Solution for Using FEMM in Electrostatic Problems with Discrete Distribution Electric Charge

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Abstract: Finite Element Method Magnetics (FEMM) is an open source software package for solving electromagnetic problems based on the finite element method. The application can numerically solve linear electrostatic problems and magnetostatic 2D problems, respectively low frequency magnetic, linear harmonic and nonlinear. FEMM is a product much used in science and engineering that, in the last 15 years, has begun to be used more and more in the academic environment. Despite the fact that FEMM can be used to solve complex problems in science and engineering, electrostatic FEMM cannot work directly with discrete electric charge distributions, that is, point electric charge. This work presents a FEMM model for simulating point electric charge that can be used in case of electrostatic problems with discrete charge distributions. The numerical solution for the electrostatic field is compared with the analytical solution. This model can be used in the case of an assembly of point electric charges with axial symmetry.

Index Terms: Teaching tools, modalities of teaching, computer-assisted training, finite element method, numerical modeling in electromagnetism.

1. Introduction

In the age of digital technology, computer-assisted training is one of the basic teaching tools of modern education [1, 2, 3]. Electromagnetic field theory is one of the most difficult courses studied by students of the specialization of electrical engineering and other related specializations. Since few problems of electromagnetic field theory have analytical solutions, the use of software packages for modeling and simulation are extremely necessary, make learning in this field much more efficient and improve the individual and team work of students [4, 5]. Since numerical modeling in electromagnetism has become a tool extensively used by engineers, scientists and researchers [7], in electrical engineering programs, lately, there is a growing emphasis on teaching design-oriented courses [6]. This category mainly includes courses that are based on the application of techniques of numerical modeling of the electromagnetic field in solving specific problems in the technique.

In the modern teaching of numerical methods for the electromagnetic field very useful are the software applications with graphical interface that allow the design, implementation, processing and post-processing of an electromagnetic field problem. For university use, and generally in the educational environment, such an application should be easy to use, offer programming support in scripting language, The ability to import and export files and further support in engineering design activity. Among applications with a graphic interface for the numerical solutions of differential equations with partial derivatives by the finite element method here: ANSYS [8, 9], Comsole Multiphysics [10, 11] and FEMM (Finite Element Method Magnetics) [12, 13, 14, 15].

FEMM is an open source analysis software package with finite elements for electrostatic, magneto-static and low frequency electromagnetism problems. It is a simple to use, good quality, accurate and low cost freeware product used

in both science and engineering. Besides electromagnetism, it helps to solve complex problems in other areas such as materials science, industry, medicine, physics, robotics, astronomy and space engineering [16, 17, 18, 19, 20, 21, 22, 23, 24].

For some 15 years, there have been a number of concerns aimed at exploring the ability of FEMM to meet the needs of electromagnetism teaching in higher education and beyond. A number of quantitative and qualitative studies and illustrative examples are intended to support the usefulness of this package of programs in the educational environment [13, 14, 15].

Although FEMM is a widely used software in electromagnetism, has no support for electrostatic problems with discrete distribution of electric charge.

In the specialized literature are offered two types of solutions for the implementation of FEMM of the load with discrete distribution [22]: the introduction of the discrete load through the properties of the node and the introduction of the discrete load as load of the inner armature of very small radius of a cylindrical capacitor implemented planar. The first solution lead to distort the shape of the electric field lines. The second solution is forced because in the planar implementation the depth parameter must be introduced, which means the length of the capacitor .

Our work presents a solution for using FEMM software in issues with discreet electric charge distribution, much closer to reality, which can help improve the learning experience in electromagnetism at both university and college levels.

Our work reinforces the idea that FEMM can also be used in classical areas [24], such as physics, where more emphasis is placed on the analytical aspect of problem solving, with electric and magnetic fields often seen as abstract notions, difficult to understand by students and students.

2. FEMM Model for Point Electric Charge

FEMM is a 2D finite element modeling tool. In general, to solve a problem with the FEMM, the problem design is defined and created in the appropriate pre-processor and then one of the FEA solvers available for that type of problem is used [25]. Finally, the results can be analyzed in the postprocessor. The application interface is easy to use and allows to define the problem adding suitable objects, symbols and materials. DXF files with problem design or expanded metadata can also be imported. The application supports LUA scripting using, which allows to schedule specific actions.

a. Electric charge point - analytical model

From the electrostatic (Coulomb's theorem) it is known that the electric field created in the air, for example by a point electric charge q at the distance r of the electric charge is in size:

$$E = \frac{q}{4\pi\varepsilon r^2}$$
(1)

And the electrical potential at a point at a distance R from the load creating the field is:

$$V = \frac{q}{4\pi\varepsilon r}$$
(2)

For example, using a Matlab script it is easy to calculate the electric field and potential at a given distance from an electric charge that creates this field-fig.1.

Matlab script for analytical solution

```
%calculation of the electric field and the electric potential of a point charge
clc,clear
r=input ( 'Distance to the electric charge (in m):' );
q=input ( 'Electric charge (in C):' );
epsilon_r=input ( 'Relative electrical permittivity:' );
epsilon_0=1/(36*pi*10^9); %electrical permittivity of vacuum in F/m
epsilon=epsilon_0*epsilon_r;
%Calculation of the electric field intensity in the considered point
E=q/(4*pi*epsilon*r^2)
%Calculation of the electric potential at the considered point
V=q/(4*pi*epsilon*r)
%Graphical representation
R=0.001:0.0005:0.04;
E=q./(4.*pi.*epsilon.*R.^2);
figure(1)
plot(R,E)% graph E=f(r)
axis([0.001 0.04 0 inf]);
xlabel('r(m)');ylabel('E(V/m)');
grid;
V=q./(4.*pi.*epsilon.*R);
figure(2)
plot(R,V)% graph E=f(r)
axis([0.001 0.04 0 inf]);
xlabel('r(m)');ylabel('V(V)');
grid;
```

Fig. 1. Matlab script for the electric charge-analytical solution

Running this script for the distance of 1cm and a positive electric charge of 1C gives the results from Fig.2 and Fig.3.

Analiytical solution

```
Distance to the electric charge (in m):0.01
Electric charge (in C):1
Relative electrical permittivity:1
E =9.0000E+13
V =9.0000E+11
```

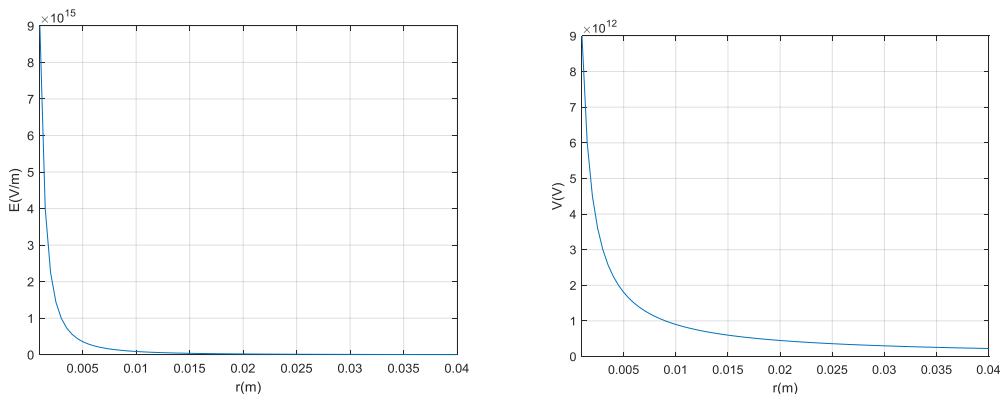Fig.2. Analytical results - electric charge

Fig. 3. Graphical representations of the analytical solution: E=f(r) și V=f(r)

b.   Electric point charge - FEMM model

Once a planar type problem has been defined, with units of measure for cm-fig length. 4, the first idea of implementing a point-of-form electric charge in FEMM starts with the point property definition, with "Add property" - fig.5. In the "Nodal Property" dialog fill in "Name" and select "Point Charge Density Property". Due to the parameter depth - Fig. 4, for total charge of 1C, at "Point Charge Density, C/m" must be entered 100 (100C/m·1cm=1C).
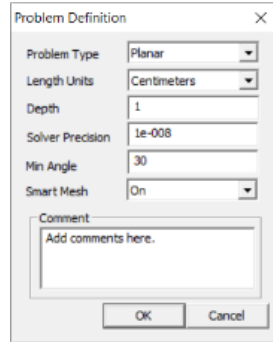
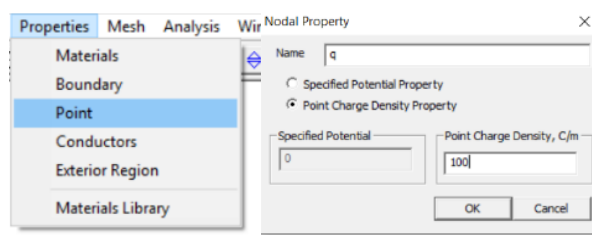Fig. 4. Defining the problem with the natural idea of implementation

Fig. 5. Electric charge simulation through "Nodal Property"

Attach the property to the creeat point and define the boundary condition (Dirichlet condition, i.e. the electrical potential is null on the border).We will use the "Asymptotic Boundary Condition" method (as described in the Appendix to the FEMM manual [25]) to mimic an unbounded geometry. -fig.6. The results are shown in Fig.7. It is noticed that the results of the numerical simulation are far from the analytical results, which means that the natural idea presented just now is far from reality.

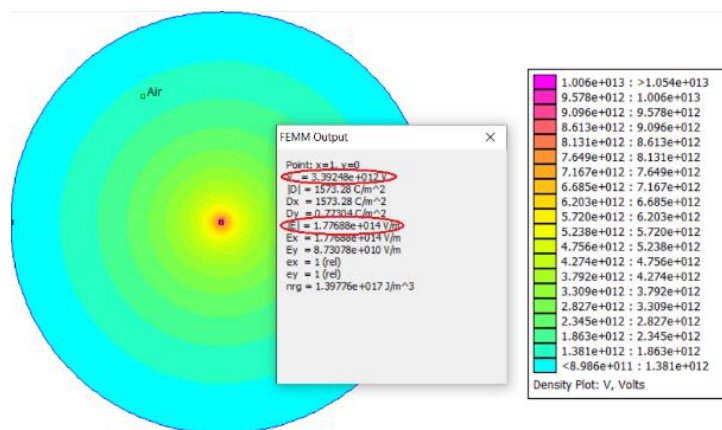Fig.6. Stages in the implementation of electric charge through "Nodal Property"

Fig. 7. Results of the implementation of electric charge through "Nodal Property"

The solution we propose starts from the observation that if in the expression of the electrical capacity of a spherical capacitor we make the outer radius to tend to infinity, we obtain the electrical capacity of a metal sphere charged with the electric charge q - table 1, whose electrical potential is determined from $C = \dfrac{q}{V}$, $V = \dfrac{q}{C} = \dfrac{q}{4\pi\varepsilon_0\varepsilon_r r}$, $dV = -Edr$,

where $E = -\dfrac{dV}{dr} = \dfrac{q}{4\pi\varepsilon_0\varepsilon_r r^2}$ as shown by the relation (1). Thus, the point electric charge can be simulated by a very small conductive metal sphere with a given charge, by defining an Axisymmetric Problem type-Fig. 8.

Table 1. Electrical capacity and electric field created by a metal sphere charged with electric charge

Spherical capacitor	$C = \dfrac{4\pi\varepsilon}{\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)}$	$E = \dfrac{q}{4\pi\varepsilon r^2}$
Metal sphere	$C = 4\pi\varepsilon R_1$	

Fig.8. Defining the problem in the proposed solution

The geometry of the problem is then constructed by placing the coordinate points (r,z) =(0,0.1) and (r,z) =(0,-0.1) that join by an arc to which the Conductors property is assigned-Fig.9.

Fig.9. Defining the conductors property "Sim_sarc" and its association to the model arc

An open border condition predefined by FEMM is used, with the Air material setting in the field of solutions -fig. 10. The FEM network is generated and the solver is launched into execution and the result is obtained from fig.11.
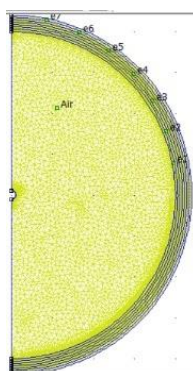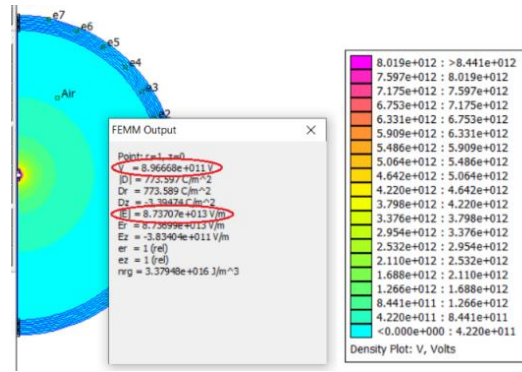

Fig. 10. The model of point electric charge

Fig. 11. Numerical results

3. Results and Discussion

Fig. 11. shows that the numerical result is very close to the analytical result. For a better concordance can be refined the mesh of finished elements - fig. 12.
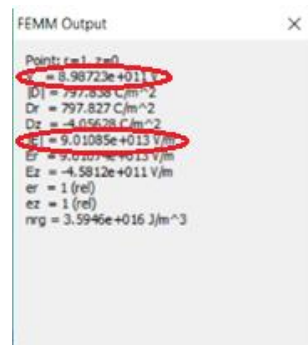

Fig. 12. Numerical results after refining the finished item network

Fig.13. shows the graphical representations of the numerical solution. Comparing these representations with the graphical representations of the analytical solution, a very good concordance is observed.
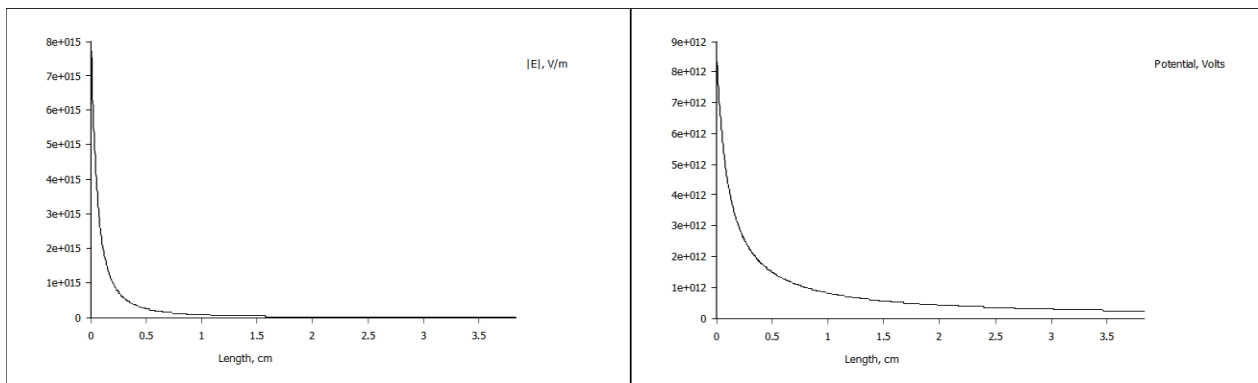

Fig. 13. Graphical representations of the numerical solution: E=f(r) și V=f(r)

The comparison between the analytical solution and the numerical solution can be better done by reading Table 2.

Table 2. Comparison between the analytical solution and the numerical solution proposed

Type of solution	Mesh size 0.1		Mesh size 0.01	
	E(V/m)	V(V/m)	E(V/m)	V(V/m)
Analytical solution	$9 \cdot 10^{13}$	$9 \cdot 10^{11}$	$9 \cdot 10^{13}$	$9 \cdot 10^{11}$
Numerical solution	$8.73707 \cdot 10^{13}$	$8.96668 \cdot 10^{11}$	$9.01085 \cdot 10^{13}$	$8.98723 \cdot 10^{11}$

For a better comparison between the analytical solution and the numerical solution, can be calculated the relative error in accord with relation:

$$E(\%) = \frac{|\text{analytical solution} - \text{numerical solution}|}{\text{analytical solution}} \cdot 100 \qquad (3)$$

In case of mesh size 0.1 the relative error is 2.92% and in case of refined mesh it obtained the relative error 0.12%. This analysis shows a very good concordance between the analytical values and the numerical values.

3.1 Use of FEMM model for point electric charge for a set of point electric charges

The model shown in the first subparagraph may be used in the case of a system of point electric charges if the system shows axial symmetry. For example, let's consider the example from din fig.14a, with $q_1 = 10\mu C$, $q_2 = 5\mu C$, $r_1 = 1cm$ and $r_2 = 2cm$.
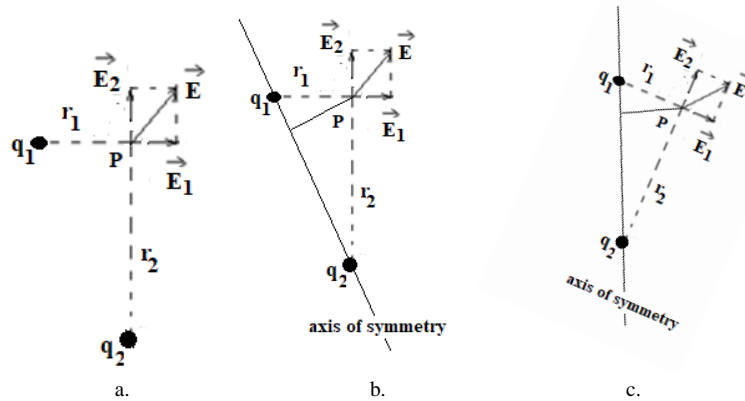

Fig. 14. System of 2 point charges

3.2 Analytical solution

The programming of the analytical solution was done in Matlab-fig 15, the solution being shown in fig. 16.

Fig. 15. Matlab script for the analytical solution -system of electric charges
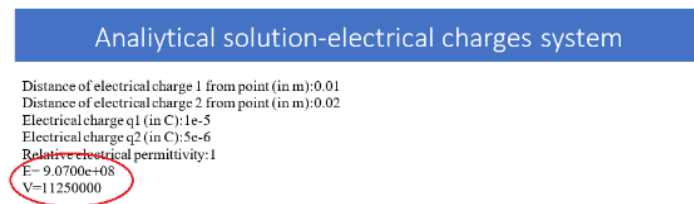
Fig. 16. Analytical results - system of electric charges

3.3 *Numerical solution*

To implement the numerical solution we choose the axis of symmetry to pass through the 2 charges, fig. 14b. Rotating fig. 14b. until the axis of symmetry reaches the upright position, fig. 14 c, the geometry of the problem to be solved using the point electric charge model shown in the previous paragraph is obtained. The numerical solution to the problem is shown in Fig. 17.

Comparing the analytical solution of the problem shown in Fig. 16 with the numerical solution represented in Fig. 17, a very good concordance is observed.
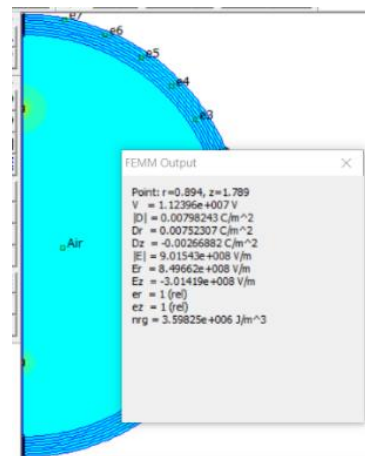

Fig. 17. Numerical results - electric charge system

4. Conclusions

The paper presents a solution for FEMM implementation of a point electric charge and the use of the proposed model in solving an electrostatic field problem for the case of a system of point electric charges. The model can be successfully used in electrostatics courses at both the university and college levels for a better understanding of the issue. The advantage of using the FEMM software package in the didactic activity is very high, given the fact that it is free and open source, easy to use and offers students the possibility to understand abstract concepts such as those of electric and magnetic fields and offers them skills in the design of electromagnetic devices, useful skills in their careers as future engineers.

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