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Comparison of Four Interval ARIMA-base Time Series Methods for Exchange Rate Forecasting

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Abstract

In today's world, using quantitative methods are very important for financial markets forecast, improvement of decisions and investments. In recent years, various time series forecasting methods have been proposed for financial markets forecasting. In each case, the accuracy of time series methods fundamental to make decision and hence the research for improving the effectiveness of forecasting models have been curried on. In the literature, Many different time series methods have been frequency compared together in order to choose the most efficient once. In this paper, the performances of four different interval ARIMA-base time series methods are evaluated in financial markets forecasting. These methods are including Auto-Regressive Integrated Moving Average (ARIMA), Fuzzy Auto-Regressive Integrated Moving Average (FARIMA), Fuzzy Artificial Neural Network (FANN) and Hybrid Fuzzy Auto-Regressive Integrated Moving Average (FARIMAH). Empirical results of exchange rate forecasting indicate that the fuzzy artificial neural network model is more satisfactory than other models.

Index Terms: Artificial Neural Networks (ANNs), Time series forecasting, Auto-Regressive Integrated Moving Average (ARIMA), Combined forecast, Exchange Rate.

1. Introduction

In today's world, using quantitative methods is transformed to undeniable exigency for forecasting the financial markets, improvement of decisions and investments. Time series forecasting is an important area of forecasting in which past observations of the same variable are collected and analyzed to develop a model describing the underlying relationship [1]. The model is then used to extrapolate the time series into the future. This modelling approach is particularly useful when little knowledge is available on the underlying data

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generating process or when there is no satisfactory explanatory model that relates the prediction variable to other explanatory variables.

Exchange rate is one of the most effective variables in financial environments and forecasting of it is very important for economic decision makers and financial managers. In exchange rate field, numerous forecasting investigations have been accomplished [2-5] that number of these investigations represent the mentioned issue importance. Nowadays, despite of obtainable numerous financial forecasting models, accurate forecasts of exchange rate are not easy task. It is the main reason that researches for obtaining more accurate results have not been stopped [6-11].

Time series forecasting is an important area of forecasting in which past observations of the same variable are collected and analyzed to develop a model describing the underlying relationship. The model is then used to extrapolate the time series into the future. This modelling approach is particularly useful when little knowledge is available on the underlying data generating process or when there is no satisfactory explanatory model that relates the prediction variable to other explanatory variables [12].

Several models have been suggested for time series forecasting, that are generally divided to linear and nonlinear models. One of the most important and widely used linear time series models is the Auto-Regressive Integrated Moving Average (*ARIMA*) model that has enjoyed fruitful applications in forecasting social, economic, engineering, foreign exchange, and stock problems. Second class of time series forecasting is nonlinear time series models. Artificial neural networks are one of these models that are able to approximate various nonlinearities in the data and are flexible computing frameworks for modelling a broad range of nonlinear problems [13].

One significant advantage of the *ANN* models over other classes of nonlinear model is that *ANNs* are universal approximators, which can approximate a large class of functions with a high degree of accuracy. No prior assumption of the model form is required in the model building process. Instead, the network model is largely determined by the characteristics of the data. Commonly used network models include multi-layer perceptron (*MLP*), Radial Basis Function (*RBF*), Probabilistic Neural Networks (*PNN*s) and General Regression Neural Networks (*GRNNs*) [14]. Single hidden layer feed-forward network is the most widely used model form for time series modelling and forecasting [15].

Forecasting accuracy is one of the most important factors to choose the forecasting method, and regardless numerous time series forecasting models, the accuracy of time series forecasting is fundamental to many decision processes and hence the research for improving and diagnosing the effectiveness of forecasting models has been never stopped. *Ture* has compared the performance of four different time series models to forecast the hepatitis *A* virus infection [16]. *Taylor et al* have compared the univariate methods for forecasting electricity demand [17]. *Kima* [18] has forecasted the international tourist flows to Australia for comparison between the direct and indirect methods. *Cho* also has compared the three different approaches to tourist arrival forecasting [19]. Some other research in this field, consist of *Weatherforda* [20] to hotel revenue management forecasting, *Smith* [21] to traffic flow forecasting and *Sfetsos* [22] to mean hourly wind speed time series forecasting.

In the financial field also is accomplished various research similar above. *Alon* has compared the performance of artificial neural networks and traditional methods to aggregate retail sales forecasting [23]. *Meade* [24] has compared the accuracy of short-term foreign exchange forecasting methods. *Leunga et al* [25] have compared the classification and level estimation models to forecasting the stock indices. *Lisi* also has compared the neural networks and chaotic models for exchange rate prediction [26]. *Tsui* [27] has compared the exchange rate and pricing behaviour between Taiwan and Japan for manufacturing industries. *Ghosh* [28] has compared the effects of exchange rate regime choice in emerging markets with advanced and low-income nations for 1999–2011. *Wang et al* [29] have compared the characterizing information flows among spot, deliverable forward and non-deliverable forward exchange rate markets: A cross-country comparison. *Razavi et al* [30] have compared the circuit patency and exchange rates between two different continuous renal replacement therapy machines.

In this paper, is compared the performance of four different interval time series methods for financial markets forecasting. The rest of the paper is organized as follows. In the next section, concepts of four time-series methods: Auto-Regressive Integrated Moving Average (*ARIMA*), Fuzzy Auto-Regressive Integrated Moving Average (*FARIMA*), Fuzzy Artificial neural Network (FANN) and Hybrid Fuzzy Auto-Regressive Integrated Moving Average (*FARIMAH*) are reviewed. Empirical result from forecasting the exchange rate (*US Dollar/Rial*) is reported in Section 3. The performance of each model is compared in section 4, and finally the conclusions are discussed.

2. Time Series Forecasting Models

There are several different approaches to time series modeling. Interval models are special class of quantitative forecasting models. These models calculate an interval as optimum forecast of independent variable. In this section are reviewed four different interval time series models.

2.1. The Auto-Regressive Integrated Moving Average (ARIMA) model

In an autoregressive integrated moving average model, the future value of a variable is assumed to be a linear function of several past observations and random errors. That is, the underlying process that generate the time series has the form

$$y_t = \theta_0 + \varphi_1 y_{t-1} + \ldots + \varphi_p y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \ldots - \theta_q \varepsilon_{t-q} \tag{1}$$

where y_t and ε_t are the actual value and random error at time period *t*, respectively; $\phi_i \quad (i = 1,2,..., p)$ and $\theta_j \quad (j = 1,2,..., q)$ are model parameters. *p* and *q* are integers and often referred to as orders of the model. Random errors, ε_t, are assumed to be independently and identically distributed with a mean of zero and a constant variance of σ^2.

The *Box–Jenkins* [31] methodology includes three iterative steps of model identification, parameter estimation and diagnostic checking. The basic idea of model identification is that if a time series is generated from an *ARIMA* process, it should have some theoretical autocorrelation properties. By matching the empirical autocorrelation patterns with the theoretical ones, it is often possible to identify one or several potential models for the given time series. *Box and Jenkins* [31] proposed to use the autocorrelation function (*ACF*) and the partial autocorrelation function (*PACF*) of the sample data as the basic tools to identify the order of the *ARIMA* model.

Once a tentative model is specified, estimation of the model parameters is straightforward. The parameters are estimated such that an overall measure of errors is minimized. This can be done with a nonlinear optimization procedure. The last step of model building is the diagnostic checking of model adequacy. This is basically to check if the model assumptions about the errors, ε_t, are satisfied.

Several diagnostic statistics and plots of the residuals can be used to examine the goodness of fit of the tentatively entertained model to the historical data. If the model is not adequate, a new tentative model should be identified, which is again followed by the steps of parameter estimation and model verification. Diagnostic information may help suggest alternative model(s). This three-step model building process is typically repeated several times until a satisfactory model is finally selected. The final selected model can then be used for prediction purposes [32].

2.2. The Fuzzy Auto-Regressive Integrated Moving Average (FARIMA) model

The parameter of *ARIMA*(p, d, q), $\varphi_1, \varphi_2, \ldots. \varphi_p$ and $\theta_1, \theta_2, \ldots. \theta_q$ are crisp. Instead of using crisp, fuzzy parameters, $\tilde{\varphi}_1, \tilde{\varphi}_2, \ldots. \tilde{\varphi}_p$ and $\tilde{\theta}_1, \tilde{\theta}_2, \ldots. \tilde{\theta}_q$, in the form of triangular fuzzy numbers are used in Fuzzy Auto-

Regressive Integrated Moving Average models [33]. A fuzzy *ARIMA*(p, d, q) model is described by a fuzzy function with a fuzzy parameter:

$$\tilde{\Phi}_p(B)W_t = \tilde{\theta}_q(B)a_t \tag{2}$$

$$W_t = (1-B)^d(Z_t - \mu) \tag{3}$$

$$\tilde{W}_t = \tilde{\varphi}_1 W_{t-1} + \tilde{\varphi}_2 W_{t-2} + ... + \tilde{\varphi}_p W_{t-p} + a_t - \tilde{\theta}_{p+1}a_{t-1} - \tilde{\theta}_{p+2}a_{t-2} - ... - \tilde{\theta}_{p+q}a_{t-q} \tag{4}$$

where $\{Z_t\}$ are observations, $\tilde{\varphi}_1, \tilde{\varphi}_2, ..., \tilde{\varphi}_p$ and $\tilde{\theta}_1, \tilde{\theta}_2, ..., \tilde{\theta}_q$, are fuzzy numbers. Eq. (4) is modified as

$$\tilde{W}_t = \tilde{\beta}_1 W_{t-1} + \tilde{\beta}_2 W_{t-2} + ... + \tilde{\beta}_p W_{t-p} + a_t - \tilde{\beta}_{p+1}a_{t-1} - \tilde{\beta}_{p+2}a_{t-2} - ... - \tilde{\beta}_{p+q}a_{t-q} \tag{5}$$

Fuzzy parameters in the form of triangular fuzzy numbers are used:

$$\mu_{\beta_i}(\beta_i) = \begin{cases} 1 - \dfrac{|\alpha_i - \beta_i|}{c_i} & if \quad \alpha_i - c_i \le \beta_i \le \alpha_i + c_i, \\ 0 & otherwise, \end{cases} \tag{6}$$

where $\mu_{\tilde{\beta}}(\beta_i)$ is the membership function of the fuzzy set that represents parameter β_i, α_i is the center of the fuzzy number, and c_i is the width or spread around the center of the fuzzy number. Using fuzzy parameters β_i in the form of triangular fuzzy numbers and applying the extension principle, it becomes clear [34] that the membership of *W* in Eq. (5) is given as

$$\mu_{\tilde{w}}(W_t) = \begin{cases} 1 - \dfrac{\left|W_t - \sum_{i=1}^{p}\alpha_i W_{t-i} - a_t + \sum_{i=p+1}^{p+q}\alpha_i a_{t+p-i}\right|}{\sum_{i=1}^{p} c_i |W_{t-i}| + \sum_{i=p+1}^{p+q} c_i |a_{t+p-i}|} & for \quad W_t \ne 0, \quad a_t \ne 0 \\ 0 & otherwise \end{cases} \tag{7}$$

Simultaneously, Z_t represents the *t*th observation, and *h*-level is the threshold value representing the degree to which the model should be satisfied by all the data points $y_1, y_2, ..., y_k$ to a certain *h*-level. A choice of the *h-level* value influences the widths *c* of the fuzzy parameters:

$$\mu_y(y_t) \ge h \qquad for\, t = 1,2,...,k \tag{8}$$

The index *t* refers to the number of nonfuzzy data used for constructing the model. On the other hand, the fuzziness *S* included in the model is defined by

$$S=\sum_{i=1}^{p}\sum_{t=1}^{k}c_i|\varphi_{ii}||W_{t-i}|+\sum_{i=p+1}^{p+q}\sum_{t=1}^{k}c_i|\rho_{i-p}||a_{t+p-i}| \tag{9}$$

where ρ_{i-p} is the autocorrelation coefficient of time lag *i-p*, φ_{ii} is the partial autocorrelation coefficient of time lag *i*. The weight of c_i depends on the relation of time lag *i* and the present observation, where the *p* of *AR* (p) is derived by *PACF* and the *q* of *MA* (q) is derived by *ACF*. Next, the problem of finding the fuzzy *ARIMA* parameters was formulated as a linear programming problem:

$$\begin{aligned}
&\textit{Minimize} \quad S=\sum_{i=1}^{p}\sum_{t=1}^{k}c_i|\varphi_{ii}||W_{t-i}|+\sum_{i=p+1}^{p+q}\sum_{t=1}^{k}c_i|\rho_{i-p}||a_{t+p-i}| \\
&\qquad \sum_{i=1}^{p}\alpha_i W_{t-i}+a_t-\sum_{i=p+1}^{p+q}\alpha_i a_{t+p-i}+(1+h)\left(\sum_{i+1}^{p}c_i|W_{t-i}|+\sum_{i=p+1}^{p+q}c_i|a_{t+p-i}|\right)\geq W_t \ t=1,2,..,k \\
&\textit{subject.to} \quad \sum_{i=1}^{p}\alpha_i W_{t-i}+a_t-\sum_{i=p+1}^{p+q}\alpha_i a_{t+p-i}+(1+h)\left(\sum_{i+1}^{p}c_i|W_{t-i}|+\sum_{i=p+1}^{p+q}c_i|a_{t+p-i}|\right)\leq W_t \ t=1,2,..,k \\
&\qquad c_i\geq 0 \qquad \textit{for}\ i=1,2,\ldots,p+q
\end{aligned} \tag{10}$$

At last, according to the Ishibuchi and Tanaka [35] opinion, the data around the model's upper bound and lower bound is deleted when the fuzzy ARIMA model has outliers with wide spread, and then reformulating the fuzzy regression model.

2.3. The Fuzzy Auto-Regressive Integrated Moving Average (FARIMA) model

The Forecasted interval of Fuzzy Auto-Regressive Integrated Moving Average models is extended in some specific data conditions [36]. According to the Ishibuchi and Tanaka opinion, forecasting interval can be too wide, when training data set includes the significant difference or outlying case [35]. In improved model, the diagnosis ability of Probabilistic Neural Networks (PNNs) [37] is used in order to recognize the more probably spaces in forecasted interval of FARIMA model. Technically, PNN is a classifier and is able to deduce the class/group of a given input vector after the training process is completed. PNN is conceptually built on the Bayesian method of classification which, given enough data, is capable of classifying a sample with the maximum probability of success [38]. The procedure of improved model is as follows:

Phase I: Fitting the FARIMA model using the available information of observations. The result of phase I is as follows:

$$\tilde{W}_t=\langle\alpha_1,c_1\rangle W_{t-1}+\ldots+\langle\alpha_p,c_p\rangle W_{t-p}+a_t-\langle\alpha_{p+1},c_{p+1}\rangle a_{t-1}-\ldots-\langle\alpha_{p+q},c_{p+q}\rangle a_{t-q}, \tag{11}$$

where $W_t=(1-B)^d(Z_t-\mu)$, α_i is the center of the fuzzy number, and c_i is the width or spread around the center of the fuzzy number. Obtained interval of *FARIMA* model is divided to *n* equal sections to use in probabilistic neural network. The subinterval which includes the real value or *n-1* other subintervals is considered as target data to neural network. Other information, include result of *FARIMA* and time series data is considered as train data.

Phase II: Designing and training one network to recognize the more probably spaces in forecasted interval of FARIMA model. The result of this phase is one interval with 1/n width and α confidence coefficient (α is the diagnosis ability of PNN in test data). In improved model, it is assumed that desired case only is one of the n divided subintervals, but generality, the each k consecutive or nonconsecutive subinterval of n divided subinterval can be selected as desired case.

2.4. The Fuzzy Artificial Neural Network (FANN) model

A hybrid model is described by a fuzzy function with a fuzzy parameter:

$$\tilde{y}_t = f(\tilde{b}_0 + \sum_{j=1}^{q} \tilde{w}_j \cdot g(\tilde{b}_{0j} + \sum_{i=1}^{p} \tilde{w}_{i,j} \cdot y_{t-i})) \tag{12}$$

where y_t are observations, $\tilde{w}_j, \tilde{w}_{i,j}, \tilde{b}_{0,} \tilde{b}_{0,j}$ are fuzzy numbers. Eq. (12) is modified as

$$\tilde{y}_t = f(\tilde{b}_0 + \sum_{j=1}^{q} \tilde{w}_j \cdot \tilde{X}_{t,j}) = f(\sum_{j=0}^{q} \tilde{w}_j \cdot \tilde{X}_{t,j}) \tag{13}$$

where $\tilde{X}_{t,j} = g(\tilde{b}_{0j} + \sum_{i=1}^{p} \tilde{w}_{i,j} \cdot y_{t-i})$. Fuzzy parameters in the form of triangular fuzzy numbers $\tilde{w}_{i,j} = (a_{i,j}, b_{i,j}, c_{i,j})$ are used:

$$\mu_{\tilde{w}_{i,j}}(w_{i,j}) = \begin{cases} \frac{1}{b_{i,j} - a_{i,j}}(w_{i,j} - a_{i,j}) & \text{if} \quad a_{i,j} \le w_{i,j} \le b_{i,j}, \\ \frac{1}{b_{i,j} - c_{i,j}}(w_{i,j} - c_{i,j}) & \text{if} \quad b_{i,j} \le w_{i,j} \le c_{i,j}, \\ 0 & \text{otherwise,} \end{cases} \tag{14}$$

where $\mu_{\tilde{w}}(w_{i,j})$ is the membership function of the fuzzy set that represents parameter $w_{i,j}$. Applying the extension principle [39], it becomes clear that the membership of $\tilde{X}_{t,j} = g(\sum_{i=0}^{p} \tilde{w}_{i,j} \cdot y_{t-i})$ in Eq. (13) is given as follows:

$$\mu_{\tilde{X}_{t,j}}\left(x_{t,j}\right)=\begin{cases}\dfrac{\left(x_{t,j}-g\left(\sum_{i=0}^{p}a_{i,j}\cdot y_{t-i}\right)\right)}{g\left(\sum_{i=0}^{p}b_{i,j}\cdot y_{t-i}\right)-g\left(\sum_{i=0}^{p}a_{i,j}\cdot y_{t-i}\right)} & if \quad g\left(\sum_{i=0}^{p}a_{i,j}\cdot y_{t-i}\right)\leq x_{t,j}\leq g\left(\sum_{i=0}^{p}b_{i,j}\cdot y_{t-i}\right),\\ \dfrac{\left(x_{t,j}-g\left(\sum_{i=0}^{p}c_{i,j}\cdot y_{t-i}\right)\right)}{g\left(\sum_{i=0}^{p}b_{i,j}\cdot y_{t-i}\right)-g\left(\sum_{i=0}^{p}c_{i,j}\cdot y_{t-i}\right)} & if \quad g\left(\sum_{i=0}^{p}b_{i,j}\cdot y_{t-i}\right)\leq x_{t,j}\leq g\left(\sum_{i=0}^{p}c_{i,j}\cdot y_{t-i}\right),\\ 0 & otherwise\end{cases} \tag{15}$$

With consider triangular fuzzy numbers $\tilde{X}_{t,j}$ with membership function as Eq. (15) and triangular fuzzy parameters w_j as follow

$$\mu_{\tilde{w}_j}\left(w_j\right)=\begin{cases}\dfrac{1}{e_j-d_j}\left(w_j-d_j\right) & if \quad d_j\leq w_j\leq e_j,\\ \dfrac{1}{e_j-f_j}\left(w_j-f_j\right) & if \quad e_j\leq w_j\leq f_j,\\ 0 & otherwise,\end{cases} \tag{16}$$

The membership function of $\tilde{y}_t=f(\tilde{b}_0+\sum_{j=1}^{q}\tilde{w}_j\cdot\tilde{X}_{t,j})=f(\sum_{j=0}^{q}\tilde{w}_j\cdot\tilde{X}_{t,j})$ is given as

$$\mu_{\tilde{Y}}\left(y_t\right)\cong\begin{cases}\dfrac{-B_1}{2A_1}+\left[\left(\dfrac{B_1}{2A_1}\right)^2-\dfrac{C_1-f^{-1}\left(y_t\right)}{A_1}\right]^{1/2} & if \quad C_1\leq f^{-1}\left(y_t\right)\leq C_3,\\ \dfrac{B_2}{2A_2}+\left[\left(\dfrac{B_2}{2A_2}\right)^2-\dfrac{C_2-f^{-1}\left(y_t\right)}{A_2}\right]^{1/2} & if \quad C_3\leq f^{-1}\left(y_t\right)\leq C_2,\\ 0 & otherwise,\end{cases} \tag{17}$$

Where

$$C_1 = \sum_{j=0}^{q}\left(d_j \cdot g\left(\sum_{i=0}^{p} a_{i,j} \cdot y_{t-i}\right)\right)$$
$$C_2 = \sum_{j=0}^{q}\left(f_j \cdot g\left(\sum_{i=0}^{p} c_{i,j} \cdot y_{t-i}\right)\right),$$
$$C_3 = \sum_{j=0}^{q}\left(e_j \cdot g\left(\sum_{i=0}^{p} b_{i,j} \cdot y_{t-i}\right)\right),$$

Now with consider threshold level h for all membership function value of observations according to Eq. (8) the nonlinear programming is given as follow [40]:

$$Min \quad \sum_{t=1}^{k}\sum_{j=0}^{q}\left(f_j \cdot g\left(\sum_{i=0}^{p} c_{i,j} \cdot y_{t-i}\right)\right)-\left(d_j \cdot g\left(\sum_{i=0}^{p} a_{i,j} \cdot y_{t-i}\right)\right)$$

$$Subject.to \quad \begin{cases} \dfrac{-B_1}{2A_1} + \left[\left(\dfrac{B_1}{2A_1}\right)^2 - \dfrac{C_1 - f^{-1}(y_t)}{A_1}\right]^{1/2} \le h \quad if \quad C_1 \le f^{-1}(y_t) \le C_3, \quad for \quad t = 1,2,\ldots,k \\ \\ \dfrac{B_2}{2A_2} + \left[\left(\dfrac{B_2}{2A_2}\right)^2 - \dfrac{C_2 - f^{-1}(y_t)}{A_2}\right]^{1/2} \le h \quad if \quad C_3 \le f^{-1}(y_t) \le C_2, \quad for \quad t = 1,2,\ldots,k \end{cases} \tag{18}$$

3. Application of Hybrid Model to Exchange Rate Forecasting

In order to demonstrate the more appropriateness and more effectiveness model of the four reviewed models, consider the following application of forecasting the exchange rate (US Dollar/Iran Rial). The information of this investigation consists of 42 daily observations from 5 Nov to 16 Des 2005 (Ref: Centre Bank of Iran (CBI)), Fig. 1.

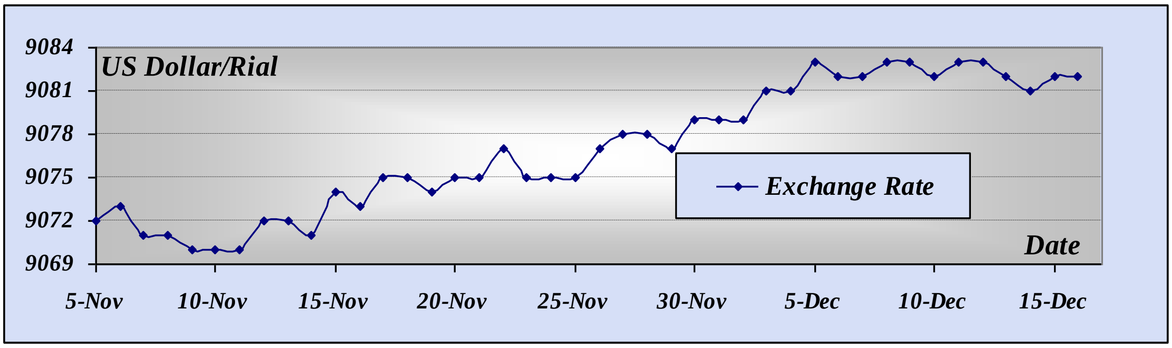

Fig. 1. Exchange Rate data from *5 Nov to 16 Des 2005*. Ref: Centre Bank of Iran (*CBI*).

3.1. The forecasts

In all models, is used the first 35 observations to formulate the model and the next 7 observations to evaluate the performance of the model.

3.1.1. The Auto-Regressive Integrated Moving Average (ARIMA) model

Using the **Eviews** package software, the best-fitted model is ARIMA (2, 1, 0) as follows. The actual value and 95% of the confidence interval of ARIMA model are given in Table. 1.

$$Z_t = 9060.5 + 0.607 Z_{t-1} + 0.421 Z_{t-2} + a_t . \tag{19}$$

Table 1. Actual value and 95% of the confidence interval of *ARIMA* model.

Date	Actual value	Lower bound	Upper bound	Date	Actual value	Lower bound	Upper bound
10- Des 11- Des 12- Des 13- Des	9082 9083 9083 9082	9074 9075 9075 9074	9090 9091 9091 9090	14- Des 15- Des 16- Des	9081 9082 9082	9073 9074 9074	9089 9090 9090

3.1.2. The Fuzzy Auto-Regressive Integrated Moving Average (FARIMA) model

Using Setting $(\alpha_0, \alpha_1, \alpha_2) = (9060.05, 0.607, 0.421)$, the fuzzy parameters obtained using Eq. (10) (with h=0) are shown in Eq. (20).

$$Z_t = 9060.5 + \langle 0.607, 0.00028 \rangle Z_{t-1} + \langle 0.421, 0.0 \rangle Z_{t-2} + a_t . \tag{20}$$

It is known from the above results that the observation of 15 Nov is located at the upper bound (outlier), so the LP constrained equation that is produced by this observation is deleted and renews phase II, let h=0 then we get the model that is in Eq. (21). The results are plotted in Fig. 4 and shown in Table 2.

$$Z_t = 9060.5 + \langle 0.607, 0.00023 \rangle Z_{t-1} + \langle 0.421, 0.0 \rangle Z_{t-2} + a_t . \tag{21}$$

Table 2. Actual value and forecasted interval of FARIMA model.

Date	Actual value	Lower bound	Upper bound	Date	Actual value	Lower bound	Upper bound
10- Des 11- Des 12- Des 13- Des	9082 9083 9083 9082	9081 9080 9081 9081	9085 9084 9085 9085	14- Des 15- Des 16- Des	9081 9082 9082	9080 9079 9080	9084 9083 9084

Using the obtained best-fitted model is ARIMA (2, 1, 0) as follow. The actual value and 95% of the confidence interval of ARIMA model are given in Table. 2.

3.1.3. The Improved FARIMA with Probabilistic Neural Networks (FARIMAH) Model

In improved model after FARIMA model is used Probabilistic Neural Networks. The optimum network is a network with five input neuron and one output neuron. The structure of designed network is given in Fig. 2.

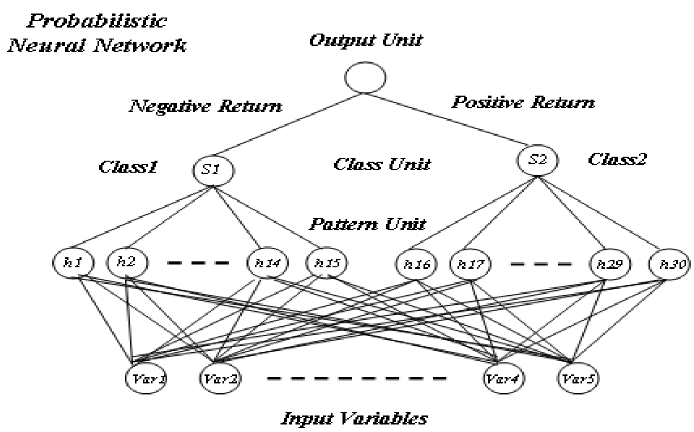

Fig. 2. The structure of designed network.

Where

Var1::Forecasted lower bond of time series in time *t* (L_t)

Var2: Forecasted upper bond of time series in time *t* (U_t)

Var3: Forecasted value of time series in time *t* ($\hat{Z}_t$)

Var4: Difference between forecasted value of time series in time *t*& *t-1* ($\hat{Z}_t - \hat{Z}_{t-1}$)

Var5: Difference between forecasted upper bond (lower bond) of time series in time *t*& *t-1* ($U_t - U_{t-1}$)

Obtained result of upper and lower bound forecasting with improved model with 100% confidence coefficient is given in Table 3.

Table 3. Actual value and forecasted interval of PNN/FARIMA model.

Date	*Actual value*	*Lower bound*	*Upper bound*	*Date*	*Actual value*	*Lower bound*	*Upper bound*
10- Des *11- Des* *12- Des* *13- Des*	*9082* *9083* *9083* *9082*	*9080* *9081* *9080* *9080*	*9083* *9084* *9083* *9083*	*14- Des* *15- Des* *16- Des*	*9081* *9082* *9082*	*9080* *9079* *9081*	*9083* *9082* *9084*

3.1.4. The Fuzzy Artificial Neural Network (FANN) model

With consider concepts artificial neural networks designing [41] and using **MATLAB7** package software, the best fitted network is $\mathbf{N}^{(3\text{-}3\text{-}1)}$. The mentioned network is shown in Fig. 3. The weights and biases of mentioned network also are given in Table 4.

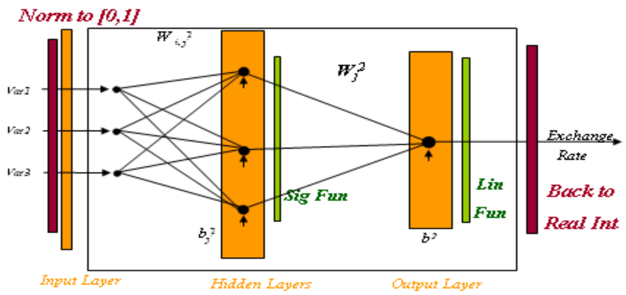

Fig. 3. Structure of the best fitted network, N(3-3-1).

Table 4. Weights and biases of final network.

Input Weights			***Hidden Weights***	***Biases***	
$W^1_{i,1}$	$W^1_{i,2}$	$W^1_{i,3}$		b^1_j	b^2
-3.3947	***1.9112***	***11.9393***	***3.3696***	***-2.7097***	***-4.0598***
-2.3734	***-0.31743***	***-28.5843***	***6.2053***	***2.3013***	
6.8024	***-3.6088***	***-27.0192***	***-1.1491***	***28.9588***	

Setting $\left(\alpha_0^*,\alpha_1^*,\alpha_2^*\right)=\left(9060.05,0.607,0.421\right)$ and $\left(\alpha_3^*,\alpha_4^*,\alpha_5^*,\alpha_6^*\right)=\left(3.37,6.205,-1.149,-4.060\right)$, the fuzzy parameters are obtained using Eq. (18) (with h=0) are shown in Eq. (22). Worthy of mention that in this case the triangular fuzzy numbers is considered symmetric, output neuron transfer function is considered linear and connection weight between hidden and input layer is considered crisp.

$$\begin{aligned}\tilde{Z}_t &= 9060.5+\langle 0.607,0.00008\rangle Z_{t-1}+\langle 0.421,0.0\rangle Z_{t-2}-\langle 4.06,0.008\rangle X_{t,0}\\ &+\langle 3.37,0.0\rangle X_{t,1}+\langle 6.205,0.0\rangle X_{t,2}-\langle 1.149,0.0\rangle X_{t,3}.\end{aligned} \tag{22}$$

Using the revised hybrid model, the future value of the gold price of the next 5 transaction days is forecasted, whose results are shown in Table 5.

Table 5. Actual value and forecasted interval of Hybrid model.

Date	***Actual value***	***Lower bound***	***Upper bound***	***Date***	***Actual value***	***Lower bound***	***Upper bound***
10- Des	***9082***	***9081***	***9084***	***14- Des***	***9081***	***9081***	***9084***
11- Des	***9083***	***9082***	***9084***	***15- Des***	***9082***	***9081***	***9084***
12- Des	***9083***	***9082***	***9084***	***16- Des***	***9082***	***9081***	***9084***
13- Des	***9082***	***9081***	***9084***				

4. Comparison the Performance of Models

In this section, based on the empirical results of this example, the predictive capabilities of the models are compared together. The information of forecasted interval width and related performance of each model is given in Table 6.

Table 6. He information of forecasted interval width and related performance of each model.

Model	Forecasted interval width	Related Performance			
		ARIMA	FARIMA	PNN/FARIMA	Fuzzy & ANNs
ARIMA	16.2	0	-	-	-
FARIMA	4.2	74.1%	0	-	-
PNN/FARIMA	3.1	80.9%	26.2%	0	-
Fuzzy & ANNs	2.5	84.6%	40.5%	19.4%	0

According to the above result between mentioned models in exchange rate forecasting, the Auto-Regressive Integrated Moving Average model has lowest performance and the hybrid artificial neural networks and fuzzy logic model has better performance than other models.

5. Conclusions

In today's world, using quantitative methods for forecasting the financial markets, improvement of decisions and investments is transformed to undeniable exigency. Nowadays, regardless numerous time series forecasting models, the accuracy of time series forecasting is fundamental to many decision processes and hence the research for improving and diagnosing the effectiveness of forecasting models has been never stopped. In this paper are compared the performance of four different interval time series methods (Auto-Regressive Integrated Moving Average (ARIMA), Fuzzy Auto-Regressive Integrated Moving Average (FARIMA), Hybrid ANNs and Fuzzy, Improved FARIMA) to exchange rate forecasting. Empirical results of exchange rate forecasting indicate that the hybrid ANNs and Fuzzy model is more satisfactory than other models.

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