I.J. Mathematical Sciences and Computing, **2017, 1, 28-37**
Published Online January 2017 in MECS (http://www.mecs-press.net)
DOI: 10.5815/ijmsc.2017.01.03

***Available online at* http://www.mecs-press.net/ijmsc**

Exact Analytical Solution of Boundary Value Problem in a Form of an Infinite Hypergeometric Series

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Abstract

This paper proposes an exact solution of the classical Graetz problem in terms of an infinite series represented by a nonlinear partial differential equation considering two space variables, two boundary conditions and one initial condition. The mathematical derivation is based on the method of separation of variables whose several stages were illustrated to reach the solution of the Graetz problem.

Index Terms: Graetz problem, Sturm-Liouville problem, Dimensionless variable, Partial differential equation.

1. Introduction

The solutions of one or more partial differential equations (PDEs), which are subjected to relatively simple limits, can be tackled either by analytical or numerical approach. There are two common techniques available to solve PDEs analytically, namely the variable separation and combination of variables. The Graetz problem describes the temperature (or concentration) field in fully developed laminar flow in a circular tube where the wall temperature (or concentration) profile is a step-function [1]. Many dynamical phenomena in nature as well as in technological applications involve nonlinear behaviour as an essential ingredient. A dynamical system not only has a State Space but also is characterized by a law that describes how the system evolves with time [2]. Several models have been suggested for time series forecasting, that are generally divided to linear and nonlinear models [3]. The simple version of the Graetz problem was initially neglecting axial diffusion, considering simple wall heating conditions (isothermal and isoflux), using simple geometry cross-section (either parallel plates or circular channels), and also neglecting fluid flow heating effects, which can be generally denoted as Classical Graetz Problem [4]. Min et al. [5] presented an exact solution for a Graetz problem with axial diffusion and flow heating effects in a semi-infinite domain with a given inlet condition. Later, the Graetz series solution was further improved by Brown [6]. Ebadian and Zhang [7] analyzed the

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convective heat transfer properties of a hydrodynamically, fully developed viscous flow in a circular tube. Lahjomri and Oubarra [8] investigated a new method of analysis and improved solution for the extended Graetz problem of heat transfer in a conduit. An extensive list of contributions related to this problem may be found in the papers of Papoutsakis et al.[9] and Liou and Wang [10]. In addition, the analytical solution proposed efficiently resolves the singularity and this methodology allows extension to other problems such as the Hartmann flow [11], conjugated problems [13] and other boundary conditions. Recently, Belhocine [13] developed a mathematical model to solve the classic problem of Graetz using two numerical approaches, the orthogonal collocation method and the method of Crank-Nicholson.

In this paper, the Graetz problem that consists of two differential partial equations will be solved using separation of variables method. The Kummer equation is employed to identify the confluent hypergeometric functions and its properties in order to determine the eigenvalues of the infinite series which appears in the proposed analytical solution.

2. Analytical Solution using Separation of Variables Method

In both qualitative and numerical methods, the dependence of solutions on the parameters plays an important role, and there are always more difficulties when there are more parameters. We describe a technique that changes variables so that the new variables are "dimensionless". This technique will lead to a simple form of the equation with fewer parameters. Let the Graetz problem is given by the following governing equation

$$(1-x^2)\frac{\partial\theta}{\partial y}=\frac{L}{PeR}\left[\frac{1}{x}\frac{\partial\theta}{\partial x}+\frac{\partial^2\theta}{\partial x^2}\right] \tag{1}$$

where, *Pe* is the Peclet number, *L* is the tube length and *R* is the tube radius.With the following initial conditions:

IC: $y = 0,\ \theta=1$

BC1: $x=0,\ \dfrac{\partial\theta}{\partial x}=0$

BC2: $x=1,\ \theta=0$

Introducing dimensionless variables as demonstrated [14] as follows:

$$x=\xi=\frac{r}{R} \tag{2}$$

$$\zeta=\frac{kz}{\rho c_p v_{\max} R^2} \tag{3}$$

$$y=\frac{z}{L} \tag{4}$$

By substituting Eq. (4) into Eq. (3) then it becomes:

$$\zeta = \frac{k\,y\,L}{\rho c_p v_{\max} R^2} \tag{5}$$

Knowing that $v_{\max} = 2\bar{u}$ Therefore,

$$\zeta = \frac{k\,y\,L}{2\bar{u}\,\rho\,c_p\,R^2} = \frac{y\,L}{\frac{2\bar{u}\,\rho c_p R}{k}.R} \tag{6}$$

Notice that the term $\frac{2\bar{u}\,\rho c_p R}{k}$ in Eq. (6) is similar to the Peclet number, *P*.
Thus, Eq. (6) can be written as

$$\zeta = \frac{yL}{PeR} \tag{7}$$

Based on Eqs. (2)-(7), ones can write the following expressions;

$$\frac{\partial\theta}{\partial x} = \frac{\partial\theta}{\partial\xi} \tag{8}$$

$$\frac{\partial^2\theta}{\partial x^2} = \frac{\partial^2\theta}{\partial\xi^2} \tag{9}$$

$$\frac{\partial\theta}{\partial y} = \frac{\partial\zeta}{\partial y}.\frac{\partial\theta}{\partial\zeta} = \frac{L}{PeR}\frac{\partial\theta}{\partial\zeta} \tag{10}$$

Now, by replacing Eqs. (8)-(10) into Eq. (1), the governing equation becomes:

$$\frac{L}{PeR}(1-\xi^2)\frac{\partial\theta}{\partial\zeta} = \frac{L}{PeR}\left[\frac{1}{\xi}\frac{\partial\theta}{\partial\theta} + \frac{\partial^2\theta}{\partial\xi^2}\right] \tag{11}$$

Eq. (4) will be reduced to:

$$(1-\xi^2)\frac{\partial\theta}{\partial\zeta} = \frac{1}{\xi}\frac{\partial\theta}{\partial\xi} + \frac{\partial^2\theta}{\partial\xi^2} \tag{12}$$

The right term in Eq. (12) can be simplified as follows:

$$\frac{1}{\xi}\frac{\partial\theta}{\partial\xi}+\frac{\partial^2\theta}{\partial\xi^2}=\frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial\theta}{\partial\xi}\right) \tag{13}$$

Finally, the equation that characterizes the Graetz problem can be written in the form of:

$$(1-\xi^2)\frac{\partial\theta}{\partial\zeta}= \frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial\theta}{\partial\xi}\right) \tag{14}$$

Now, using an energy balance method in the cylindrical coordinates, Eq. (14) can be decomposed into two ordinary differential equations. This is done by assuming constant physical properties of a fluid and neglecting axial conduction and in steady state. By imposing initial conditions as given below:

at $\zeta = 0$, $\theta = 1$
at $\xi = 1$, $\theta = 0$
at $\xi = 0$, $\frac{\partial\theta}{\partial\xi}=0$

and dimensionless variables are defined by:

$$\theta = \frac{T_\omega - T}{T_\omega - T_0},\ \xi=\frac{r}{r_1} \text{ and } \zeta=\frac{kz}{\rho c_p v_{\max} r_1^2}$$

while the separation of variables method is given by

$$\theta = Z(\zeta)R(\xi) \tag{15}$$

Finally, Eq. (14) can be expressed as follows:

$$\frac{dZ}{Z}=-\beta^2\zeta \tag{16}$$

and

$$\xi\frac{d^2R}{d\xi^2}+\frac{dR}{d\xi}+\beta^2\xi(1-\xi^2)R=0 \tag{17}$$

where β^2 is a positive real number and represents the intrinsic value of the system.
The solution of Eq. (16) can be given as:

$$Z=c_1 e^{-\beta^2\zeta} \tag{18}$$

where c_1 is an arbitrary constant. In order to solve Eq. (17), transformations of dependent and independent variables need to be made by taking:

(I) $v = \beta\xi^2$

(II) $R(v) = e^{-v/2} S(v)$

Thus, Eq. (17) is now given by;

$$v\frac{d^2S}{dv^2} + (1-v)\frac{dS}{dv} - \left(\frac{1}{2} - \frac{\beta}{4}\right)S = 0 \tag{19}$$

Eq. (19) is also called as confluent hypergeometric as cited in [15] and it is commonly known as Kummer equation.

2.1. Theorem of Fuchs

A homogeneous linear differential equation of the second order is given by

$$y'' + P(Z)y' + Q(Z)y = 0 \tag{20}$$

If P(Z) and Q(Z) admit a pole at point $Z=Z_0$, it is possible to find a solution developed in the whole series provided that the limits on $\lim_{Z\to Z_0}(Z - Z_0)P(Z)$ and $\lim_{Z\to Z_0}(Z - Z_0)^2 Q(Z)$ exist.

The method of Frobenius seeks a solution in the form of

$$y(Z) = Z^{\lambda}\sum_{n=0}^{\infty} a_n Z^n \tag{21}$$

where, λ is a coefficient to be determined whilst properties of the hypergeometric functions are defined by;

$$F(\alpha,\beta,Z) = 1 + \frac{\alpha}{\beta}Z + \frac{\alpha(\alpha+1)}{\beta(\beta+1)}\frac{Z^2}{2!} + \cdots + \frac{\alpha(\alpha+1)(\alpha+2)\cdots(\alpha+n-1)}{\beta(\beta+1)(\beta+2)\cdots(\theta+n-1)}\frac{Z^n}{n!} + \cdots +$$

$$F(\alpha,\beta,\gamma) \text{ converge } \forall Z \tag{22}$$

Using derivation against Z, the function is now become

$$\frac{d}{dZ}[F(\alpha,\beta,Z)] = \frac{\alpha}{\beta}\left[1 + + \frac{(\alpha+1)}{(\beta+1)}Z + \frac{(\alpha+1)(\alpha+2)}{(\beta+1)(\beta+2)}\frac{Z^2}{2!}\right.$$
$$\left. + \cdots + \frac{\alpha(\alpha+1)(\alpha+2)\cdots(\alpha+n-1)}{\beta(\beta+1)(\beta+2)\cdots(\alpha+n-1)}\frac{Z^n}{n!} + \cdots + -\right]$$

$$\frac{\alpha}{\beta}F(\alpha+1,\beta+1,Z) \tag{23}$$

From Eq. (23), ones will get;

$$\frac{d}{q\xi}F\left(\frac{1}{2}-\frac{\beta_n}{4},1,\beta_n\xi^2\right)=(-\beta_n{}^2)\xi\left(\frac{1}{2}-\frac{1}{\beta_n}\right)F\left(\frac{3}{2}-\frac{\beta_n}{4},2,\beta_n\xi^2\right) \tag{24}$$

Thus, the solution of Eq. (17) can be obtained by:

$$R=c_2 e^{-\beta\xi^2/2}F\left(\frac{1}{2}-\frac{\beta}{4},1,\beta\xi^2\right) \tag{25}$$

$$F\left(\frac{1}{2}-\frac{\beta_n}{4},1,\beta_n\xi^2\right)=0 \tag{26}$$

where n = 1, 2, 3 ... and eigenvalues β_n are the roots of Eq. (26). Since the system is linear, the general solution can be determined using superposition approach:

$$\theta=\sum_{n=1}^{\infty}c_n e^{-\beta^2\zeta}.e^{-\beta_n\xi^2/2}.F\left(\frac{1}{2}-\frac{\beta_n}{4},1,\beta_n\xi^2\right) \tag{27}$$

The constants in Eq. (27) can be sought using orthogonality property of the Sturm-Liouville systems after the initial condition is being applied as stated below;

$$c_n=\frac{\left(\frac{1}{2}-\frac{1}{\beta_n}\right)e^{-\beta_n/2}.F\left(\frac{3}{2}-\frac{\beta_n}{4},2,\beta_n\right)}{\int_0^1(\xi-\xi^3)e^{-\beta_n\xi^2}.\left[F\left(\frac{1}{2}-\frac{\beta_n}{4},1,\beta_n\xi^2\right)\right]^2 d\xi} \tag{28}$$

The integral in the denominator of Eq. (28) can be evaluated using numerical integration.
For the Graetz problem, it is noticed that:

$$(\xi-\xi^3)\frac{\partial\theta}{\partial\zeta}=\frac{\partial}{\partial\xi}\left(\xi\frac{\partial\theta}{\partial\xi}\right) \tag{29}$$

where $(\xi-\xi^3)$ is the function of the weight / β_n eigenvalues

B.C $\xi=1$, $\theta=0$
B.C $\xi=0$, $\theta=1$
IC $\zeta=0$, $\theta=1$

$$\theta = \sum_{n=1}^{\infty} C_n e^{\beta_n^2 \zeta} G_n(\xi) = \sum_{n=1}^{\infty} C_n e^{\beta_n^2 \zeta} e^{-\beta_n \xi^2/2} F\left(\frac{1}{2} - \frac{\beta_n}{4}, 1, \beta_n \xi^2\right) \tag{30}$$

$G_n(\xi) = e^{-\beta_n \xi^2/2} F\left(\frac{1}{2} - \frac{\beta_n}{4}, 1, \beta_n \xi^2\right)$ is the function of the weight

Sturm-Liouville problem.

$$\frac{1}{\xi}\frac{d}{d\xi}\left(\xi \frac{dG_n}{d\xi}\right) - (1-\xi^2)\beta_n^2 G_n = 0 \tag{31}$$

$$\frac{dG_n}{d\xi} = 0 \text{ for } \xi = 0 \text{ , } G_n = 0 \text{ for } \xi = 1 \tag{32}$$

IC $\zeta = 0, \theta = 1$

$$\theta(\zeta = 0) = 1 = \sum_{n=1}^{\infty} C_n e^{-\beta_n \xi^2/2} F\left(\frac{1}{2} - \frac{\beta_n}{4}, 1, \beta_n \xi^2\right) \tag{33}$$

Relation of orthogonality

$$\int_0^1 W(x) Y_i(x) Y_j(x) = 0 \text{ , } (i \neq j) \tag{34}$$

$$\int_0^1 (\xi - \xi^3) e^{-\beta_n \xi^2/2} F\left(\frac{1}{2} - \frac{\beta_n}{4}, 1, \beta_n \xi^2\right) e^{-\beta_m \xi^2/2} F\left(\frac{1}{2} - \frac{\beta_m}{4}, 1, \beta_m \xi^2\right) d\xi = 0 \text{ , } (\forall n \neq m) \tag{35}$$

$$\begin{aligned} \frac{\partial \theta}{\partial \xi} &= \sum_{n=1}^{\infty} C_n e^{\beta_n^2 \zeta} (-\beta_n \xi) e^{-\beta_n \xi^2/2} F\left(\frac{1}{2} - \frac{\beta_n}{4}, 1, \beta_n \xi^2\right) \\ &\quad + \sum_{n=1}^{\infty} C_n e^{\beta_n^2 \zeta} (-\beta_n{}^2) \xi \left(\frac{1}{2} - \frac{1}{\beta_n}\right) e^{-\beta_n \xi^2/2} F\left(\frac{3}{2} - \frac{\beta_n}{4}, 2, \beta_n \xi^2\right) \end{aligned} \tag{36}$$

$$\frac{\partial \theta}{\partial \zeta} = \sum_{n=1}^{\infty} C_n e^{\beta_n^2 \zeta} (-\beta_n{}^2) e^{-\beta_n \xi^2/2} F\left(\frac{1}{2} - \frac{\beta_n}{4}, 1, \beta_n \xi^2\right) \tag{37}$$

By considering $F\left(\frac{1}{2} - \frac{\beta_n}{4}, 1, \beta_n\right)$, Eq. (29) and onwards can be given as;

$$\int_0^1 (\xi-\xi^3)\frac{\partial\theta}{\partial\zeta}d\xi=\int_0^1\frac{\partial}{\partial\xi}\left(\xi\frac{\partial\theta}{\partial\xi}\right)=\xi\frac{\partial\theta}{\partial\xi}\bigg|_0^1 \tag{38}$$

$$\int_0^1 (\xi-\xi^3)\frac{\partial\theta}{\partial\zeta}d\xi=\frac{\partial\theta}{\partial\xi}\bigg|_{\xi=1} \tag{39}$$

$$\begin{aligned}&\int_0^1 (\xi-\xi^3)\left[\sum_{n=1}^{\infty}C_n e^{\beta_n^2\zeta}(-\beta_n{}^2)e^{-\beta_n\xi^2/2}F\left(\frac{1}{2}-\frac{\beta_n}{4},1,\beta_n\xi^2\right)\right]d\xi\\ &\qquad\sum_{n=1}^{\infty}C_n e^{\beta_n^2\zeta}(-\beta_n{}^2)e^{-\beta_n\ /2}F\left(\frac{1}{2}-\frac{\beta_n}{4},1,\beta_n\right)\\ &\qquad\quad\sum_{n=1}^{\infty}C_n e^{\beta_n^2\zeta}(-\beta_n{}^2)\left(\frac{1}{2}-\frac{1}{\beta_n}\right)e^{-\beta_n\ /2}F\left(\frac{3}{2}-\frac{\beta_n}{4},2,\beta_n\right)\end{aligned} \tag{40}$$

$$\begin{aligned}&\sum_{n=1}^{\infty}C_n e^{\beta_n^2\zeta}(-\beta_n{}^2)e^{-\beta_n\xi^2/2}F\left(\frac{1}{2}-\frac{\beta_n}{4},1,\beta_n\xi^2\right)\\ &\qquad\quad\sum_{n=1}^{\infty}C_n e^{\beta_n^2\zeta}(-\beta_n{}^2)\left(\frac{1}{2}-\frac{1}{\beta_n}\right)e^{-\beta_n\ /2}F\left(\frac{3}{2}-\frac{\beta_n}{4},2,\beta_n\right)\end{aligned} \tag{41}$$

By combining Eqs. (39), (40) and (41), the equation can be reduced to;

$$\int_0^1(\xi-\xi^3)e^{-\beta_n\xi^2/2}F\left(\frac{1}{2}-\frac{\beta_n}{4},1,\beta_n\xi^2\right)d\xi=\left(\frac{1}{2}-\frac{1}{\beta_n}\right)e^{-\beta_n\ /2}F\left(\frac{3}{2}-\frac{\beta_n}{4},2,\beta_n\right) \tag{42}$$

Let’s multiply Eq. (1) by Eq. (43) and then integrate Eq. (44),

$$(\xi-\xi^3)e^{-\beta_m\xi^2/2}F\left(\frac{1}{2}-\frac{\beta_m}{4},1,\beta_m\xi^2\right) \tag{43}$$

$$\begin{aligned}&\int_0^1(\xi-\xi^3)e^{-\beta_m\xi^2/2}F\left(\frac{1}{2}-\frac{\beta_m}{4},1,\beta_m\xi^2\right)d\xi\\ &\qquad=\sum_{n=1}^{\infty}\ C_n\int_0^1(\xi-\xi^3)e^{-\beta_m\xi^2/2}F\left(\frac{1}{2}-\frac{\beta_m}{4},1,\beta_m\xi^2\right).\\ &\qquad\qquad\qquad e^{-\beta_n\xi^2/2}F\left(\frac{1}{2}-\frac{\beta_n}{4},1,\beta_n\xi^2\right)d\xi\end{aligned} \tag{44}$$

The outcomes of multiplication and integration process will produce the following:

(i) If $(n \neq m)$ the result is equal to zero (0)
(ii) If $(n = m)$ the result is

$$\int_0^1 (\xi-\xi^3)e^{-\beta_n\xi^2/2}F\left(\frac{1}{2}-\frac{\beta_n}{4},1,\beta_n\xi^2\right)d\xi = C_n\int_0^1 (\xi-\xi^3)e^{-\beta_n\xi^2}\left[F\left(\frac{1}{2}-\frac{\beta_n}{4},1,\beta_n\xi^2\right)\right]^2 d\xi \tag{45}$$

Substituting Eq. (42) into Eq.(45), the equation becomes;

$$\left(\frac{1}{2}-\frac{1}{\beta_n}\right)e^{-\beta_n/2}F\left(\frac{3}{2}-\frac{\beta_n}{4},2,\beta_n\right) = C_n\int_0^1 (\xi-\xi^3)e^{-\beta_n\xi^2}\left[F\left(\frac{1}{2}-\frac{\beta_n}{4},1,\beta_n\xi^2\right)\right]^2 d\xi \tag{46}$$

And the constants C_n can be obtained by;

$$C_n = \frac{\left(\frac{1}{2}-\frac{1}{\beta_n}\right)e^{-\beta_n/2}F\left(\frac{3}{2}-\frac{\beta_n}{4},2,\beta_n\right)}{\int_0^1 (\xi-\xi^3)e^{-\beta_n\xi^2}\left[F\left(\frac{1}{2}-\frac{\beta_n}{4},1,\beta_n\xi^2\right)\right]^2 d\xi} \tag{47}$$

3. Conclusion

In this paper, an exact solution of the Graetz problem is successfully obtained using the method of separation of variables. The hypergeometric functions are employed in order to determine the eigenvalues and constants, C_n and later to a find solution for the Graetz problem. The mathematical method performed in this study can be applied to the prediction of the temperature distribution in steady state thermally laminar heat transfer based on the fully developed velocity for fluid flow through a circular tube. In future work extensions, we recommend performing the Graetz solution by separation of variables in a variety of ways of accommodating non-Newtonian flow, turbulent flow, and other geometries besides a circular tube. It will be also interesting to solve the equation of the Graetz problem using numerical methods such as finite difference or orthogonal collocation for comparison purposes with the proposed exact solution. These numerical works will be carried out and included in the upcoming papers.

References

[1] Shah RK, London AL, Laminar Flow Forced Convection in Ducts, Chap.V, Academic Press, New York, 1978.

[2] Khashei M, Montazeri MA, Bijari M., "Comparison of Four Interval ARIMA-base Time Series Methods for Exchange Rate Forecasting", International Journal of Mathematical Sciences and Computing, Vol. 1, No. 1, Pages 21-34, 2015.

[3] Younis A. Shah, Irshad.A. Mir, Uzair M. Rathea, "Quantum Mechanics Analysis: Modeling and Simulation of some simple systems", International Journal of Mathematical Sciences and Computing, Vol.2, No. 1, Pages 23-40, 2016.

[4] Braga, N. R.., de Barros, L. S., Sphaier, L. A.: Generalized Integral Transform Solution of Extended Graetz Problems with Axial Diffusion ICCM2014 28-30th July, Cambridge, England pp.1-14 (2014)

[5] Min, T., Yoo, J. Y., Choi, H.: Laminar convective heat transfer of a bingham plastic in a circular pipei. Analytical approach—thermally fully developed flow and thermally developing flow (the Graetz problem extended). International Journal of Heat and Mass Transfer, 40(13):3025–3037, (1997).

[6] Brown, G.M.: Heat or mass transfer in a fluid in laminar flow in a circular or flat conduit, AIChE J. 6 179–183 (1960).

[7] Ebadian MA, Zhang HY, An exact solution of extended Graetz problem with axial heat conduction International Journal of Heat and Mass Transfer 32(9):1709-1717 1989.

[8] J. Lahjomri , A. Oubarra, Analytical Solution of the Graetz Problem With Axial Conduction Journal of Heat Transfer, 1, (1999) ,1078 -1083.

[9] Papoutsakis, E., Damkrishna, D., Lim, H. C, "The Extended Graetz. Problem with Dirichlet Wall Boundary Conditions," Appl. Sci. Research, Vol. 36, pp.13-34, 1980.

[10] Liou, C. T., Wang, F. S., "A Computation for the Boundary Value Problem of a Double Tube Heat Exchanger," Numerical Heat Transfer, Part A., Vol.17, pp. 109-125, 1990.

[11] Lahjomri, .J., Oubarra, A., and Alemany, A., "Heat transfer by laminar Hartmann flow in thermal entrance eregion with a step change in wall temperatures: The Graetz problem extended," International Journal of Heat and Mass Transfer 45(5):1127-1148,2002.

[12] Fithen, R. M„ Anand, N. K., "Finite Element Analysis of Conjugate Heat Transfer in Axisymmetric Pipe Flows," Numerical Heat Transfer, Vol. 13, pp.189-203,1988.

[13] Belhocine.A. Numerical study of heat transfer in fully developed laminar flow inside a circular tube, International Journal of Advanced Manufacturing Technology, pp.1-12 (2015).

[14] Huang C. R., M. Matloz, Wen Dow-Pan and William Snyder, "Heat Transfer to a Laminar Flow in a Circular Tube", AIChE Journal, 5, 833, 1984.

[15] Slater L. J., "Confluent Hypergeometric Functions", Cambridge University Press, 1960.

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How to cite this paper: Ali Belhocine,"Exact Analytical Solution of Boundary Value Problem in a Form of an Infinite Hypergeometric Series", International Journal of Mathematical Sciences and Computing(IJMSC), Vol.3, No.1, pp.28-37, 2017.DOI: 10.5815/ijmsc.2017.01.03