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Mathematical Model for Adaptive Technology in E-learning Systems

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Abstract: The emergence of a large number of e-learning platforms and courses does not solve the problem of improving the quality of education. This is primarily due to insufficient implementation or lack of mechanisms for adaptation to the individual parameters of the student. The level of adaptation in modern e-learning systems to the individual characteristics of the student makes the organization of human-computer interaction relevant. As the solution of the problem, a mathematical model of the organization of human-computer interaction was proposed in this work. It is based on the principle of two-level adaptation that determines the choice of the most comfortable module for studying at the first level. The formation of an individual learning path is performed at the second level. The problem of choosing an e-module is solved using a fuzzy logic. The problem of forming a learning path is reduced to the problem of linear programming. The input data are the characteristics of the quality of student activity in the education system. Based on the proposed model the computer technology to support student activities in modular e-learning systems is developed. This technology allows increasing the level of student's cognitive comfort and optimizing the learning time. The most important benefit of the proposed approach is to increase the average score and increase student satisfaction with learning.

Index Terms: Computer Science, E-Learning, Adaptation, Fuzzy Logic, Optimization, Mathematical Model.

1. Introduction

The development of information technology has led to the possibility of mass introduction of e-learning through an e-learning system. Features of modern e-learning system are a large number of electronic resources. However, the accumulation of e-learning modules (e-Module) often does not have the desired effect. There are some problems in modern e-learning system. The most important are the low level of adaptation to the individual characteristics of the student and the inability to predict learning outcomes.

In the works of many scientists in human-computer interaction elements of the theory and practice of training operators of various automated systems have been developed [1-5]. However, the specificity of modern e-learning system excludes the full solution of the problems of choosing options for organizing human-computer interaction based on known methods and models. If there are several options of algorithms for the implementation of educational and cognitive activities, the learner (hereinafter "student") is needed to choose the most rational option, taking into account the existing constraints. Research aim is to improve the efficiency of human-machine interaction in learning systems. The criterion of effectiveness is student satisfaction and assessment of the quality of learning outcomes. Following research objectives would facilitate the achievement of this aim:

1. To conduct a meaningful analysis of the task of choosing the option of organizing human-computer interaction in e-learning system.

2. Develop a mathematical model for choosing the most comfortable learning module taking into account knowledge about learning styles.
3. Develop a mathematical model for optimizing the learning path taking into account the existing constraints.
4. To develop a computer system for organizing human-computer interaction in the e-learning system based on the designed mathematical model.

Its solution will improve the quality of human-computer interaction in e-learning system and rationally use learning time.

2. Related Works Analysis

The study relates to two main questions:

1. How to compare the modules available in the system with different types of presentation of educational material and individual student preferences?
2. How to form an individual learning path from the selected modules?

An option to overcome these difficulties can be the use of adaptation technologies [6].

In work [7] it is shown that the most applicable adaptation technologies are adaptation to the characteristics of the student and to the learning style. When organizing an adaptive dialogue, knowledge of the learning style can be applied. The theory and practice of learning styles is of great interest, but there are some controversies. One of them is that there is no single definition for the term “learning style”. In general, learning style refers to different approaches and learning paths. Papers [8, 9] provide an overview of learning styles and recommendations regarding them. An example of use is the VAK model [10]. According to this model, individuals are divided into categories according to their preferences for the perception of information.

One should note that, despite significant theoretical developments in this direction, in practice, it is still a problem to prepare various versions of e-Modules and their ratio to specific students for adapting educational material.

In work [11] a recommendation system is proposed. This system forms a user profile and, using data mining technologies, recommends learning content. In work [12] use Formalizing Logic Based Rules for recommendation educational material. In work [13] proposes an architecture for the recommendation of courses to a learner based on his/her profile. The profile of a learner is created by applying k-means algorithm to learner’s interaction data in MOODLE.

However, all of these works are suitable for quantitative data. In the task of choosing the basic type of the module according to the VAK model, qualitative indicators are used (for example, the student's preferences in terms of the degree of the presence of a kinesthetic component in the module). Therefore, a method for processing quality indicators is needed. The most suitable for this is fuzzy logic. In work [14] a fuzzy logic approach to assess web learner’s joint skills was proposed. The system will check the learner’s knowledge levels to provide the appropriate content.

In this article was proposed to define a Degree of Cognitive Comfort (DCC). It shows the degree to which the student's preferences match the parameters of the module. We propose to use fuzzy inferences system for calculation DCC.

The influence of the use of adaptive path technology on learning outcomes was investigated in [15]. It was indicated that the use of this technology made it possible to improve learning outcomes and save time.

To solve the problem of forming a learning path, various mathematical tools are used: Petri nets [16], graph theory [17], artificial intelligence technologies [4, 5, 18, 19], decision trees [20] and others. However, the application of these approaches does not make it possible to obtain a predicted value of the number of points in the final testing.

The formation of a learning path as an optimization problem was considered in papers [11-23]. However, these works did not take into account such individual psychophysiological indicators of a student as cognitive comfort [24] and functional state [25], which significantly affect learning outcomes.

The possibility of using various methods of organizing a dialogue in e-learning system in the formation of an individual learning path has not been investigated. Alternative options for organizing a dialogue in the e-learning system are due to the discrete nature of the educational material, different levels of knowledge of the e-Module fragments and the possibility of carrying out various methods of self-control.

To solve this problem, the concept of an agent-manager for e-learning support was proposed in [26-27]. The agent-manager allows you to comprehensively automate the main areas of quality assurance in e-learning:

1) Conduct an examination of the quality of e-Module,
2) Determine the individual psychophysiological parameters of the student,
3) Optimize human-machine interaction with the subsequent formation of an individual learning path.

Methods and models for items 1 and 2 are proposed in [26-30]. The problem of optimizing human-computer interaction remains open.

To solve this issue, the expediency of using the functional-structural theory of ergo-technical systems (FST ETS) [31] as a basic methodology was proved in [32]. The possibilities and limitations of the existing methods for their application for the task of organizing the student's educational activity in e-learning system are analyzed and the application of the apparatus of functional networks (FN) and the generalized structural method is substantiated.

The FST has developed a large number of tasks to optimize activities in human-machine systems [2, 31]. However, the specific limitations associated with the features of e-learning system, technological and morphological limitations were not taken into account. There are also no approaches to solving the optimization problem when it is possible to change the activity algorithm. Another disadvantage of this method is the lack of adequate input data. In the works [2], averaged data were used that do not take into account the individual characteristics of a person.

A possible solution to these problems may be the use of artificial neural networks to create an expert system for assessing the indicators of the quality of student activity.

All this suggests that it is appropriate to conduct research on the development of mathematical models for choosing options for the organization of human-computer interaction in e-learning system. This will be the main objective of this research study. This is important because the formation of an individual path based on quantitative indicators of the quality of student activity significantly improves the results of human-computer interaction.

We proposed a recommendation system that recommends bases e-Modules and create an individual learning path with its.

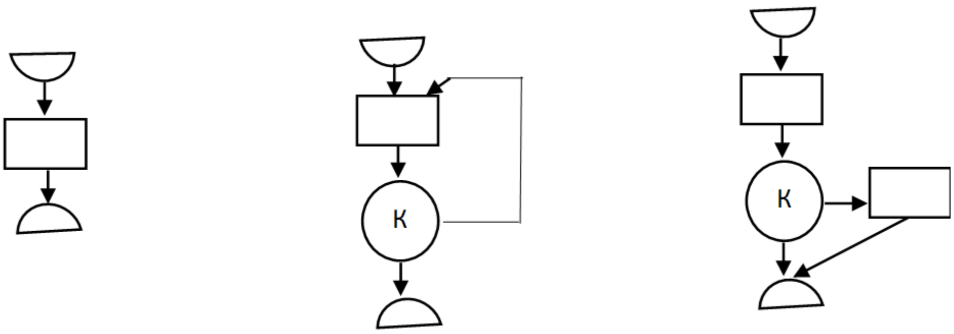
3. Mathematical Model of the Organization of Human-Computer Interaction

3.1. Substantive analysis of the tasks of choosing options for organizing dialogue human-computer interaction

As a rule, educational material is divided into separate modules [32]. The allocation of the levels of knowledge of the learning material determines the number of submodules. A sequence of submodules and self-control tests form an individual learning path. Self-control is a test of the quality of assimilation of educational material, which a student performs after studying a sub-module. Many options for organizing educational activities are created by choosing different variants of self-control (Fig. 1). The points obtained during the self-test do not count towards the final grade. Different methods of self-control require the allocation of different amounts of time and provide different levels of quality in learning activities.

Let us highlight the following features of the e-Module structure:

1. We have a learning module with different ways of organizing self-control and a reserve of learning time. All materials of the module are mandatory for study. You need to choose the option of self-control, which will provide the maximum score on the final test and the time for its implementation will not exceed the available time.



A *B* *C*

Fig.1. Possible variants for self-control: A - study of educational material, no self-control; B - study of educational material and testing with subsequent re-study of the topic in case of a low level of test results; C - study of educational material and testing with further study of only problematic parts of the topic in case of the low level of testing results.

2. In the structure of the module, submodules are distinguished according to the levels of knowledge of the educational material. It is necessary to choose those submodules and self-control that would provide the maximum score on the final testing. Moreover, the level of knowledge of the material should correspond to the student's desired level of assessment (for example, satisfactory or good, or excellent), and the time for completing the module should not exceed the available time.

Thus, the principle of adaptation can be formulated. At the first stage, choose an e-Module, the interaction with which provides the student with the maximum cognitive comfort of learning. At the second stage, choose a method of dialogue interaction, taking into account the time limit and highlighting the levels of knowledge of the educational material.

3.2. Formalized statement of the task of organizing human-computer interaction

We introduce the following notations: BP – set of e-Module, $BP = \{bp_m\}$, $m = \overline{1,M}$, M – the amount of basic e-Module, m – the number of e-Module; bp_m – m-th e-Module, CC – matrix of values of Degrees of Cognitive Comfort (DCC), $CC = \{c_{im}\}$, $m = \overline{1,M}$, $i = \overline{1,ES}$, cc_{im} – values of DCC of the i-th student for m-th basic e-Module, ES – number of students.

The first level of adaptation: The task is to choose the basic e-Module. At the first level, it is necessary to choose e-Module with a modality vector that will be closest to the human modality vector that provides the maximum DCC:

$$\text{MAX: } F_{cc}(m) \tag{1}$$

where m – the number of the basic e-Module.

The second level of adaptation: The task is to get the optimal plan for human-computer interaction. We introduce a binary controlled variable:

$$x_{kl} = \begin{cases} 1, \text{if for the } k\text{-}th \text{ level of complexity, the } l\text{-}th \text{ variant of } self-control \text{ is selected;} \\ 0, \text{in all other cases.} \end{cases} \tag{2}$$

Then, taking into account the introduced designations and the variable x_{kl}, you can set the objective function:

$$\text{MAX: } F_p(X_m). \tag{3}$$

Subject to:

$$\begin{gathered} T(X_m) \le T_0, \\ P(X_m) \ge p_0, \\ U(X_m) \le u_0, \\ x_{kl} \in \{0,1\},\ m = \overline{1,M}, \end{gathered} \tag{4}$$

where

$F_p(X_m)$ – assessment in points of the quality of studying the i-th submodule of the m-th module,
X_m – a variable that determines the variant of dialogue interaction,
$T(X_m)$ – implementation time of the dialogue interaction X_m,
T_o – the maximum allowable time for the implementation of dialogue procedures,
$P(X_m)$ – the level of knowledge assessment,
p_o – the minimum allowable level of knowledge assessment,
$U(X_m)$ – the level of knowledge of the submodule,
u_o – the maximum allowable level of knowledge,
$k = (\overline{1,k_0})$ – a variable that characterizes the level of knowlege of the fragment of the process of studying the m-th module,
$l = \overline{1,l_0}$ – a variable that characterizes the variant of self-control,
l_0 – the number of possible methods of dialogue interaction.

The solution of problem (3) – (4) allows you to determine the optimal variant of human-computer interaction and predict the result of passing the overall final control.

3.3. Selection of the basic e-Module for the implementation of training procedures

Let us define the DCC indicator in the form of some subjective assessment $D \in [0;1]$. The greater the value of this criterion, the greater the priority of the module to present it to a particular student. The simulation is based on the assumption: the more the parameters of the module meet the requirements of the student, the greater the value of *D*.

The correspondence model will represent a functional mapping of $R = (Pm, Pe) \rightarrow D \in [0;1]$, where $Pm = \{pm_i\}, i = \overline{(1, l)}$ – module parameters, $Pe = \{pe_i\}, i = \overline{(1, l)}$ – student preferences.

The set of specific parameters analyzed in each case depends on many factors. In this paper, we will limit the parameter of the module “Style of presentation of information” with the following components: verbal, visual, aural, and kinesthetic.

Let *D* denote the integral indicator of the DCC. To assess this indicator, we use the following information:

X – DCC for the visual component assessed taking into account the following particular indicators: x_1 – the degree of manifestation of the visual component of the module, x_2 – the student’s preferences for the visual component,

Y – DCC for the verbal component assessed taking into account the following particular indicators: y_1 – the amount of text material in the module, y_2 – the student’s preferences for the verbal component,

Z – DCC for the aural component taking into account the following particular indicators: z_1 – the degree of manifestation of the aural component in the module, z_2 – student's preference for the aural component,

V – DCC for the kinesthetic component assessed taking into account the following local indicators: v_1 –the number of interactive elements in the module, v_2 – the student's preference for the kinesthetic component.

The task of the assessment is to obtain the value of the student's DCC when working with a module with known parameters. The general scheme for solving the problem is a sequence of the following actions:

a) Assessment of the parameters of the e-Module and the student according to the selected indicators,
b) Fuzzy inference procedure,
c) Calculation of the DCC.

In the case of several available modules, ranking by DCC is performed.

The peculiarity of the module's indicators is that they are qualitative in nature, that is, they do not have an exact quantitative measurement. It is advisable to evaluate the indicators using the “thermometer principle”. Student parameters can either be determined automatically from the test results, or by directly entering their preferences.

Fuzzy inference. Fig. 2 shows the hierarchy of compliance indicators in the form of an output tree, which corresponds to the system of relations (5) - (9).

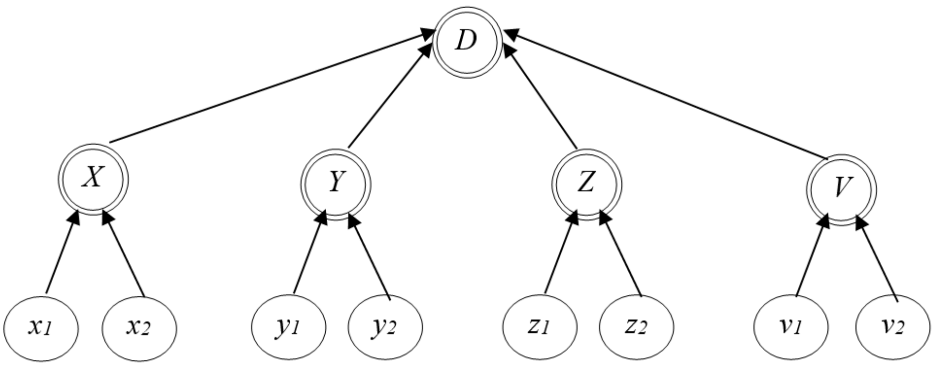


Fig.2. Hierarchical tree of logical inference.

$$R = f_R(X, Y, Z, V), \tag{5}$$

$$X = f_X(x_1, x_2), \tag{6}$$

$$Y = f_Y(y_1, y_2), \tag{7}$$

$$Z = f_Z(z_1, z_2), \tag{8}$$

$$V = f_V(v_1, v_2). \tag{9}$$

This ratio puts into correspondence fuzzy logic equations that allow determining the level of the indicator R by the maximum of the membership function:

$$\mu^{D_j}(X,Y,Z,V) = \max_{p=\overline{1,q_j}} \left\{ \min_{i=\overline{1,g}} \left[\mu^{X^{jp}}(X), \mu^{Y^{jp}}(Y), \mu^{Z^{jp}}(Z), \mu^{V^{jp}}(V) \right] \right\}, \tag{10}$$

$$\mu^{X_j}(x_1, x_2) = \max_{p=\overline{1,e_j}} \left\{ \min_{i=\overline{1,l}} \left[\mu^{x_i^{jp}}(x_i) \right] \right\}, \tag{11}$$

$$\mu^{Y_j}(y_1, y_2) = \max_{p=\overline{1,g_j}} \left\{ \min_{i=\overline{1,m}} \left[\mu^{y_i^{jp}}(y_i) \right] \right\}, \tag{12}$$

$$\mu^{Z_j}(z_1, z_2) = \max_{p=\overline{1,h_j}} \left\{ \min_{i=\overline{1,n}} \left[\mu^{z_i^{jp}}(z_i) \right] \right\}, \tag{13}$$

$$\mu^{V_j}(v_1, v_2) = \max_{p=\overline{1,t_j}} \left\{ \min_{i=\overline{1,k}} \left[\mu^{v_i^{jp}}(v_i) \right] \right\}. \tag{14}$$

The fuzzy inference algorithm has the form:

1. The vector of values of input variables is fixed ($x_1^*, x_2^*, y_1^*, y_2^*, z_1^*, z_2^*, v_1^*, v_2^*$).
2. The value of the membership function of terms-estimates of input variables is determined.
3. Using relations (10) - (14), the membership functions of the terms-estimates of the initial quantity corresponding to the vector of values of the input variables are calculated.
4. An estimate is determined, for which the membership function is maximal:

$$\mu^{D_j^*}(X, Y, Z, V) = \max_{j=\overline{1,r}} [\mu^{D_j}(X, Y, Z, V)] \to D = D_j^*. \tag{15}$$

The fuzzy number at the output of the algorithm becomes crisp:

$$\tilde{D} = \left\{ \frac{\mu^{D_1}(X,Y,Z,V)}{D_1}, \frac{\mu^{D_2}(X,Y,Z,V)}{D_2}, \frac{\mu^{D_3}(X,Y,Z,V)}{D_3}, \frac{\mu^{D_4}(X,Y,Z,V)}{D_4} \right\}. \tag{16}$$

Assessment of the integral indicator. Suppose that the linguistic variables x_i, y_i, z_i, v_i are estimated by fuzzy terms H – low, C – medium, B – high, determined using the Gaussian membership function.

Using fuzzy terms, let us define knowledge about relation (5) – (9) in the form of knowledge matrices.

Table 1 presents a fragment of the matrix for relation (5). Each group of lines reflects a conditional statement that connects the fuzzy values of the input and output variables.

For example, Table 1 shows that the following statement: **IF** ($X = B$) **AND** ($Y = B$) **AND** ($Z = B$) **AND** ($V = B$) **THEN** $R = B$ is a condition of high conformity of the module.

Table 1. Fuzzy knowledge base for the integrated indicator DCC (fragment)

X	***Y***	***Z***	***V***	***D***
B	B	B	B	B
C	C	C	C	C
H	H	H	H	H

Fuzzy logic equations, which are put in accordance with Table 1, make it possible to assess the integral indicator of compliance for fixed values of local indicators. Fuzzy inference for a hierarchical system of indicators occurs by fuzzy inference for intermediate peaks with the subsequent transfer of clear values of these variables to fuzzy systems of the next level of the hierarchy.

3.4. Obtaining the optimal plan of human-machine interaction in e-learning system

Enter the notation:

X_m is a matrix of possible variants for the process of human-machine interaction in the environment of the basic m-th e-Module, $m = \overline{1, M}$, where M is the number of the e-Module. The elements of the matrix x_{kl} define the objects for constructing an individual learning path in the basic e-Module. The variable $k = \overline{1, k_0}$ characterizes the level of knowledge of the submodule of the basic e-Module. The variable $l = \overline{1, l_0}$ characterizes the variant of constructing dialogue procedures, where l_0 is the number of possible options for organizing the control procedure.

B_m is a matrix that contains learning scores in points in the environment of the basic e-Module. The elements of the matrix b_{kl} ($k = \overline{1, k_0}$, $l = \overline{1, l_0}$) are formed because of calculating individual predictive values for each student.

T_m is a matrix that contains the value of time to study the submodules of the basic e-Module. The elements of the matrix t_{kl} ($k = \overline{1, k_0}$, $l = \overline{1, l_0}$) are formed as a result of calculating individual predictive values for each student.

You can set the objective function, which determines, for the m-th e-Module, the number of points obtained for the correct answers during self-monitoring if the dialogue interaction took place according to the variant x_{kl}:

$$\text{MAX:} \sum_{k=1}^{k_0} \sum_{l=1}^{l_0} b_{kl} \cdot x_{kl} \tag{17}$$

Constraints on the mathematical expectation of the execution time of the dialogue procedure for studying the m-th basic e-Module according to the l-th variant with a knowledge level of k:

$$\sum_{k=1}^{k_0} \sum_{l=1}^{l_0} T_{kl} \cdot x_{kl} \le T_0. \tag{18}$$

Constraints on the level of knowledge (U_0 – the maximum allowable knowledge level):

$$\sum_{k=1}^{k_0} x_{kl} \le U_0 - k + 1,\ k = (\overline{1, k_0}),\ \ l = (\overline{1, l_0}). \tag{19}$$

There is only one way to interact at each step:

$$\sum_{l=1}^{lo} x_{kl} = 1\,; \tag{20}$$

$$x_{kl} \in \{0,1\}. \tag{21}$$

Thus, the solution of problem (17)-(20) allows us to determine the optimal plan of human-machine interaction $X=(x_{kl})$, ($k = (\overline{1, k_0})$, $l = (\overline{1, l_0})$).

If there is a sufficiently complete database of the results of the actual human-machine interaction, the problem of obtaining the original data is reduced to the problem of approximation. To solve this problem, the apparatus of artificial neural networks (ANN) is used.

An artificial neural network of the MIMO type (multi-input multi-output) with one hidden layer has been developed. The parameters of the student (level of motivation, input control, cognitive comfort, and functional state), parameters and features of the structure of the module (level of knowledge of the module, method of self-control, etc.) are set as inputs. Interaction results (training time, learning result in points) are given as outputs. The setting of weights of ANN in the training sample “known input” – “known output” was based on the algorithm of inverse error propagation. The ANN’s ability to generalize makes it possible to obtain data that are absent in the training sample.

3.5. *An example of modeling the organization of human-computer interaction in e-learning system*

The input data for the first stage of adaptation are the parameters of the e-Module (Fig. 3) and the parameters of the student (Fig. 4). Since they are of a qualitative nature, the measurements are carried out according to the method [29] on the thermometer scale. The modality test was used to determine the student parameters [33].

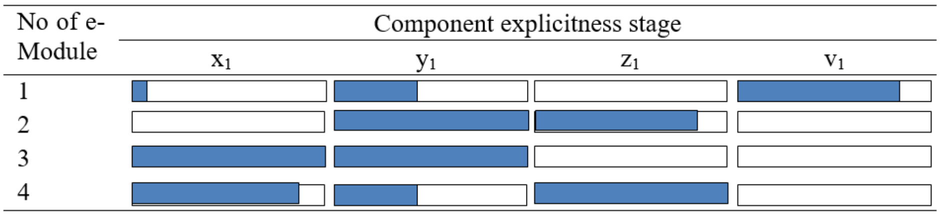


Fig.3. Evaluation of e-learning module parameters.

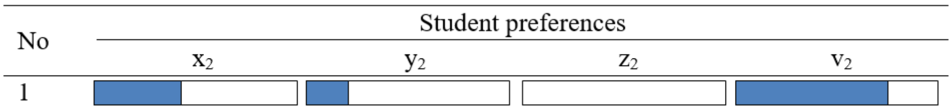


Fig.4. Evaluation of student parameters.

Because of applying the model according to item 4.1, we obtained the following values of DCC (Table 2).

Table 2. DCC

No of e-Module	1	2	3	4
DCC	0.63	0.39	0.22	0.29

Therefore, the module No1 is chosen for the basic e-Module.

The second stage of adaptation makes it possible to obtain an optimal plan for choosing an option for dialogue interaction using various methods of self-control (Fig. 1). Table 3 shows the input data for the optimization problem for the selected three levels of knowledge of the e-Module.

Table 3. Input data for the optimization problem

Levels of knowledge of the module	Score, points	Time, min.
1 2 3	$B = \begin{pmatrix} 8 & 9 & 9 \\ 8 & 9 & 9 \\ 8 & 10 & 10 \end{pmatrix}$	$T = \begin{pmatrix} 10 & 18 & 15 \\ 9 & 19 & 18 \\ 9 & 19 & 20 \end{pmatrix}$

It is necessary to obtain an individual student learning path for different values of time reserves $T_0=\{30, 45, 60\}$.

Table 4 shows the results of solving the optimization problem for the input data (Table 3) and various time reserves T_0.

The greater the reserve of available time, the greater the level of quality of training can be achieved. The obtained matrices correspond to the learning paths shown in Fig. 5. Possible options for the organization of self-control correspond to Fig.1 (for $l = 1$ – Fig.1, A; for $l = 2$ – Fig. 1, B; for $l=3$ – Fig. 1, C).

Table 4. The results of solving the optimization problem

Time reserve $T_{0,}$ min	The optimal plan	Forecast of points scored per module / possible number of points	Time forecast, min
30	$X=\begin{pmatrix}1&0&0\\1&0&0\\1&0&0\end{pmatrix}$	24/30	28
45	$X=\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}$	27/30	43
60	$X=\begin{pmatrix}0&0&1\\0&1&0\\0&0&1\end{pmatrix}$	28/30	54

The path (Fig. 5, a) can be used with minor time reserves. It does not provide self-control but provides the lowest level of learning quality.

The path (Fig. 5, b) requires more time, but provides an opportunity to refine fragments of educational material with an insufficient level of quality of learning, which improves the quality of learning.

The path (Fig. 5, c) provides the highest level of learning quality through re-study of the material in case of low self-control results but requires even more time.

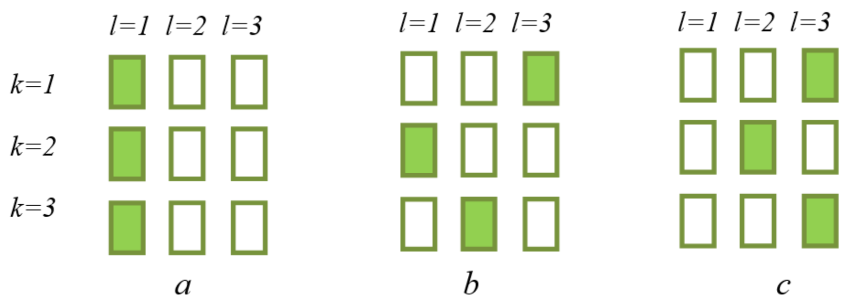


Fig.5. Variants of individual learning paths for different time reserves: *a* – T_0=30; *b* – T_0=45; *c* – T_0=60, where *k* is the level of knowledge, and *l* is a variant of self-control.

4. Results and Discussions

Computer technology for the organization of human-computer interaction in e-learning system is implemented by the software package “Modelling qualimetric complex of dialogue interaction in the student-computer system”.

Basic functional requirements for the software package:

- quality examination and evaluation of e-Module parameters;
- determination of the parameters of students;
- determination of the degree of compliance of the e-Module parameters with the student's preferences;
- determination of the current functional state of the student;
- preparation of initial data is the definition of probabilistic-temporal indicators of the quality of training for the assessment of algorithms of activity;
- optimization is the choice of an individual learning path in the environment of basic e-Module;
- maintaining a database of the results of dialogue interaction in the e-learning system.

Fig. 6 shows the composition of software and information tools required to automate the task of organizing human-computer interaction in e-learning system.

To study the effectiveness of the developed models and computer technology, the experiments were carried out based on the Sumy National Agrarian University. In the e-learning system MOODLE, a course "Informatics" was developed. This course contained four options for basic modules of educational content, three levels of knowledge and three options for conducting self-monitoring of learning outcomes. Fig. 7 shows the user interface in the formation of an individual learning path.

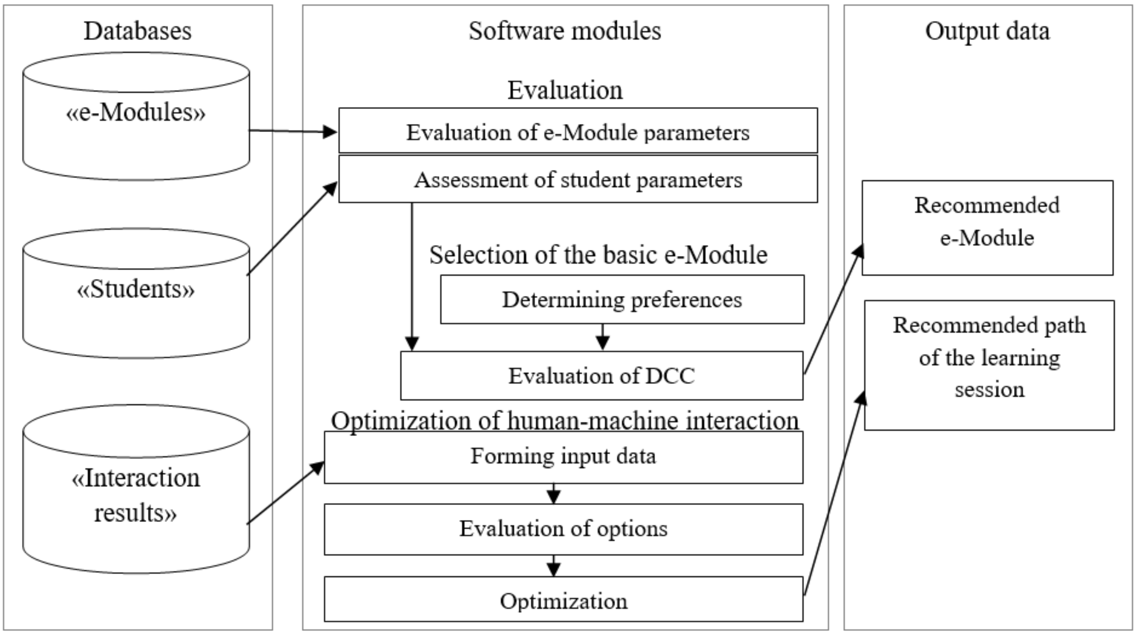


Fig.6. A set of information and software tools for organizing human-computer interaction in the e-learning system.

Test A corresponds to the method of self-control (Fig. 1, B), which provides a complete re-study of the material in the case of an insufficient number of correct answers to test questions.

Test B corresponds to the method of self-control (Fig. 1, C), which provides additional study of only problematic fragments of educational material in the case of an insufficient number of correct answers to test questions.

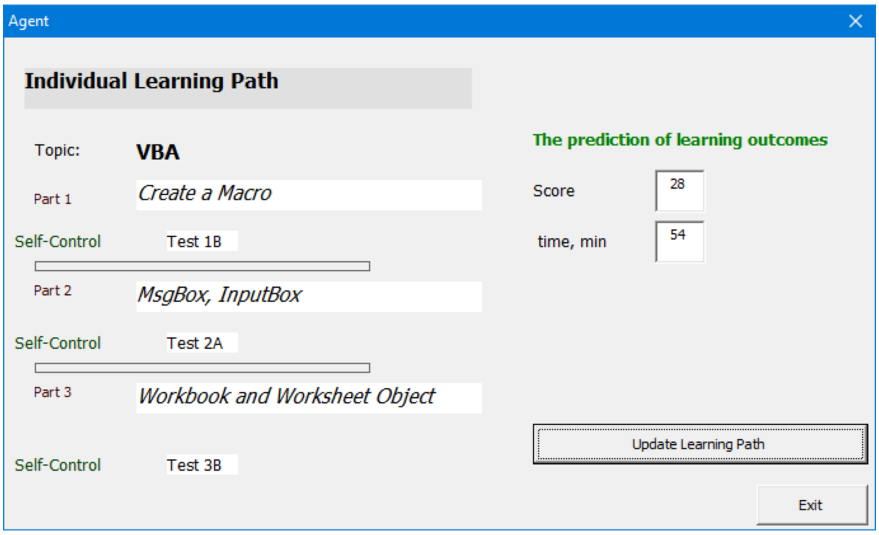


Fig.7. Window for the formation of an individual learning path.

Two groups of students were selected for the experiment. The experimental group was trained with the support of a recommendation system. The control group was trained using traditional technology. In the experimental group, at the first step, VAK-testing of students was carried out. Then these students were asked to look through samples of basic modules and choose the most comfortable one for their study. The coincidence of the student's choice and the recommendation of the system was 86.73%. The second step determined the individual learning path. The average score in the experimental group raised from 72.32 to 81.43. An important indicator is a decrease in student refusals from a learning session and an increase in learning satisfaction.

We also applied this recommendation system for the course "Information and Cybersecurity Standards" is being developed on the TalentLMS cloud learning platform (Fig.8). We are currently conducting a study on the effectiveness of system implementation for this platform.

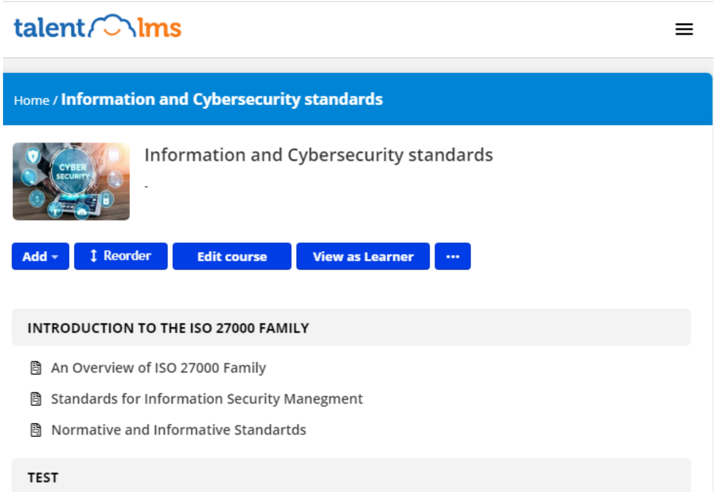


Fig.8. The course "Information and Cybersecurity Standards".

The learning materials of the course are formed using the knowledge of learning styles according to the VAK model. So, for example, in Fig. 9 shows a fragment of a course for a Visual learning style. The Auditory style is implemented as an audio version of the learning material.

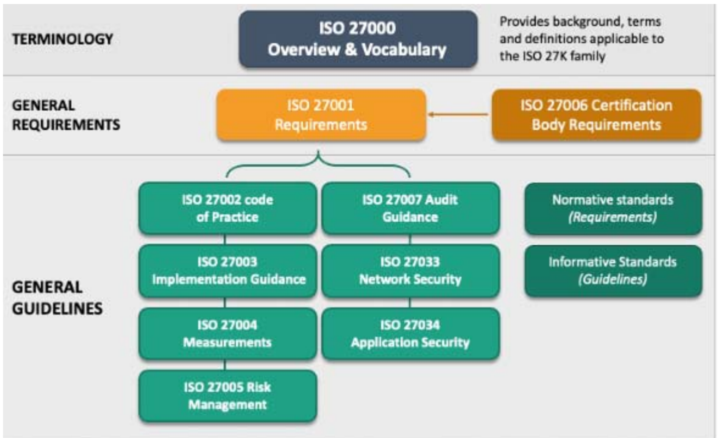


Fig.9. Fragment of a course for a Visual learning style.

The formulation of the problem of choosing options for organizing human-machine interaction in the e-learning system as an optimization problem has been developed. The problem was solved by implementing the principle of two-level adaptation. At the first level, we select an electronic module, the parameters of which are most consistent with the student's requirements. At the second level, we form an adaptive individual learning path in the environment of this module.

In contrast to the known optimization models [1, 13, 14], which use the indicator "probability of error-free execution", the model uses the assessment of learning quality in points. The advantage of the proposed approach is that the developed model takes into account the desired level of learning knowledge and features of the structure of the e-Module.

In contrast to the [14] models, which use average human performance, a model based on artificial neural networks has been developed. The advantage of this model is the use of the actual results of human-machine interaction, which are accumulated in e-learning system database and taking into account the individual psychophysiological parameters of the student.

We can identify the following main limitations inherent in this study: only algorithmic discrete student activity is considered, the course has a modular structure, student performance indicators are unchanged when working with e-Module, control is carried out in the form of closed-type tests.

However, we should note that the actual learning results might not match the values determined by the model. Based on this, it is necessary to develop a technology for the possible reconstruction of the individual learning path, taking into account the new actual values.

5. Conclusions and Future Work

1. We carried out a meaningful analysis of the problem of choosing a variant of organizing human-computer interaction in e-learning system. It has been determined that alternative options for organizing a dialogue in e-learning system are due to the discrete nature of the educational material, the features of the structure of the e-Module and the possibility of organizing the methods of human-computer interaction through the use of various methods of self-control.

2. A mathematical model of the organization of human-computer interaction, based on the principle of two-level adaptation, has been developed. At the first level, the basic e-Module is selected. The apparatus of fuzzy logic is chosen as the main tool for mathematical formalization [34]. A fuzzy inference model was developed to quantify the degree of compliance. At the second level, the search for the optimal plan of human-computer interaction in the environment of the basic e-Module is performed. The problem comes down to a linear programming problem. An artificial neural network provides the formation of input data. According to the obtained optimal plan and taking into account the existing restrictions, the parameters of the students and the features of the modular structure, an individual learning path is formed.

3. We have developed a computer technology, which, unlike the known ones, allows us, within the framework of a single system, to automate comprehensively all the processes necessary for organizing high-quality human-computer interaction in e-learning system:

- comprehensive consideration of factors influencing the performance of students;
- quality assessment of e-learning modules;
- automatic forecasting of learning outcomes taking into account individual features;
- automatic selection of the optimal variant of human-computer interaction.

The most important benefit of the proposed approach is improving the average score and increasing student satisfaction in learning.

The direction of further research is to develop a model of human-machine interaction, which will take into account the actual learning outcomes and update the proposed individual path. It is also necessary to develop agent-manager software that implements cloud computing.

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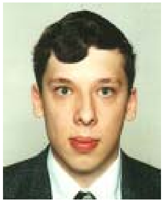

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