)(\alpha\lambda + \psi + q)(p\,x_0 + p^2\,x_1 + p^3\,x_2 + \ldots) - p\psi = 0$$

$$= x_0' + px_1' + p^2x_2' + \beta p\,x_0y_0 + \beta p^2\,x_0y_1 + \beta p^3\,x_0y_2 + \beta p^2x_1y_0 + \beta p^3\,x_1y_1 + \beta p^4\,x_1y_2 + \beta p^3\,x_2y_0 + \beta p^4\,x_2y_1 + \beta p^5\,x_2y_2 + (\alpha\lambda + \psi + q)\,(p\,x_0 + p^2\,x_1 + p^3\,x_2) - p\psi = 0$$ …… (21)

Collecting the coefficient powers of p we have

P^0: $\int x_0 = 0 = x(0) = S_0$ …… (22)

P^1: $x_1' + \beta\,x_0y_0 + (\alpha\lambda + \psi + q)\,x_0 - \rho = 0$ (23)

$$x_1' = (\rho - \beta x_0y_0 - (\alpha\lambda + \psi + q)\,x_0)$$

Integrating both sides we have with respect to t

$x_1 = (\rho - \beta S_0 I_0 - (\alpha\lambda + \psi + q) S_0)t + C_1$ …….. (24)

$x_1(0) = 0$, *therefore* $C_1 = 0$

$x_1 = (\rho - \beta S_0 I_0 - (\alpha\lambda + \psi + q) S_0)$ …….. (25)

$p^2: x_2' + \beta x_0 y_1 + \beta x_1 y_0 + (\alpha\lambda + \psi + q) x_1 = 0$ (26)

$\int x_2 = \int -[\beta(x_0 y_1 + x_1 y_0) + (\alpha\lambda + \psi + q) x_1]tdt$ (27)

Substituting x_1 *and* y_1 *in the above we have,*

$x_2 = -[\beta x_0(\alpha\lambda x_0 + \beta x_0 I_0 - (\sigma + \psi + \delta + u)I_0) + \beta y_0(\rho - \beta x_0 y_0 - (\alpha\lambda + \psi + q)x_0) +$

$(\alpha\lambda + \psi + q)(\rho - \beta x_0 y_0 - (\alpha\lambda + \psi + q) x_0]\, tdt$ (28)

$x_2 = -\frac{t^2}{2}[\beta S_0(\alpha\lambda S_0 + \beta S_0 I_0 - (\sigma + \psi + \delta + u)I_0) + \beta I_0(\rho - \beta I_0 y_0 - (\alpha\lambda + \psi + q) S_0) +$

$(\alpha\lambda + \psi + q)(\psi - \beta S_0 I_0 - (\alpha\lambda + \psi + q) S_0] + C_3$ …….. (29)

Since $x_2(0) = 0$, *therefore* $C = 0$ (30)

$x_2 = -\frac{t^2}{2}[\beta S_0(\alpha\lambda S_0 + \beta S_0 I_0 - (\sigma + \psi + \delta + u)I_0) + \beta I_0(\rho - \beta I_0 y_0 - (\alpha\lambda + \psi + q) S_0) +$

$(\alpha\lambda + \psi + q)(\psi - \beta S_0 I_0 - (\alpha\lambda + \psi + q) S_0] + C_3$ (31)

Therefore, the approximate solution for S(t)

$S(t) = S_0 + (\rho - \beta S_0 I_0 - (\alpha\lambda + \psi + q)S_0)t - -\frac{t^2}{2}[\beta S_0(\alpha\lambda S_0 + \beta S_0 I_0 - (\sigma + \psi + \delta + u)I_0) + \beta I_0(\rho - \beta I_0 y_0$
$-(\alpha\lambda + \psi + q) S_0) + (\alpha\lambda + \psi + q)(\psi - \beta S_0 I_0 - (\alpha\lambda + \psi + q) S_0] + C$…….. (32)

For*I(t)*

$\frac{dI}{dt} - \alpha\lambda S - \beta SI + (\sigma + u + \delta + \psi)I = 0$ …….. (33)

$I = y_0 + py_1 + p^2 y_2 + \ldots$ …….. (34)

$\frac{dI}{dt} = y_0' + py_1' + p^2 y_2' + \ldots$ …….. (35)

Applying Homotopy perturbation to (b) and multiplying each contributory rate with p

$$\frac{dI}{dt} - p\frac{dI}{dt} + p\left[\frac{dI}{dt} - \alpha\lambda S - \beta SI + (\sigma + u + \delta + \psi)I\right] = 0 \ \ldots\ldots\ldots\ldots (2.21) \quad (36)$$

After collecting like terms we have,

$$(1-p)\frac{dI}{dt} + p[\frac{dI}{dt} - \alpha\lambda S - \beta SI + (\sigma + u + \delta + \psi)I] = 0 \ \ldots\ldots\ldots \quad (37)$$

Multiplying each contributory rate with p and substituting I and $\frac{dI}{dt}$ *below*

$$(1-p)(y_0' + py_1' + p^2y_2' + \ldots) + p\,[(y_0' + py_1' + p^2y_2' + \ldots) - \alpha\lambda(x_0 + px_1 + p^2x_2 + \ldots) - \beta(x_0 + px_1 + p^2x_2 + \ldots)(y_0 + py_1 + p^2y_2 + \ldots) + (\sigma + u + \delta + \psi)(y_0 + py_1 + p^2y_2 + \ldots)] = 0 \ \ldots\ldots\ldots \quad (38)$$

Expanding further we have,

$$y_0' + py_1' + p^2y_2' - py_0' - p^2y_1' - p^3y_2' + py_0' + p^2y_1' + p^3y_2' - \alpha\lambda(px_0 + p^2x_1 + p^3x_2 + \ldots) - \beta(px_0y_0 + p^2x_0y_1 + p^3x_0y_2 + p^2x_1y_0 + p^3x_1y_1 + p^4x_1y_2 + p^3x_2y_0 + p^4x_2y_1 + p^5x_2y_2 + \ldots.) + (\sigma + u + \delta + \psi)(py_0 + p^2y_1 + p^3y_2 + \ldots) = 0 \ \ldots\ldots\ldots \quad (39)$$

Expanding further we have,

$$y_0' + py_1' + p^2y_2' - py_0' - p^2y_1' - p^3y_2' + py_0' + p^2y_1' + p^3y_2' - \alpha\lambda(px_0 + p^2x_1 + p^3x_2 + \ldots) - \beta(px_0y_0 + p^2x_0y_1 + p^3x_0y_2 + p^2x_1y_0 + p^3x_1y_1 + p^4x_1y_2 + p^3x_2y_0 + p^4x_2y_1 + p^5x_2y_2 + \ldots.) + (\sigma + u + \delta + \psi)(py_0 + p^2y_1 + p^3y_2 + \ldots) = 0 \ \ldots\ldots\ldots \quad (40)$$

Applying cancellation to the positive and negative power of p we have,

$$z_0' + pz_1' + p^2z_2' - \varepsilon(py_0 + p^2y_1 + p^3y_2 + \ldots) + (\phi + z)(pz_0 + p^2z_1 + p^3z_2 + \ldots) = 0 \ \ldots\ldots\ldots \quad (41)$$

Collecting the powers of *p*

P^0: $\int z_0' = 0 = z(0) = B_0 \ \ldots\ldots\ldots$ (42)

P^1: $z_1' - \varepsilon y_0 + (\phi + z)z_0 = 0 \ \ldots\ldots\ldots$ (43)

$$\int z_1' = \int[\varepsilon y_0 - (\phi + z)z_0]\,dt \quad (44)$$

$$z_1 = [\varepsilon y_0 - (\phi + z)z_0]t + C_1 \quad (45)$$

Since $z_1(0) = 0$, *therefore* $C_1 = 0$ (46)

$$z_1 = [\varepsilon I_0 - (\phi + z)B_0]t \ \ldots\ldots\ldots \quad (47)$$

P^2: $z_2' - \varepsilon y_1 + (\phi + z)z_1 = 0 \ \ldots\ldots\ldots$ (48)

$$\int z_2{}' = \int [\varepsilon y_1 - (\phi + z) z_1] \mathrm{t}\mathrm{d}t \tag{49}$$

Substituting x_1 *and* z_1 *in the above we have,*

$$\int z_2{}' = \int [\varepsilon\ (\alpha\lambda x_0 + (\beta x_0 - (\sigma + u + \delta + \psi)) y_0) - (\phi + z)[\varepsilon y_0 - (\phi + z) z_0] \mathrm{t}\mathrm{d}t \tag{50}$$

$$z_2 = \frac{t^2}{2} [\varepsilon(\alpha\lambda S_0 + \beta S_0 I_0 - (\sigma + u + \delta + \psi) I_0) - (\phi + z)[\varepsilon I_0 - (\phi + z) B_0] + C_2 \tag{51}$$

$z_2\ (t) = 0$, so $C_2 = 0$ (52)

$$z_2 = \frac{t^2}{2} [\varepsilon(\alpha\lambda S_0 + \beta S_0 I_0 - (\sigma + u + \delta + \psi) I_0) - (\phi + z)[\varepsilon I_0 - (\phi + z) B_0] \ldots\ldots\ldots \tag{53}$$

For the approximate solution of B(t) we have

$$B(t) = B_0 + [\varepsilon I_0 - (\phi + z) B_0] \mathrm{t} + \frac{t^2}{2} [\varepsilon(\alpha\lambda\, S_0 + \beta S_0 I_0 - (\sigma + u + \delta + \psi) I_0) - (\phi + z)[\varepsilon I_0 - (\phi + z) B_0] \tag{54}$$

For R(t),

$$\frac{dR}{dt} - (\sigma + u) I + uR - qS = 0 \ldots\ldots\ldots \tag{55}$$

$R = q_0 + pq_1 + p^2 q_2 + \ldots \ldots\ldots\ldots$ (56)

$$\frac{dR}{dt} = q_0{}' + pq_1{}' + p^2 q_2{}' + \ldots \ldots\ldots\ldots \tag{57}$$

$$\frac{dR}{dt} - p\frac{dR}{dt} + p\left[\frac{dR}{dt} - (\sigma + u) I + uR - qS\right] = 0 \ldots\ldots\ldots\ldots \tag{58}$$

After collecting like terms we have,

$$(1 - \mathrm{p})\frac{dR}{dt} + \mathrm{p}[\frac{dR}{dt} - (\sigma + u) I + uR - qS] = 0 \ldots\ldots\ldots\ldots \tag{59}$$

Substituting R and $\frac{dR}{dt}$ below, we have

$$(1 - \mathrm{p})(q_0{}' + pq_1{}' + p^2 q_2{}' + \ldots) + p[(q_0{}' + pq_1{}' + p^2 q_2{}' + \ldots) - (\sigma + u)(y_0 + py_1 + p^2 y_2 + \ldots) + \psi\ (q_0 + pq_1 + p^2 q_2 + \ldots) - q(x_0 + px_1 + p^2 x_2 + \ldots)] = 0 \ldots\ldots\ldots\ldots \tag{60}$$

Expanding this further,

$$q_0{}' + pq_1{}' + p^2 q_2{}' - pq_0{}' - p^2 q_1{}' - p^3 q_2{}' + pq_0{}' + p^2 q_1{}' + p^3 q_2{}' - (\sigma + u)(py_0 + p^2 y_1 + p^3 y_2 + \ldots) + \psi(pq_0 + p^2 q_1 + p^3 q_2 + \ldots) - q(px_0 + p^2 x_1 + p^3 x_2 + \ldots) = 0 \tag{61}$$

Applying cancellation we have,

$q_0' + pq_1' + p^2q_2' - (\sigma+u)(py_0 + p^2y_1 + p^3y_2 + \ldots) + \psi(pq_0 + p^2q_1 + p^3q_2 + \ldots) - q(px_0 + p^2x_1 + p^3x_2 + \ldots) = 0$ …….. (62)

Collecting the coefficient powers of p, we have

P^0: $\int q_0' = 0 = q(0) = R_0$ …….. (63)

P^1: $q_1' - (\sigma+u)y_0 + \psi q_0 - qx_0 = 0$ …….. (64)

$$\int q_1' = \int[(\sigma+u)y_0 - \psi q_0 + qx_0]dt \quad (65)$$

$$q_1(t) = [(\sigma+u)y_0 - \psi q_0 + qx_0]t + C_1 \quad (66)$$

Where $q_1(t) = 0$ therefore $C_1 = 0$

$$q_1(t) = [(\sigma+u)I_0 - \psi R_0 + qS_0]t \quad \ldots\ldots\ldots \quad (67)$$

P^2: $q_2' - (\sigma+u)y_1 + \psi q_1 - qx_1 = 0$ (68)

$$\int q_2' = \int[(\sigma+u)y_1 - \psi q_1 + qx_1]dt \quad (69)$$

$$\int q_2' = \int[(\sigma+u)y_1 - \psi q_1 + qx_1]dt \quad (70)$$

Substituting x_1 *and* $y_1$as previously derived

$$\int q_2' = \int[(\sigma+u)(\alpha\lambda x_0 + \beta x_0 y_0 - (\sigma+u+\delta+\psi)y_0) - \psi[(\sigma+u)y_0 - \psi q_0 + qx_0] + q(\psi - \beta S_0 I_0 + (\alpha\lambda + \psi + q)x_0)]dt \quad (71)$$

$$q_2 = \frac{t^2}{2}[(\sigma+u)(\alpha\lambda S_0 + \beta S_0 I_0 - (\sigma+u+\delta+\psi)I_0) - \psi[(\sigma+u)I_0 - \psi R_0 + qS_0] + q(\psi - \beta S_0 I_0 - (\alpha\lambda + \psi + q)S_0)] + C_3 \quad (72)$$

At t=0, $q_2(0) = C_3 = 0$,

$$q_2 = \frac{t^2}{2}[(\sigma+u)(\alpha\lambda S_0 + \beta S_0 I_0 - (\sigma+u+\delta+\psi)I_0) - \psi[(\sigma+u)I_0 - \psi R_0 + qS_0] + q(\rho - \beta S_0 I_0 - (\alpha\lambda + \psi + q)S_0)] \ldots\ldots\ldots \quad (73)$$

The approximate system of R(t) is given as follows:

$$R(t) = R_0 + [(\sigma+u)I_0 - \psi R_0 + qS_0]t + \frac{t^2}{2}[(\sigma+u)(\alpha\lambda S_0 + \beta S_0 I_0 - (\sigma+u+\delta+\psi)I_0) - \psi[(\sigma+u)I_0 - \psi R_0 + qS_0] + q(\rho - \beta S_0 I_0 + (\alpha\lambda + \psi + q)S_0)] \ldots\ldots\ldots \quad (74)$$

4. Equilibria

Referring to the previous equations (3.1-3.4) that was solved using homotopy perturbation we say:

$$\frac{dS}{dt} = \rho H - \alpha \frac{B}{K+B} S - \beta SI - \psi S - qS \ldots\ldots\ldots \quad (75)$$

$$\frac{dI}{dt} = \alpha \frac{B}{K+B} S + \beta SI - (\sigma + \delta + \psi)I - uI \ldots\ldots\ldots \quad (76)$$

$$\frac{dB}{dt} = \varepsilon I - \phi B - zB \ldots\ldots\ldots \quad (77)$$

$$\frac{dR}{dt} = \sigma I - \psi R + uI + qS \ldots\ldots\ldots \quad (78)$$

At the equilibrium state

$$\frac{dS}{dt} = \frac{dI}{dt} = \frac{dB}{dt} = \frac{dR}{dt} = 0 \quad (79)$$

all vanish, equating the rhs of the above equation (3.18) to zero. Then the steady state of the system above therefore satisfy the following algebraic system of equation. Assuming a closed population where H = 1.

We reduce this further by

$$\lambda = \frac{B}{K+B} \quad (80)$$

$$\rho - \alpha\lambda S - \beta SI - \psi S - qS = 0 \ldots\ldots \quad (81)$$

$$\alpha\lambda S + \beta SI - (\sigma + \delta + \psi)I - uI = 0 \ldots\ldots \quad (82)$$

$$\varepsilon I - \phi B - zB = 0 \ldots\ldots\ldots \quad (83)$$

$$\sigma I - \psi R + uI + qS = 0 \ldots\ldots \quad (84)$$

From (82)

$$I = \frac{\alpha\lambda S}{(\sigma - \beta S + \psi + \delta + u)} \ldots\ldots\ldots \quad (85)$$

From (84)

$$R = \frac{(\sigma+u)I + qS}{\psi} \ldots\ldots\ldots \quad (86)$$

From (81)

$$S = \frac{\rho}{(\beta I + \alpha\lambda + q + \psi)} \ldots\ldots\ldots \quad (87)$$

From (83)

$$B = \frac{\varepsilon I}{(\phi+z)} \ldots\ldots\ldots \quad (88)$$

For

$$\lambda = \frac{B}{K+B}, \quad (89)$$

Substituting it in

$$I = \frac{\alpha BS}{(K+B)(\sigma - \beta S+u+\delta+\psi)} \ldots\ldots\ldots \quad (90)$$

Hence

$$S = \frac{\rho}{\beta\left[\frac{\alpha BS}{(K+B)(\sigma-\beta S+u+\delta+\psi)}\right]+\alpha\left[\frac{B}{K+B}\right]+q+\psi} \ldots\ldots\ldots \quad (91)$$

At Disease Free State B =I = 0 where no outbreak occurs, which means the state where the susceptible and the infected are assumed to be zero.

$$S = \frac{\rho}{q+\psi} \ldots\ldots\ldots \quad (92)$$

Analogously,

$$R = \frac{qS}{\psi} \ldots\ldots\ldots \quad (93)$$

On substituting S in R we have

$$R = \frac{q}{(q+\psi)} \ldots\ldots\ldots \quad (94)$$

Therefore the Disease Free Equilibrium is given thus as:

$$(S^0, I^0, R^0, B^0) = \left(\frac{\rho}{q+\psi}, 0, 0, \frac{q}{(q+\psi)}\right) \ldots\ldots\ldots \quad (95)$$

5. The Stability Analysis

If we introduce a small fraction of infected people into this population will there be a Disease Free State? Recently, it was discovered that, only 10% cases of cholera shows up with vomiting and diarrhea symptoms but most often the rate at which the infected sheds *Vibrio Cholerae* to the environment increases the rate of spread bringing about a pandemic situation. Akor (2007).We obtain the determinant of the Jacobian as follows:

The Jacobian matrix is therefore given by

$$J(S,I,B,R) = \begin{bmatrix} -(\alpha\lambda + \beta SI + \psi + q) & \beta S & -\frac{\alpha s\kappa}{(B+\kappa)^2} & 0 \\ \frac{\alpha B}{(B+\kappa)} + \beta I & -(\sigma+\psi+\delta+u)+\beta S & \frac{\alpha s\kappa}{(B+\kappa)^2} & 0 \\ 0 & \varepsilon & -(\varphi+z) & 0 \\ q & (\sigma+u) & 0 & -\psi \end{bmatrix} \quad (96)$$

The equation above when we apply the values of S, I, B and R derived at the Disease Free State gives the following.

$$J\left(\frac{\psi}{q+\psi}, 0, 0, \frac{q}{q+\psi}\right) = \begin{bmatrix} -(\alpha\lambda + \psi + q) & \beta S & -\frac{\alpha s}{\kappa} & 0 \\ 0 & -(\sigma+\psi+\delta+u)+\beta S & \frac{\alpha s}{\kappa} & 0 \\ 0 & \varepsilon & -(\varphi+z) & 0 \\ q & (\sigma+u) & 0 & -\psi \end{bmatrix} \quad (97)$$

We define our the above thus:

$$d_1 = (\alpha\lambda + \psi + q) \quad (98)$$

$$d_2 = (\sigma + \psi + \delta + u) \quad (99)$$

$$d_3 = (\phi + z) \quad (100)$$

$$d_4 = (\sigma + u) \quad (101)$$

6. Reproduction Number (R_0)

Reproduction Number as defined by Fraser et al (2011), is the number of secondary infections generated by a primary infection in susceptible class of the population. This parameter is used in determining the likeliness of an infectious disease spreading through a given population or becoming extinct after some time in a population also known *as R_0 Number*. From this, we have some basic assumptions about R_0 number: If $R_0<1$ the infection will die out in the long run. If $R_0>1$ the infection will spread in the population.From the above Jacobian Matrix we have that

$$J = \begin{bmatrix} -d_1 & \beta S & -\frac{\alpha s}{\kappa} & 0 \\ 0 & \beta S - d_2 & \frac{\alpha s}{\kappa} & 0 \\ 0 & \varepsilon & -d_3 & 0 \\ q & d_4 & 0 & -\psi \end{bmatrix} \quad (102)$$

To evaluate this further, using the *Van Den Driessche and Watmough Method* the associated next generation

matrices. Therefore we adopt the later as follows. The Reproduction Number (R_0) is the spectral radius of the next generation matrix derived from the infected classes i.e. $R_0 = \rho(K)$. ρ = spectral radius and K is the next generation matrix given as $K = FV^{-1}$

$$F = \begin{bmatrix} \beta S & 0 \\ 0 & 0 \end{bmatrix} \tag{103}$$

$$V = -\begin{bmatrix} -d_2 & 0 \\ \varepsilon & -d_3 \end{bmatrix} = \begin{bmatrix} d_2 & 0 \\ -\varepsilon & d_3 \end{bmatrix} \tag{104}$$

We see that F is derived from the Infected Classes (I and B) and V from the transition term i.e. the remaining term after the evaluating F from the infected classes.Therefore we find V^{-1} (transposing V and multiplying by the adjoint matrix)

$$V^{-1} = \frac{1}{d_2 d_3}\begin{bmatrix} d_3 & 0 \\ -\varepsilon & d_2 \end{bmatrix} = \begin{bmatrix} \frac{d_3}{d_2 d_3} & 0 \\ \frac{-\varepsilon}{d_2 d_3} & \frac{d_2}{d_2 d_3} \end{bmatrix} = \begin{bmatrix} \frac{1}{d_2} & 0 \\ \frac{-\varepsilon}{d_2 d_3} & \frac{1}{d_2} \end{bmatrix} \tag{105}$$

$$FV^{-1} = \begin{bmatrix} \beta S & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \frac{1}{d_2} & 0 \\ \frac{-\varepsilon}{d_2 d_3} & \frac{1}{d_2} \end{bmatrix} = \begin{bmatrix} \frac{\beta S}{d_2} & 0 \\ 0 & 0 \end{bmatrix} \tag{106}$$

Therefore, we determine the spectral radius we consider the determinant of $[K - \lambda I] = 0$ and the largest Eigen value is the spectral radius.

$$\begin{bmatrix} \frac{\beta S}{d_2} & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = 0 \tag{107}$$

$$\begin{bmatrix} \frac{\beta S}{d_2} - \lambda_1 & 0 \\ 0 & -\lambda_2 \end{bmatrix} = 0 \tag{108}$$

$$\frac{\beta S}{d_2} - \lambda_1 = 0 \tag{109}$$

$$\lambda_1 = \frac{\beta S}{d_2} \tag{110}$$

$$\therefore \lambda_2 = 0 \tag{111}$$

Thus since the largest of the Eigen value after is the Spectral radius,

$$\lambda_1 = \frac{\beta S}{d_2} \tag{112}$$

Since the largest of the eigen value is λ_1 then the Effective Reproduction Number *(R_0)* is $\frac{\beta S}{d_2}$. We further justify stability at the Disease Free State by saying the Reproduction Number, $R_0 < 1$.

Therefore considering the Jacobian previously derived we have

$$J(S,I,B,R)=\begin{bmatrix} -d_1 & \beta S & -\frac{\alpha s}{\kappa} & 0 \\ 0 & \beta S-d_2 & \frac{\alpha s}{\kappa} & 0 \\ 0 & \varepsilon & -d_3 & 0 \\ q & d_4 & 0 & -\psi \end{bmatrix} \quad (113)$$

Therefore, we use Row operations to transform all the values below the diagonal matrix to zero.

$$\text{New } R_{3,2} = \text{Old } R_{3,2} - \left(\frac{\varepsilon}{\beta S - d_2}\right) R_{3,2} \quad (114)$$

$$= \varepsilon - \left(\frac{\varepsilon}{\beta S - d_2}\right)(\beta S - d_2) \quad (115)$$

$$= \varepsilon - \varepsilon$$

$$= 0$$

$$\text{New } R_{3,3} = \text{Old } R_{3,3} - \left(\frac{\varepsilon}{\beta S - d_2}\right) R_{2,3} \quad (116)$$

$$= -d_3\left[-\left(\frac{\varepsilon}{\beta S - d_2}\right)\right] + \frac{\alpha S}{k} \quad (117)$$

$$= -d_3 - \frac{\alpha \varepsilon S}{k(\beta S - d_2)} \quad (118)$$

$$= -\left(d_3 + \frac{\alpha \varepsilon S}{k(\beta S - d_2)}\right) \quad (119)$$

$$= -\left[\frac{\kappa d_3(\beta S - d_2) + \alpha \varepsilon S}{\kappa d_3(\beta S - d_2)}\right]$$

$$J(S,I,R,B) = \begin{bmatrix} -d_1 & \beta S & -\frac{\alpha s}{\kappa} & 0 \\ 0 & \beta S - d_2 & \frac{\alpha s}{\kappa} & 0 \\ 0 & 0 & -\left[\frac{\kappa d_3(\beta S - d_2) + \alpha \varepsilon S}{\kappa d_3(\beta S - d_2)}\right] & 0 \\ q & d_4 & 0 & -\psi \end{bmatrix} \quad (120)$$

$$\text{New } R_{4,1} = \text{Old } R_{4,1} + \left(\frac{q}{d_1}\right) R_{1,1} \quad (121)$$

$$= q + \left(\frac{q}{d_1}\right)(-d_1) \quad (122)$$

$$= q - q$$

$$= 0$$

$$\text{New } R_{4,2} = \text{Old } R_{4,2} + \left(\frac{q}{d_1}\right)\beta S \tag{123}$$

$$= d_4 + \frac{q\beta S}{d_1} = \frac{d_1 d_4 + q\beta S}{d_1} \tag{124}$$

$$\text{New } R_{4,3} = \text{Old } R_{4,3} + \left(\frac{q}{d_1}\right) - \frac{\alpha S}{\kappa} \tag{125}$$

$$= 0 + \left(-\frac{\alpha q S}{\kappa d_1}\right) = -\frac{\alpha q S}{\kappa d_1} \tag{126}$$

Updating the results of the new entries in a Jacobian Matrix

$$J(S, I, B, R) = \begin{bmatrix} -d_1 & \beta S & -\frac{\alpha S}{\kappa} & 0 \\ 0 & \beta S - d_2 & \frac{\alpha S}{\kappa} & 0 \\ 0 & 0 & -\left[\frac{\kappa d_3 (\beta S - d_2) + \alpha \varepsilon S}{\kappa d_3 (\beta S - d_2)}\right] & 0 \\ 0 & \frac{d_1 d_4 + q\beta S}{d_1} & \frac{-\alpha q S}{\kappa d_1} & -\psi \end{bmatrix} \tag{127}$$

$$\text{New } R_{4,2} = \text{Old } R_{4,2} - \left[\frac{d_1 d_4 + q\beta S}{d_1 \beta S}\right]\beta S \tag{128}$$

$$\text{New } R_{4,2} = \text{Old } R_{4,2} - \left[\frac{d_1 d_4 + q\beta S}{d_1 \beta S}\right]\beta S \tag{129}$$

$$= \frac{d_1 d_4 + q\beta S}{d_1} - \frac{d_1 d_4 + q\beta S}{d_1} \tag{130}$$

$$= 0$$

$$\text{New } R_{4,3} = \text{Old } R_{4,3} + \left(\frac{q}{d_1}\right)\beta S \tag{131}$$

$$= d_4 - \left[\frac{d_1 d_4 + q\beta S}{d_1 \beta S}\right] - \frac{\alpha S}{\kappa} \tag{132}$$

$$= \frac{-\alpha q S}{\kappa d_1} + \frac{\alpha S[d_1 d_4 + q\beta S]}{\kappa d_1 \beta S} \tag{133}$$

$$= \frac{\alpha[d_1 d_4 + q\beta S] - \alpha\beta q S}{\kappa d_1 \beta} \tag{134}$$

Updating the latest entries we have

$$J = \begin{bmatrix} -d_1 & \beta S & -\frac{\alpha s}{\kappa} & 0 \\ 0 & \beta S - d_2 & \frac{\alpha s}{\kappa} & 0 \\ 0 & 0 & -\left[\frac{\kappa d_3 (\beta S - d_2) + \alpha \varepsilon S}{\kappa d_3 (\beta S - d_2)}\right] & 0 \\ 0 & \frac{d_1 d_4 + q\beta S}{d_1} & \frac{\alpha(d_1 d_4 + q\beta S) - \alpha\beta qS}{\kappa d_1 \beta} & -\psi \end{bmatrix} \quad (135)$$

New $R_{4,3}$ = Old $R_{4,3}$+ $(\frac{q}{d_1})\beta S$

$$= d_4 + -\left[\frac{\kappa(\beta S - d_2)[\alpha d_1 d_4 + \alpha\beta qS) - \alpha\beta qS]}{\kappa d_1 \beta[\kappa d_3(\beta S - d_2) + \alpha\varepsilon S]}\right]\left[\frac{\kappa d_3 (\beta S - d_2) + \alpha\varepsilon S}{\kappa d_3 (\beta S - d_2)}\right] \quad (136)$$

After cancellation it reduces to:

$$= \frac{\alpha d_1 d_4}{\kappa d_1 \beta} - \frac{\alpha d_1 d_4}{\kappa d_1 \beta} \quad (137)$$

$$= 0$$

Updating the new entries and taking the characteristic matrix $|A - \lambda I|$ =0

$$J = \begin{bmatrix} -d_1 & \beta S & -\frac{\alpha s}{\kappa} & 0 \\ 0 & \beta S - d_2 & \frac{\alpha s}{\kappa} & 0 \\ 0 & 0 & -\left[\frac{\kappa d_3 (\beta S - d_2) + \alpha\varepsilon S}{\kappa d_3 (\beta S - d_2)}\right] & 0 \\ 0 & 0 & 0 & -\psi \end{bmatrix} - \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{bmatrix} = 0 \quad (138)$$

$$J = \begin{bmatrix} -(d_1 + \lambda_1) & \beta S & -\frac{\alpha s}{\kappa} & 0 \\ 0 & (\beta S - d_2) - \lambda_2 & \frac{\alpha s}{\kappa} & 0 \\ 0 & 0 & -\left[\frac{\kappa d_3 (\beta S - d_2) + \alpha\varepsilon S}{\kappa d_3 (\beta S - d_2)} + \lambda_3\right] & 0 \\ 0 & 0 & 0 & -(\psi + \lambda_4) \end{bmatrix} \quad (139)$$

$$-d_1 - \lambda_1 = 0$$
$$\Rightarrow \lambda_1 = -d_1 \prec 0..... \quad (140)$$

$$\beta S - d_2 - \lambda_2 = 0$$
$$\Rightarrow \lambda_2 = \beta S - d_2 \neq 0 \quad \ldots\ldots\ldots \tag{141}$$

$$-\left[\frac{\kappa d_3(\beta S - d_2) + \alpha\varepsilon S}{\kappa d_3(\beta S - d_2)}\right] - \lambda_3 = 0 \Rightarrow$$
$$\lambda_3 = -\left[\frac{\kappa d_3(\beta S - d_2) + \alpha\varepsilon S}{\kappa d_3(\beta S - d_2)}\right] \prec 0 \ldots\ldots\ldots \tag{142}$$

$$-\psi - \lambda_4 = 0$$
$$\Rightarrow \lambda_4 = -\psi \prec 0 \ldots\ldots\ldots\ldots \tag{143}$$

From (141)

$$\beta S - d_2 = \lambda_2 \neq 0$$
$$\lambda_2 \prec 0$$
$$\Rightarrow iff \beta S \prec d_2$$
$$\frac{\beta S}{d_2} \prec \frac{d_2}{d_2}$$
$$\frac{\beta S}{d_2} \prec 1 \tag{144}$$

Where $R_0 < 1$ (145)

Hence since the Reproduction Number $R_0 < 1$ Disease Free Equilibrium is locally stable.

7. Results

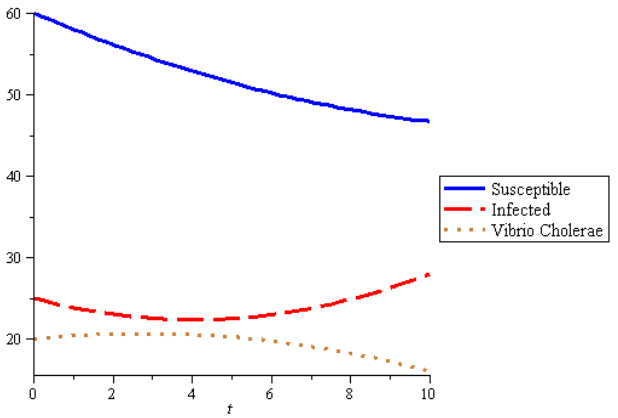

Fig. 4.1: The Phase Portrait of equation (3.3.1) without Control Strategy.
Taking S=60, I=25, B=20, ρ=0.02, ψ=0.01, β=0.001, α= 0.015,
λ=0.01, q=0.04, σ=0.1, e=0.1, φ=0.1, z=0.01, δ=0.002

Fig.1. Phase Portrait of the System (3.3) with Respect to Time without Applying Containment Parameters. S=60, I=25, B=20, R=15

Table 3. Showing the Population Dynamics When No Containment Parameter is used: For S=60, I=25 and B=20

Time	S(t)	I(t)	B(t)
0	60.00000000	25.00000000	20.0000000
1	57.98644418	23.72491183	20.1404500
2	56.12377670	22.05447300	20.6418000
3	54.41997580	21.55906425	20.6940500
4	52.85110680	20.38189200	20.6940500
5	51.85110980	18.94279563	20.2612500
6	50.01819903	17.82682570	19.7762000
7	49.07376458	16.74267900	19.1120500
8	48.11642720	15.69035680	18.2688000
9	47.30997818	14.66985783	17.2464500
10	46.65444175	13.68132500	16.0450000

Table 4. The Population Dynamics using Different Values of Vaccination Parameter Q, on the Susceptible.

Time	q(0)	q(0.01)	q(0.02)	q(0.04)
0	60.00000000	60.000000000	60.000000000	60.000000000
1	57.98644418	57.41093418	56.84042418	55.71840418
2	56.1237767	55.01973670	53.93969670	51.85160700
3	54.4199758	52.82790000	50.09781758	48.39961575
4	52.8511068	50.83494680	48.91478680	45.36242800
5	51.8511098	49.04085438	46.9147868	42.74010438
6	50.01819903	47.4456303	44.92527300	40.53246300
7	49.07376458	46.04927458	43.31878458	39.00104561
8	48.1164272	44.8517872	41.9711472	38.73968575
9	47.30997818	43.85316818	40.88235818	36.39854175
10	46.65444175	43.0534175	40.0524175	35.8504175

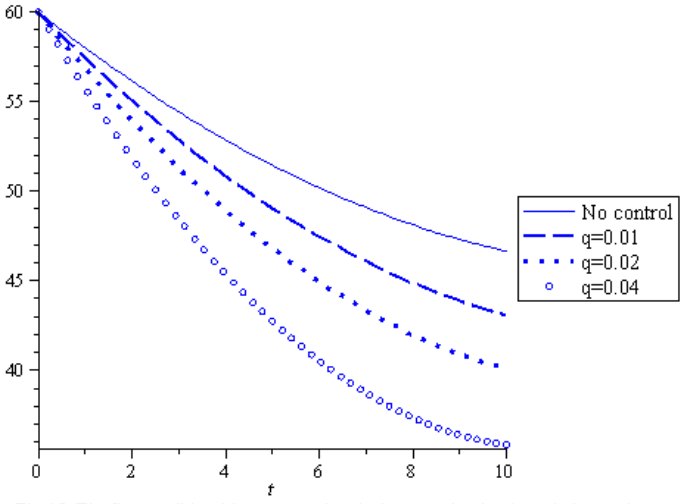

Fig 4.2: The Susceptible without control and when vaccination is varied over time.
Taking S=60, I=25, B=20, ρ=0.02, ψ=0.01, β=0.001, α= 0.015,
λ=0.01, q=0.04, σ=0.1, ε=0.1, φ=0.1, z=0.01, δ=0.002

Fig.2. Graph of the Susceptible S(t) Against to Time for Different Values of Vaccination Parameter q.

Table 5. The Population Dynamics using Different Values of Therapeutic Treatment Parameter U on the Infected.

Time	u(0.01)	u(0.02)	u(0.04)
0	25.00000000	25.00000000	25.00000000
1	23.48050183	23.5708273	22.78811683
2	22.0040073	21.5708273	20.73446730
3	20.57051643	19.97086143	18.83905143
4	19.18002920	18.4473092	17.10186920
5	17.83254563	17.0017063	15.52292063
6	16.52806570	16.0614457	14.10220570
7	15.26658943	14.33513443	12.83972443
8	14.0481168	13.1172368	11.73547680
9	12.872264783	11.97575283	10.78946283
10	11.7401825	10.9106825	10.0016825

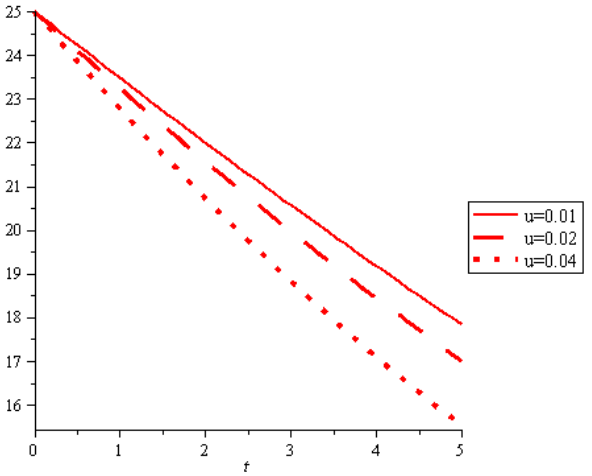

Fig. 4.3: Plots I (t) against time for different values of therapeutic treatment parameter

Fig.3. Graph of the Infected, I (t) Against Time for Different Values of Therapeutic Treatment Parameter u.

Table 6 The Population Dynamics using Different Values of Water Sanitation Parameter Z.

Time	Z	z(0.01)	z(0.02)	z(0.04)
0	20.00000	20.00000	20.000000	20. 00000000
1	20.14045	20.38105	20.029450	19.65645000
2	20.6418	20.17055	19.917800	19.2258000
3	20.69405	19.90320	19.665050	18.7080500
4	20.69405	19.90320	18.736230	18.103200
5	20.26125	19.47375	18.736250	17.411250
6	19.77620	18.88220	18.060200	16.632200
7	19.11205	18.12825	17.243050	15.766050
8	18.26880	17.21280	16.284800	14.812800
9	17.24645	16.13495	15.185450	13.772450
10	16.04500	14.89500	13.945000	12.645000

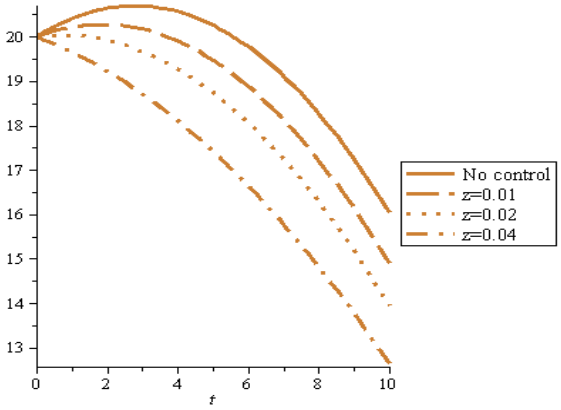

Fig4 .4: Plot of B(t) against time showing the death rate of the vibrios in water for different values of water treatment parameter.

Fig.4. Plot of B (t) Against to Time Showing Death Rate of the Vibrios in Water for Different Values Water Treatment.

Table 7. The Population of the Recovered using Different Values of Vaccination parameter q.

Time	**q(0.01)**	**q(0.02)**	**q(0.04)**
0	15.000000	15.000000	15.000000
1	17.857255	18.23481	19.57192
2	20.529020	21.63924	23.78768
3	23.015295	24.61329	27.64728
4	25.316080	27.35696	31.15072
5	27.431375	28.87025	34.29800
6	29.361180	32.15316	37.08912
7	31.105495	34.20569	39.52408
8	32.664320	36.02784	41.60288
9	34.037655	37.61961	43.32552
10	35.225500	38.98100	44.69200

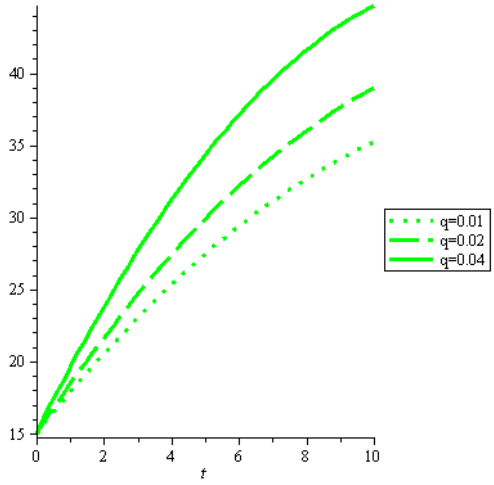

Fig 4.5 Showing varied recovery rates using vaccines

Fig.5. Plot Showing Varied Recovery Rates using Vaccines From 1% to 4% showing that an Improved use of Vaccines on the Susceptible Increase the Level of Immunity of the People.

Table 8. The Population of the Recovered using Different Values of Therapeutic Parameter u.

Time	**u(0.01)**	**u(0.02)**	**u(0.04)**
0	15.0000000	15.0000000	15.0000000
1	17.253495	17.7282900	18.17288
2	19.313968	20.213160	20.99152
3	21.181455	22.45261	23.45592
4	22.85592	24.45261	25.56608
5	24.337375	26.20725	27.32200
6	25.625830	27.71844	28.72368
7	26.721255	28.98210	29.77112
8	27.623680	30.01056	30.46432
9	28.334095	30.79149	30.80328
10	28.849500	31.3290	31.7882

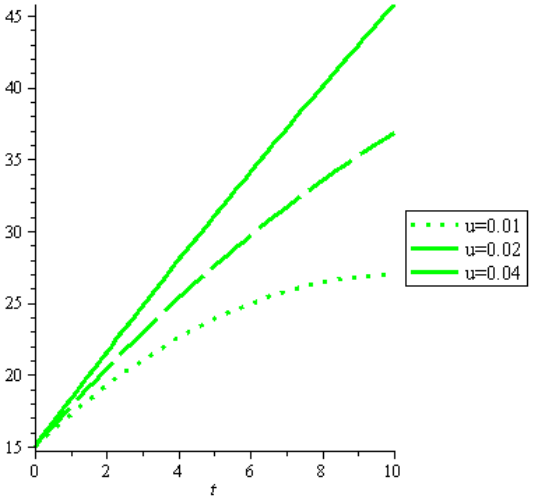

Fig. 4.6: Showing the varied recovery rates using therapy

Fig.6. Plot of Varied Recovery Rates using Therapy from 1% to 4% Showing an Improved use of Therapy on the Infected Leads to Higher Recovery. For S= 60, I=25, B=20, R=15

Table 9. Showing a Healthy Population Dynamics When All the 3 Interventions Have Been Used

Time	S(t)	I(t)	B(t)	R(t)
0	60.00000000	25.000000000	20. 00000000	15. 00000000
1	56.84042193	22.788116830	19.656450000	19.571920000
2	53.93968770	20.734467300	19.225800000	23.787680000
3	51.29779733	18.839051430	18.708050000	27.787680000
4	48.91475080	17.101860514	18.103200000	31.150720000
5	46.79054813	15.522920630	17.411250000	34.298000000
6	44.92518930	14.102205700	16.632200000	37.089120000
7	43.31867433	12.839722443	15.766050000	39.524080000
8	41.97100320	11.735476800	14.812800000	41.602880000
9	40.88217593	10.789462830	13.772450000	43.325520000
10	40.05219250	10.001682500	12.645000000	44.692000000

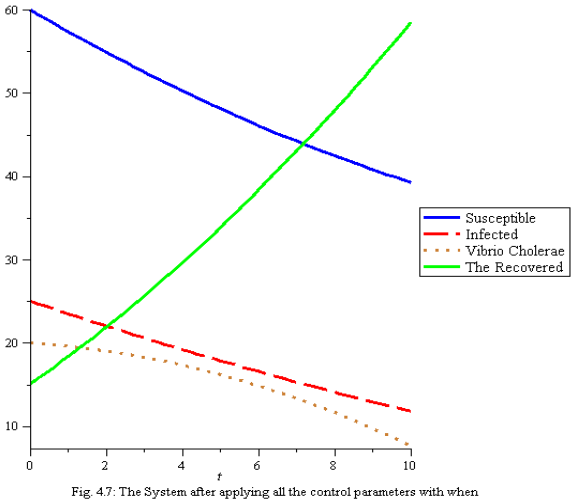

Fig. 4.7: The System after applying all the control parameters with when

Fig.7. An Immune Human Population after Applying All the Containment Parameters.

8. Discussion of Results

Figure 4.1 shows the combined system of the Susceptible S(t), the Infected I(t), the Vibrio Cholerae in water and B(t), showing that the susceptible will decrease due as a result vulnerability to the cholera epidemics by contaminated water, interaction of the susceptible people without taking health precaution such as personal hygiene and water sanitation as they relate with the infected. The Infected decreased to a certain point because of the natural death rate of the of the infected and death due to the disease and increased because more people from the susceptible class for instance, those who had taken a certain level of the vibrios in their system (10^6 cells of v.cholerae) from contaminated sources are getting infected and those who through relationship with the infected without taken adequate health precautions are getting infected.

Figure 4.2 displays the susceptible population, with improved use of vaccination.

Figure 4.3 displays the infected population dynamics with improved use of therapy

Figure 4.4 displays the dynamics of the vibrios in water with improved water leading to more death of the vibrios. This means that availability of portable water increases the immunity of people to the prevalent epidemic.

Figure 4.5 displays the recovery rates ranging vaccination from 1% – 4% showing that with increase use of vaccination on the susceptible class more people will become more immune to the disease.

Figure 4.6 displays the recovery rates using varied level of therapeutic treatment on the infected from 1% to 4% revealing showing that with improved therapy more people leave the infected class to the recovered class.

Figure 4.7 shows a wholesome system with dynamics of the population over time showing decreased level of susceptibility, infectivity, decreased population of the vibrios due to water sanitation and increased number of immune people in the recovered class which connotes a more healthy population.

9. Conclusion

The use of Homotopy Perturbation helped us to derive the approximate solution of each of the four (4) compartments considered in this work, which resulted in a quadratic equation which was varied with respect to time on a computer symbolic package, Maple and the results were presented graphically and shown with tables. The approximate solution derived was used to present the model graphically to enhance better understanding of

the dynamics of the interaction between the four classes. It is obvious from our result that the use of containment options control to cholera epidemics works better in saving more lives.

10. Recommendation

Cholera is known to the third cause of death from infectious disease worldwide after HIV/AIDS and respiration infection. So we therefore recommend that this work can be improved upon by researchers in the field of mathematics in order to test and validate its effectiveness on other diseases.

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