I.J. Modern Education and Computer Science, **2021, 6, 25-29**
Published Online December 2021 in MECS (http://www.mecs-press.org/)
DOI: 10.5815/ijmecs.2021.06.03

Bond Graph Modelling of a Rotary Inverted Pendulum on a Wheeled Cart

Jessica A. Onwuzuruike
Department of Electric and Electronics Engineering, Nile University of Nigeria, Abuja, 900108, Nigeria
Email: jessica.ada21@yahoo.com

Suleiman U. Hussein
Department of Electric and Electronics Engineering, Nile University of Nigeria, Abuja, 900108, Nigeria
Email: elsuligh@gmail.com

Received: 01 July 2020; Revised: 12 August 2020; Accepted: 26 September 2020; Published: 08 December 2021

Abstract: There are some systems that are yet to be modelled using certain methods. One of them is Rotary Inverted Pendulum (RIP) on a wheeled cart which is yet to be modeled using the bond graph technique. Therefore, this work explored the bond graph technique for this system. Using this technique, which uses the concept of energy (power) transfer between elements in a system, the system was modeled. Then, the state space equations of the system, which give the first-order differential equations, were derived. It was observed that the system has five state variables because of the five integrally causal storage elements.

Index Terms: Bond graph, wheeled rotary inverted pendulum, state space

1. Introduction

An inverted pendulum is one with its mass on the top and pivot on the bottom. This pendulum is not stable as compared to the normal pendulum which has its pivot on the top and mass on the bottom. Therefore inverted pendulum must be actively balanced for it to remain upright [1].

The wheeled inverted pendulum (WIP) is frequently realised with the pivot attached on a moving load, chart or wheel that can move horizontally. There are different designs for various wheeled inverted pendulum in [2–6], after which they focused mainly on the control and stability of the WIP. Due the unsteady nature of the inverted pendulum controlling its movement was presented in most literature.

Rotary inverted pendulum (RIP) consist of a DC motor connected to an arm unit and a pendulum [7]. The arm unit is driven by a DC motor which has a control system to balance the pendulum in the upright position [7]. A variety of RIPs have been modelled, simulated and controlled. Since RIPs are under actuated mechanical systems, they have been used as an experimental system to describe and test diverse types of control algorithms owing to their nonlinear and unstable behaviour [8].

Numerous applications of the inverted pendulum include robotics for 2D/3D robot walking, stability, fall prevention [2,9,10] some of which model the standing human behaviour.

2. Related Works

Modelling a Quanser Rotary Inverted Pendulum with bond graph technique and computation of the mathematical model was done in [11], where the RIP system was further divided into smaller subsystems and modelled separately. Then the different models were added up to give a final model of the Quanser RIP. BondSim software was used for the modelling and simulation in [11].

In [12] a robotic leg was modelled using bond graph. This was achieved by connecting simpler, similar models. Formulating a set of simple design with similar dynamic behaviour to the human leg in various phases of motion was the aim of the work.

Bond Graph method was used to model an inverted pendulum on cart in [13]. State space equation for the system was obtained and PID controller was designed was done in [13]. MATLAB was used in the development and analysing the PID controller.

The research on inverted pendulum with applications in balancing robot , the balance control problem of rockets launching [7], has gained interest over the last decade at research, industrial and hobby level around the world. A new proposed system which includes the characteristics of a RIP and two wheeled robot, referred to as TWRIP has been recommended in [8] as a new research area. The TWRIP can be used for testing of higher order intelligent controllers [8].

Modelling of rotary inverted pendulum on a wheeled cart using bond graph technique has not yet been explored. This work therefore focused on obtaining system equations that totally describe the rotary inverted pendulum on a wheeled cart using bond graph.

3. Modelling of the System Using Bond Graph

The proposed Rotary Inverted Pendulum on a wheeled cart is presented in Fig. 1. This wheeled is a multi-energy domain system because it has mechanical and electrical components. Bond graph technique is an appropriate tool used in modelling such systems[14].

Bond graph uses the concept of energy or power transfer between elements in a system and a diagrammatic representation is made with bonds and elements to illustrate this power transfer [15]. Elements could be single port as in resistive element (R), compliance element (C), inertial element (I), source of effort (SE) and source of flow (SF) or two-port elements as in transformers (TF) and gyrators (GY) [14,15]. The power in an element is a product of its effort and flow, bonds are used to represent this exchange of power between elements.

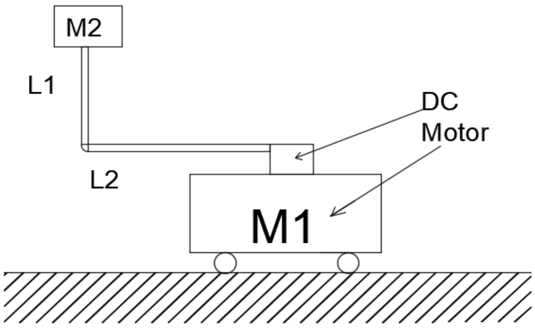

Fig. 1. Proposed rotary inverted pendulum on a wheeled cart.

The rotary inverted pendulum on a wheeled cart consists of a chart with mass M_1 been moved by a DC motor.

Let the radii of the wheels be r rotational inertia be J_1 , rotational damping of the driven wheels R_1 .

Elements of the cart's DC motor considered include voltage source E_1 , armature inductances L_{a1} , and reactance R_{a1}.

The second DC motor that controls the rotating arm L_2 have voltage source E_2 , armature inductances and reactance L_{a2} and R_{a2} respectively. The rotational inertia and damping of L_2 are J_2 and R_2 respectively and R_3 , J_3 are the gear resistances and inertia of the rotating arm. The arm L_1 has a rotational motion of J_4 and g is the acceleration due to gravity.

Bond graph of the proposed Rotary Inverted Pendulum on a wheeled cart is shown in Fig. 2.

The gyrator modulus of GY1 and GY2 is K_1 and K_2 respectively. TF1 is the driven wheel with a transformation ratio of r^{-1}. The gear ratio, TF2, for the rotating arm is N_1 / N_2. Transformation ratio of MTF1, MTF2, MTF3 are L_2 , $L_1 \cos\theta$ and $-L_1 \sin\theta$ respectively. Where θ is the angle that L_1 is been displaced.

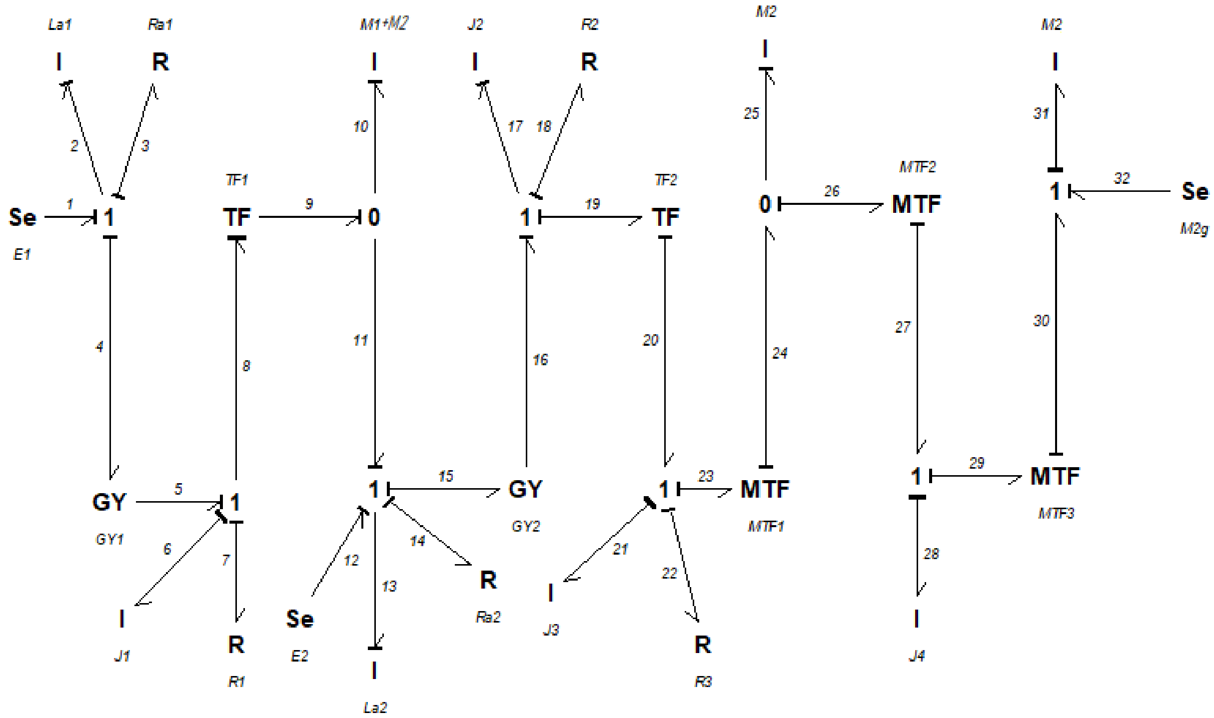

Fig. 2. Bond graph of the proposed rotary inverted pendulum on a wheeled cart

4. State Space Equation

State variables are momentums of the integrally causal storage element which are P_2, P_{10}, P_{13}, P_{21} and P_{25}. P_2 is the momentum for the armature inductance of the cart's DC motor. Momentum of the sum of both masses is P_{10}. P_{13} is the armature inductance of the arm's DC motor momentum. P_{21} and P_{25} are the momentums of the arm's gear inertia and the horizontal component of mass M_2 respectively .The differential equation for the system as derived from the bond graph in Fig. 2 is given by Equation (2) in the form of (1).

$$\dot{x} = Ax + Bu \tag{1}$$

$$\begin{bmatrix} \dot{P}_2 \\ \dot{P}_{10} \\ \dot{P}_{13} \\ \dot{P}_{21} \\ \dot{P}_{25} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} \end{bmatrix} \begin{bmatrix} P_2 \\ P_{10} \\ P_{13} \\ P_{21} \\ P_{25} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} \tag{2}$$

Where

$$A_{11} = \frac{-R_{a1}}{L_{a1}},\ A_{12} = \frac{-K_1 N_2}{N_1 (M_1 + M_2)},\ A_{13} = \frac{-K_1 N_2}{N_1 L_{a2}}$$

$$A_{21} = \frac{N_1 N_2 L_{a2} K_1 (M_1 + M_2)}{L_{a1}\left(N_1^2 L_{a2} (M_1 + M_2) + N_1^2 N_2^2 J_1 (M_1 + M_2) + N_2^2 J_1 L_{a2}\right)}$$

$$A_{22} = \frac{-R_1 N_2 L_{a2}}{N_1^2 L_{a2}\left(M_1 + M_2\right) + N_1^2 N_2^2 J_1\left(M_1 + M_2\right) + N_2^2 J_1 L_{a2}}$$

$$A_{23} = \left(R_{a2} J_1 - R_1 N_2^2 L_{a2}\right)\left(\frac{M_1 + M_2}{L_{a2}\left(N_1^2 L_{a2}\left(M_1 + M_2\right) + N_1^2 N_2^2 J_1\left(M_1 + M_2\right) + N_2^2 J_1 L_{a2}\right)}\right)$$

$$A_{24} = \frac{K_2 J_1\left(M_1 + M_2\right)}{rJ_3\left(N_1^2 L_{a2}\left(M_1 + M_2\right) + N_1^2 N_2^2 J_1\left(M_1 + M_2\right) + N_2^2 J_1 L_{a2}\right)}$$

$$A_{31} = A_{21},\ A_{32} = A_{22}$$

$$A_{33} = \left(R_{a2} J_1 - R_1 N_2^2 L_{a2}\right)\left(\frac{M_1 + M_2}{L_{a2}\left(N_1^2 L_{a2}\left(M_1 + M_2\right) + N_1^2 N_2^2 J_1\left(M_1 + M_2\right) + N_2^2 J_1 L_{a2}\right)}\right) - \frac{R_{a2}}{L_{a2}}$$

$$A_{34} = \frac{K_2 J_1\left(M_1 + M_2\right)}{rJ_3\left(N_1^2 L_{a2}\left(M_1 + M_2\right) + N_1^2 N_2^2 J_1\left(M_1 + M_2\right) + N_2^2 J_1 L_{a2}\right)} - \frac{K_2}{rJ_3}$$

$$A_{43} = \frac{K_2 r J_3^2\left(M_2 + M_2 L_1^4 \sin^2\theta\cos^2\theta + L_1^2 J_4 \cos^2\theta\right)}{L_{a2}\left(J_3\left(r^2 J_3 + J_2\right)\left(M_2 + M_2 L_1^4 \sin^2\theta\cos^2\theta + L_1^2 J_4\cos^2\theta\right) + L_1^4 L_2^2 r^2 M_2^2 \sin^2\theta\cos^2\theta + L_1^2 L_2 J_4 J_3 r^2 M_2\right)}$$

$$A_{44} = \frac{J_3\left(R_2 - r^2 R_3\right)\left(M_2 + M_2 L_1^4 \sin^2\theta\cos^2\theta + L_1^2 J_4\cos^2\theta\right)}{J_3\left(r^2 J_3 + J_2\right)\left(M_2 + M_2 L_1^4 \sin^2\theta\cos^2\theta + L_1^2 J_4\cos^2\theta\right) + L_1^4 L_2^2 r^2 M_2^2 \sin^2\theta\cos^2\theta + L_1^2 L_2 J_4 J_3 r^2 M_2}$$

$$A_{53} = \left[\frac{\left(L_1^4 L_2 M_2 \sin^2\theta\cos^2\theta + L_1^2 L_2 J_4\cos^2\theta\right)\left(K_2 r J_3^2\left(M_2 + M_2 L_1^4\sin^2\theta\cos^2\theta + L_1^2 J_4\cos^2\theta\right)\right)}{L_{a2}\left(J_3\left(r^2 J_3 + J_2\right)\left(M_2 + M_2 L_1^4\sin^2\theta\cos^2\theta + L_1^2 J_4\cos^2\theta\right) + L_1^4 L_2^2 r^2 M_2^2\sin^2\theta\cos^2\theta + L_1^2 L_2 J_4 J_3 r^2 M_2\right)}\right]$$
$$\times\left[\frac{M_2}{M_2 + M_2 L_1^4\sin^2\theta\cos^2\theta + L_1^2 J_4\cos^2\theta}\right]$$

$$A_{54} = \frac{\left(R_2 - r^2 R_3\right)\left(M_2 + M_2 L_1^4\sin^2\theta\cos^2\theta + L_1^2 J_4\cos^2\theta\right)}{J_3\left(r^2 J_3 + J_2\right)\left(M_2 + M_2 L_1^4\sin^2\theta\cos^2\theta + L_1^2 J_4\cos^2\theta\right) + L_1^4 L_2^2 r^2 M_2^2\sin^2\theta\cos^2\theta + L_1^2 L_2 J_4 J_3 r^2 M_2}$$

$$A_{14} = A_{15} = A_{25} = A_{35} = A_{45} = A_{51} = A_{52} = A_{55} = 0$$

$$B_1 = E_1$$

$$B_2 = \frac{-J_1\left(M_1 + M_2\right)E_2}{rJ_3\left(N_1^2 L_{a2}\left(M_1 + M_2\right) + N_1^2 N_2^2 J_1\left(M_1 + M_2\right) + N_2^2 J_1 L_{a2}\right)}$$

$$B_3 = E_2 - \frac{J_1\left(M_1 + M_2\right)E_2}{rJ_3\left(N_1^2 L_{a2}\left(M_1 + M_2\right) + N_1^2 N_2^2 J_1\left(M_1 + M_2\right) + N_2^2 J_1 L_{a2}\right)}$$

$$B_4 = \frac{M_2^2 g L_1^2 L_2 r^2 J_3^2 \sin\theta\cos\theta}{J_3\left(r^2 J_3 + J_2\right)\left(M_2 + M_2 L_1^4\sin^2\theta\cos^2\theta + L_1^2 J_4\cos^2\theta\right) + L_1^4 L_2^2 r^2 M_2^2\sin^2\theta\cos^2\theta + L_1^2 L_2 J_4 J_3 r^2 M_2}$$

$$B_5 = \left[\frac{\left(L_1^4 L_2 M_2\sin^2\theta\cos^2\theta + L_1^2 L_2 J_4\cos^2\theta\right)\left(M_2^2 g L_1^2 L_2 r^2 J_3^2\sin\theta\cos\theta\right)}{J_3\left(r^2 J_3 + J_2\right)\left(M_2 + M_2 L_1^4\sin^2\theta\cos^2\theta + L_1^2 J_4\cos^2\theta\right) + L_1^4 L_2^2 r^2 M_2^2\sin^2\theta\cos^2\theta + L_1^2 L_2 J_4 J_3 r^2 M_2}\right]$$
$$\times\left[\frac{M_2}{M_2 + M_2 L_1^4\sin^2\theta\cos^2\theta + L_1^2 J_4\cos^2\theta}\right]$$

5. Conclusion

The paper aimed at exploring the bond graph technique for Rotary Inverted Pendulum (RIP) on a wheeled cart system. The aim was achieved by first, modelling the system using bond graph technique, and then deriving the state space equations of the system. Based on the number of integrally causal storage elements in the system, the system has five state variables. Since the state space representation of the system is now readily available different controllers, such as Linear Quadratic Regulator (LQR), Linear Quadratic Gaussian (LQG), Proportional Integral Derivative (PID) and a combination with artificial intelligences can be applied to the system to control and analyze its performance.

References

[1] Agarana, M. C., and Ajayi, O. O. "Dynamic Modeling and Analysis of Inverted Pendulum Using Lagrangian-Differential Transform Method." 2017.

[2] Dai, F., Gao, X., Jiang, S., Guo, W., and Liu, Y. "A Two-Wheeled Inverted Pendulum Robot with Friction Compensation." Mechatronics, Vol. 30, 2015, pp. 116–125.

[3] Cui, R., Guo, J., and Mao, Z. "Adaptive Backstepping Control of Wheeled Inverted Pendulums Models." Nonlinear Dynamics, Vol. 79, No. 1, 2015, pp. 501–511.

[4] Huang, J., Ri, S., Liu, L., Wang, Y., Kim, J., and Pak, G. "Nonlinear Disturbance Observer-Based Dynamic Surface Control of Mobile Wheeled Inverted Pendulum." IEEE Transactions on Control Systems Technology, Vol. 23, No. 6, 2015, pp. 2400–2407.

[5] Sultan, K., and Mirza, A. "Inverted Pendulum, Analysis, Design and Implementation." Visionaries Document, 2003.

[6] Muehlebach, M., and D'Andrea, R. "Nonlinear Analysis and Control of a Reaction-Wheel-Based 3-D Inverted Pendulum." IEEE Transactions on Control Systems Technology, Vol. 25, No. 1, 2016, pp. 235–246.

[7] Yang, X., and Zheng, X. "Swing-up and Stabilization Control Design for an Underactuated Rotary Inverted Pendulum System: Theory and Experiments." IEEE Transactions on Industrial Electronics, Vol. 65, No. 9, 2018, pp. 7229–7238.

[8] Hamza, M. F., Yap, H. J., Choudhury, I. A., Isa, A. I., Zimit, A. Y., and Kumbasar, T. "Current Development on Using Rotary Inverted Pendulum as a Benchmark for Testing Linear and Nonlinear Control Algorithms." Mechanical Systems and Signal Processing, Vol. 116, 2019, pp. 347–369.

[9] Kelly, M., and Ruina, A. Non-Linear Robust Control for Inverted-Pendulum 2D Walking. Presented at the 2015 IEEE International Conference on Robotics and Automation (ICRA), 2015.

[10] Shahbazi, M., Babuška, R., and Lopes, G. A. "Unified Modeling and Control of Walking and Running on the Spring-Loaded Inverted Pendulum." IEEE Transactions on Robotics, Vol. 32, No. 5, 2016, pp. 1178–1195.

[11] Roman, M., Bobasu, E., and Sendrescu, D. Modelling of the Rotary Inverted Pendulum System. In 2008 IEEE International Conference on Automation, Quality and Testing, Robotics, No. 2, 2008, pp. 141–146.

[12] Ragusila, V., and Emami, M. R. "Modelling of a Robotic Leg Using Bond Graphs." Simulation Modelling Practice and Theory, Vol. 40, 2014, pp. 132–143.

[13] Pacheco, F. E. An Inverted Pendulum Cart Modeled Using the Bond Graph Approach. Presented at the 2017 IEEE Second Ecuador Technical Chapters Meeting (ETCM), 2017.

[14] Khaouch, Z., Zekraoui, M., Kouider, N., and Mabrouki, M. "Mechatronic Control Model of an Inverted Pendulum." IOSR Journal of Electrical and Electronics Engineering, Vol. 12, pp 24-30, 2017

[15] Banerjee, S. Dynamics for Engineers. John Wiley & Sons, 2005.

Authors' Profiles

Jessica Adanma Onwuzuruike received her Bachelor of Engineering degree in Electrical and Electronic Engineering from Michael Okpara University of Agriculture, Abia State, Nigeria in 2015. She received her Master's degree in Electrical and Electronics Engineering from Nile University of Nigeria, Abuja, Nigeria in 2019. Presently she is pursuing a Ph.D. degree in Electrical and Electronics Engineering from Nile University of Nigeria, Abuja, Nigeria.

Suleiman Usman Hussein received his Bachelor of Engineering degree in Electrical and Electronic Engineering from the Federal University of Agriculture, Makurdi, Nigeria in 2006 and M.Sc. Degree in Applied Instrumentation and Control from Glasgow Caledonian University, Glasgow, UK in 2012. He received his Ph.D. in Control Engineering from Ahmadu Bello University, Zaria, Nigeria in 2017. He is currently a lecturer in the Department of Electrical and Electronics Engineering, Nile University of Nigeria, Abuja, Nigeria and also work with National Space Research and Development Agency, Abuja, Nigeria. His research interest include Linear and Nonlinear Systems, System Modeling, Instrumentation, Optimal Control, Classical Control, Adaptive Control, Wireless Sensor Network, and Artificial Intelligence.

How to cite this paper: Jessica A. Onwuzuruike, Suleiman U. Hussein, " Bond Graph Modelling of a Rotary Inverted Pendulum on a Wheeled Cart", International Journal of Modern Education and Computer Science(IJMECS), Vol.13, No.6, pp. 25-29, 2021.DOI: 10.5815/ijmecs.2021.06.03