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Representation of Fuzzy Matrices Based on Reference Function

Mamoni Dhar
Assistant Professor, Science College, Kokrajhar-783370,Assam, India
Email: mamonidhar@rediffmail.com

***Abstract*—** Fuzzy matrices in the present form do not meet the most important requirement of matrix representatiom in the form of reference function without which no logical result can be expected. In this article, we intend to represent fuzzy matrices in which there would be the use of reference function. Our main purpose is to deal specially with complement of fuzzy matrices and some of its properties when our new definition of complementation of matrices is considered. For doing these the new definition of complementation of fuzzy sets based on reference function plays a very crucial role. Further, a new definition of trace of a fuzzy matrix is introduced in this article which is in accordance with newly defined fuzzy matrices with the help of reference function and thereby efforts have been made to establish some of the properties of trace of fuzzy matrices.

***Index Terms*—** Membership Value, Reference Function, Boolean Matrices

I. Introduction

Matrices with entries in [0, 1] and matrix operation defined by fuzzy logical operations are called fuzzy matrices. All fuzzy matrices are matrices but every matrix is not a fuzzy matrix. Fuzzy matrices play a fundamental role in fuzzy set theory. They provide us with a rich framework within which many problems of practical applications of the theory can be formulated. Fuzzy matrices can be successfully used when fuzzy uncertainty occurs in a problem. These results are extensively used for cluster analysis and classification problem of static patterns under subjective measure of similarity. On the other hand, fuzzy matrices are generalized Boolean matrices which have been studied for fruitful results. And the theory of Boolean matrices can be back to the theory of matrices with non negative contents, for which most famous classical results were obtained 1907 to 1912 by Parren and Frobenius. So the theory of fuzzy matrices is interesting in its own right. An important connection between fuzzy sets and fuzzy matrices has been recognized and this has led us to define fuzzy matrices in a quite different way. This will inevitably play an important role in any problem area that involves complementation of fuzzy matrices. It should be noted at this point that the concept of trace of a fuzzy matrix has also been investigated in terms of the newly introduced method of fuzzy matrix representation. Further some of the properties of trace of fuzzy matrix according to the suggested definition have also been proposed in this article. Along with these the trace of transpose of fuzzy matrix is also defined and some properties associated with this have also been discussed.

In case of fuzzy matrices min or max operations are defined in order to get the resulting matrix as a fuzzy matrix, Kandasamy [1].

Clearly under max or min operation, the resulting matrix is again a fuzzy matrix which is in some way analogus to our usual addition.

Here since we would like to deal with some properties of trace of matrix which involves addition and multiplication and so we would like to define addition and multiplication of two matrices in our way.

Let us consider the following examples for illustration purposes to make the matter clear and simple.The representation of fuzzy matrix A, which are defined in accordance with the existing definition would be the following:

$$A = \begin{pmatrix} 0.3 & 0.7 & 0.8 \\ 0.4 & 0.5 & 0.3 \\ 0.6 & 0.1 & 0.4 \end{pmatrix}$$

which is a fuzzy matrix of order 3

We would like to represent the matrix in the following manner taking into consideration of reference function.

$$A = \begin{pmatrix} (0.3, 0) & (0.7, 0) & (0.8, 0) \\ (0.4, 0) & (0.5, 0) & (0.3, 0) \\ (0.6, 0) & (0.1, 0) & (0.4, 0) \end{pmatrix}$$

We shall call the matrix

$$A = \begin{pmatrix} (0.3 - 0) & (0.7 - 0) & (0.8 - 0) \\ (0.4 - 0) & (0.5 - 0) & (0.3 - 0) \\ (0.6 - 0) & (0.1 - 0) & (0.4 - 0) \end{pmatrix}$$

As the membership value matrix which is same as the one that is usually referred to in the literature of fuzzy matrices (see for example[1]).

Similarly, the representation of the complement of the fuzzy matrix A which is denoted by A^c and this if defined in terms of reference function would be the following

$$A^c = \begin{pmatrix} (1,0.3) & (1,0.7) & (1,0.8) \\ (1,0.4) & (1,0.5) & (1,0.3) \\ (1,0.6) & (1,0.1) & (1,0.4) \end{pmatrix}$$

Then the matrix obtained from so called membership value would be the following

$$A^c = \begin{pmatrix} (1-0.3) & (1-0.7) & (1-0.8) \\ (1-0.4) & (1-0.5) & (1-0.3) \\ (1-0.6) & (1-0.1) & (1-0.4) \end{pmatrix}$$

If it is calculated then this matrix is same as the usual matrix complement. The complement of a fuzzy matrix is used to analyse the complement nature of a system. For example if A represents the crowdness of a network in a particular time period, then its complement represents the clearness of the network at that time period. So we can say that dealing with complement of a fuzzy matrix is as important as usual matrices.

If we define the complement of a fuzzy matrix in the way indicated and then there is the need to define the operation of addition and multiplication of two fuzzy matrices which keep pace with the new definition and we shall discuss about these in the following sections.

The paper is organized as follows: Section II describes the way in which addition and multiplication of fuzzy matrices are defined and some numerical examples to show its justification are also presented. Section III describes some properties of fuzzy matrix multiplication. Section IV introduces the new definition of trace of fuzzy matrices. Again in this section some properties of trace of fuzzy matrices are also presented and numerical examples are cited for illustration purposes. Finally, Section V presents our conclusions.

II. Addition And Multiplication of Fuzzy Matrices

(i) Addition of fuzzy matrices:

Two fuzzy matrices are conformable for addition if the matrices are of same order. That is to say, when we wish to find addition of two matrices, the number of rows and columns of both the matrices should be same. If A and B be two matrices of same order then their addition can be defined as follows

$$A+B=\{\max(a_{ij},b_{ij}),\min(r_{ij},r'_{ij})\} \tag{1}$$

where a_{ij} stands for the membership function of the fuzzy matrix A for the ith row and jth column and r_{ij} is the corresponding reference function and b_{ij} stands for the membership function of the fuzzy matrix B for the ith row and jth column where r'_{ij} represents the corresponding reference function.

Again we can see that if we define the addition of two fuzzy matrices in the aforesaid manner and thereafter if we use the new definition of complementation of fuzzy matrices which is in accordance with the definition of complementation of fuzzy sets as introduced by Baruah [2, 3, 4] and later on used in the works of Dhar [5, 6, 7, 8, 9 & 10] we would be able to arrive at the following result;

$$(A+B)^c = A^c + B^c \tag{2}$$

Example1.

$$A = \begin{pmatrix} 0.3 & 0.7 & 0.8 \\ 0.4 & 0.5 & 0.3 \\ 0.6 & 0.1 & 0.4 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 1 & 0.2 & 0.3 \\ 0.8 & 0.5 & 0.2 \\ 0.5 & 1 & 0.8 \end{pmatrix}$$

be two fuzzy matrices of order 3

Then

$$A+B=[C_{ij}]$$

Where

$$C_{11}=\{\max(a_{11},b_{11}),\min(r_{11},r'_{11})\}$$

= {max (0.3, 1), min (0,0)}

= (1,0)

$$C_{12}=\{\max(a_{12},b_{12}),\min(r_{12},r'_{12})\}$$

= {max (0.7, 0.2), min (0,0)}

=(0.7,0)

$$C_{13}=\{\max(a_{13},b_{13}),\min(r_{13},r'_{13})\}$$

= {max (0.8, 0.3), min (0, 0)}

= (0.8, 0)

Proceeding in the above manner, we get

$$A+B=\begin{pmatrix}(1,0) & (0.7,0) & (0.8,0)\\ (0.8,0) & (0.5,0) & (0.3,0)\\ (0.6,0) & (1,0) & (0.8,0)\end{pmatrix}$$

Again we have

$$A^c=\begin{pmatrix}(1,0.3) & (1,0.7) & (1,0.8)\\ (1,0.4) & (1,0.5) & (1,0.3)\\ (1,0.6) & (1,0.1) & (1,0.4)\end{pmatrix}$$

and

$$B^c=\begin{pmatrix}(1,1) & (1,0.2) & (1,0.3)\\ (1,0.8) & (1,0.5) & (1,0.2)\\ (1,0.5) & (1,1) & (1,0.8)\end{pmatrix}$$

be the two fuzzy complement matrices. Proceeding similarly we get the sum of these two matrices as

$$A^c+B^c=\begin{pmatrix}(1,1) & (1,0.7) & (1,0.8)\\ (1,0.8) & (1,0.5) & (1,0.3)\\ (1,0.6) & (1,1) & (1,0.8)\end{pmatrix}$$

Again proceeding in the similar way, we get

$$(A+B)^c=\begin{pmatrix}(1,1) & (1,0.7) & (1,0.8)\\ (1,0.8) & (1,0.5) & (1,0.3)\\ (1,0.6) & (1,1) & (1,0.8)\end{pmatrix}$$

Hence we have the following result

$$(A+B)^c = A^c + B^c$$

(ii) Multiplication of fuzzy matrices

Now after finding addition of fuzzy matrices, we shall try to find the multiplication of two fuzzy matrices .The product of two fuzzy matrices under usual matrix multiplication is not a fuzzy matrix. It is due to this reason; a conformable operation analogus to the product which again happens to be a fuzzy matrix was introduced by many researchers which can be found in fuzzy literature. However, even for this operation the product AB to be defined if the number of columns of the first fuzzy matrix A is equal to the number of rows of the second fuzzy matrix B. In the process of finding multiplication of fuzzy matrices, if this condition is satisfied then the multiplication of two fuzzy matrices A and B, will be defined and can be represented in the following form:

$$AB=\{\max\min(a_{ij},b_{ji}),\min\max(r_{ij},r'_{ji}\} \qquad (3)$$

Example:

$$A^c=\begin{pmatrix}(1,0.3) & (1,0.7) & (1,0.8)\\ (1,0.4) & (1,0.5) & (1,0.3)\\ (1,0.6) & (1,0.1) & (1,0.4)\end{pmatrix}$$

$$B^c=\begin{pmatrix}(1,1) & (1,0.2) & (1,0.3) & (1,0.6)\\ (1,0.8) & (1,0.5) & (1,0.2) & (1,0.9)\\ (1,0.5) & (1,1) & (1,0.8) & (1,0.7)\end{pmatrix}$$

Then the product would be defined as

$$A^cB^c=\begin{pmatrix}C11 & C12 & C13 & C14\\ C21 & C22 & C23 & C24\\ C31 & C32 & C33 & C34\end{pmatrix}$$

$$C_{11}=\{\max\min(a_{11},b_{11}),\min\max(r_{11},r'_{11})\}$$

= [max {min (1,1), min(1, 1), min(1,1)},

min {max(0.3,1), max(0.7,0.8), max(0.8,0.5)}]

={max(1,1,1), min(1.0.8,0.8)}

= (1, 0.8)

$$C_{12}=\{\max\min(a_{12},b_{12}),\min\max(r_{12},r'_{12})\}$$

= [max {min (1.1), min (1, 1), min (1,1)} ,

min {max(0.3,0.2), max (0.7, 0.5), max(0.8, 1)}]

= {max (1, 1, 1), min (0.3, 0.7,1)}

= (1, 0.3)

$$C_{13}=\{\max\min(a_{13},b_{13}),\min\max(r_{13},r'_{13})\}$$

=[{ max{min(1.1), min (1,1), min(1,1)} ,

min{max(0.3,0.3), max(0.7, 0.2), max(0.8, 0.8)}]

= {max (1, 1, 1), min (0.3, 0.7,0.8)}

= (1,0, 3)

$$C_{14}=\{\max\min(a_{14},b_{14}),\min\max(r_{14},r'_{14})\}$$

= [{max {min (1.1), min (1, 1), min (1, 1),

Min {max(0.3,0.6),max(0.7,0.9), max(0.8, 0.7)}]

= {max (1, 1, and 1), min (0.6, 0.9, and 0.8)

= (1, 0.6)

Proceeding similarly we get the product of the two matrices as

$$A^cB^c = \begin{pmatrix} (1,0.8) & (1,0.3) & (1,0.3) & (1.0.6) \\ (1,0.5) & (,0.4) & (1,0.4) & (1,0.6) \\ (1,0.5) & (1,0.5) & (1,0.2) & (1,0.6) \end{pmatrix}$$

Multiplication of matrices in the aforesaid manner would lead us to write some properties of fuzzy matrices about which we shall discuss in the following section. But before proceeding further, we would like to mention one thing that since we have defined complementation of fuzzy matrices in a manner which is different from the existing way of representation of complementation of a fuzzy matrix, it would be helpful if we try to establish the properties with the help of complementation of fuzzy matrices. In the following section, we have cited some numerical examples for the purpose of showing the properties of multiplication of fuzzy matrices.

III. Properties of Fuzzy Matrix Multiplication

In this section, we shall consider some of the properties of multiplication of fuzzy matrices.

Property1:

Multiplication of fuzzy matrices is associative, if conformability is assured i.e A (BC) = (AB) C if A, B, C are $m \times n$, $n \times p, p \times q$ matrices respectively. The same result would hold if we consider the complementation fuzzy matrices in our manner.Here we would like to cite an example with the complementation of fuzzy matrices for illustration purposes.

Example:

$A = \begin{pmatrix} 0.1 & 0.3 \\ 0.5 & 0.7 \end{pmatrix}$, $B = \begin{pmatrix} 0.5 & 0.2 \\ 0.7 & 0.8 \end{pmatrix}$ and

$$C = \begin{pmatrix} 0.1 & 1 \\ 0.9 & 0.6 \end{pmatrix}$$

be three fuzzy matrices then their complement would be defined as

$$A^c = \begin{pmatrix} (1,0.1) & (1,0.3) \\ (1,0.5) & (1,0.7) \end{pmatrix}$$

$$B^c = \begin{pmatrix} (1,0.5) & (1,0.2) \\ (1,0.7) & (1,0.8) \end{pmatrix}$$

And

$$C^c = \begin{pmatrix} (1,0.1) & (1,1) \\ (1,0.9) & (1,0.6) \end{pmatrix}$$

Then we have

$$A^cB^c = \begin{pmatrix} (1,0.5) & (1,0.2) \\ (1,0.5) & (1,0.5) \end{pmatrix}$$

and

$$B^cC^c = \begin{pmatrix} (1,0.5) & (1,0.6) \\ (1,0.7) & (1,0.8) \end{pmatrix}$$

Consequently, we get

$$A^c(B^cC^c) = (A^cB^c)C^c \tag{4}$$

Property2:

Multiplication of fuzzy matrices is distributive with respect to addition of fuzzy matrices. That is, A (B+C) =AB+AC, where A, B, C are $m \times n$, $n \times p, p \times q$ matrices respectively. Here we shall show the following if the complementation of the matrices are considered.

$$A^c(B^c + C^c) = A^cB^c + A^cC^c \tag{5}$$

Property3

Multiplication of fuzzy matrices is not always commutative. That is to say that whenever A^cB^c and B^cA^c exist and are matrices of same type, it is not necessary that

$$A^cB^c = B^cA^c \tag{6}$$

If we consider the above set of matrices, then we get

$$A^cB^c = \begin{pmatrix} (1,0.5) & (1,0.2) \\ (1,0.5) & (1,0.5) \end{pmatrix}$$

And similarly

$$B^cA^c = \begin{pmatrix} (1,0.5) & (1,0.5) \\ (1,0.7) & (1,0.7) \end{pmatrix}$$

Which shos that

$$A^cB^c \neq B^cA^c$$

IV. Trace of a Matrix:

Our aim in this section is quite modest: to illustrate the way in which the trace of a fuzzy matrix is defined thereafter to represent the different properties with citing suitable numerical examples.

Let A be a square matrix of order n. Then the trace of the matrix A is denoted by tr A and is defined as

$$trA = (\max \mu_{ii}, \min r_{ii}) \tag{7}$$

where μ_{ii} stands for the membership functions lying along the principal diagonal and r_{ii} refers to the refence function of the corresponding membership functions.

In the following theorem we have given some fundamental properties of trace of a matrix. Moreover, to make the matter clear and simple, we have presented some numerical examples.

Theorem**:** Let A and B be two fuzzy square matrices each of order n and λ be any scalar such that $0 \le \lambda \le 1$.

Then

(i) tr (A+B) = trA+tr B

(ii) tr (λA)= $\lambda tr(A)$

(iii) tr A= tr A', where A' is the transpose of A.

(i) Proof

We have from the proposed definition of trace of fuzzy matrices

$$trA = \{\max(a_{ii}), \min(r_{ii})\}$$

and

$$trB = \{\max(b_{ii}), \min(r'_{ii})\}$$

Then

A+B= C(say), where $C = [C_{ij}]$

Now according to the definition of addition of two fuzzy matrices, we have

$$C_{ij} = [\max(a_{ij}, b_{ij}), \min(r_{ij}, r'_{ij})] \qquad (7)$$

Again according to our definition of trace of a fuzzy matrix, we shall get

$$tr(C) = [\max\{\max(a_{ii}, b_{ii})\}, \min\{\min(r_{ii}, r'_{ii})\}]$$

$$= [\max\{\max(a_{ii}), \max(b_{ii})\}, \min\{\min(r_{ii}), \min(r'_{ii})\}]$$

=trA+trB

Conversely,

$$trA + trB = [\max\{\max a_{ii}, \max b_{ii}\} \min\{\min r_{ii}, \min r'_{ii}\}]$$

$$= [\max\{\max(a_{ii}, b_{ii})\} \min\{\min(r_{ii}, r'_{ii})\}]$$

=tr(A+B)

This proves the result

trA+trB=tr(A+B) (8)

Here is the numerical example for the above result.

Example:

Let us consider the following two matrices for illustration purposes

$$A = \begin{pmatrix} (0.3,0) & (0.7,0) & (0.8,0) \\ (0.4,0) & (0.5,0) & (0.3,0) \\ (0.6,0) & (0.1,0) & (0.4,0) \end{pmatrix}$$

and

$$B = \begin{pmatrix} (1,0) & (0.2,0) & (0.3,0) \\ (0.8,0) & (0.5,0) & (0.2,0) \\ (0.5,0) & (1,0) & (0.8,0) \end{pmatrix}$$

The addition of two matrices will yield the following result

$$A + B = \begin{pmatrix} (1,0) & (0.7,0) & (0.8,0) \\ (0.8,0) & (0.5,0) & (0.3,0) \\ (0.6,0) & (1,0) & (0.8,0) \end{pmatrix}$$

Using the definition of trace of fuzzy matrices, we see the following results:

tr A={ max(0.3, 0.5, 0.4), min(0, 0, 0)

= (0.5, 0)

tr B= { max (1, 0.5, 0.8), min (0, 0, 0)}

= (1, 0)

Thus we have

tr A+ tr B={ max(0.5,1),min(0,0)}

= (1,0)

and

tr (A+B)= { max(1, 0.5, 0.8), min(0.0,0)}

= (1, 0)

Hence the result

tr (A+B) = tr A+ tr B (9)

(ii) To show

$tr(\lambda A) = \lambda trA$, $0 \le \lambda \le 1$

$$tr(\lambda A) = \{\max(\lambda a_{ii}), \min(\lambda r_{ii})\}$$

$$= \lambda\{\max(a_{ii}), \min(r_{ii})\}$$

$$= \lambda tr(A)$$

Example:

$$A=\begin{pmatrix}(0.3,0) & (0.7,0) & (0.8,0)\\ (0.4,0) & (0.5,0) & (0.3,0)\\ (0.6,0) & (0.1,0) & (0.4,0)\end{pmatrix}$$

and

$$\lambda = .5$$

Then

$$\lambda A=\begin{pmatrix}(0.15,0) & (0.35,0) & (0.40,0)\\ (0.20,0) & (0.25,0) & (0.15,0)\\ (0.30,0) & (0.05,0) & (0.20,0)\end{pmatrix}$$

$tr(\lambda A) =$ {max(0.15, 0.25, 0.20), min (0,0,0)

= (0.25, 0)

Again

tr A =(0.5, 0)

and hence

tr A= 0.5(0.5, 0)=(0.25, 0)

The same result will hold if we consider the complements of the above fuzzy matrices. For the fuzzy matrix A, the complement of the fuzzy matrix would be the following

$$A^c=\begin{pmatrix}(1,0.3) & (1,0.7) & (1,0.8)\\ (1,0.4) & (1,0.5) & (1,0.3)\\ (1,0.6) & (1,0.1) & (1,0.4)\end{pmatrix}$$

Now the trace of the above matrix would be calculated as

$trA^c =$ {max (1, 1, 1), min (0.3, 0.5,0.4)}

= (1, 0.3)

Here if we consider another fuzzy matrix B whose complement can be represented in the manner

$$B^c=\begin{pmatrix}(1,1) & (1,0.2) & (1,0.3)\\ (1,0.8) & (1,0.5) & (1,0.2)\\ (1,0.5) & (1,1) & (1,0.8)\end{pmatrix}$$

Then the trace of this matrix will be the following

$trB^c =$ {Max (1,1,1), min (1,0.5,0.8)

= (1, 0.5)

Using the definition of addition of two fuzzy matrices, we get

$$A^c+B^c=\begin{pmatrix}(1,0.3) & (1,0.2) & (1,0.3)\\ (1,0.4) & (1,0.5) & (1,0.2)\\ (1,0.5) & (1,0.1) & (1,0.4)\end{pmatrix}$$

$tr(A^c+B^c) =$ {max(1,1,1), min (0.3,0.5,0.4)}

= (1, 0.3)

Thus the first property is satisfied and proceeding similarly, we shall see that the properties of trace of matrices are satisfied in case of complementation of fuzzy matrices also.

Again if A' be the transpose of the fuzzy square matrix A, which is obtained by interchanging the rows and columns of a given fuzzy matrix, then we must have

Example:

$$A'=\begin{pmatrix}(0.3,0) & (0.4,0) & (0.6,0)\\ (0.7,0) & (0.5,0) & (0.1,0)\\ (0.8,0) & (0.3,0) & (0.4,0)\end{pmatrix}$$

$trA' =$ {max (0.3, 0.5, 0.4), min(0, 0, 0)}

= (0.5, 0)

Thus we get

$$trA' = trA \tag{10}$$

V. Conclusions

This article deals with finding new definition of trace of fuzzy matrices. For doing so addition and multiplication of matrices are to be defined accordingly. For which it was required to define addition and multiplication of fuzzy matrices with the help of reference function which is different from the existing definitions. The reference function plays a key role in defining complementation of fuzzy sets and hence the presence of it is essential in dealing with complementation of fuzzy matrices. The main purpose of the article is to propose a new definition of trace of a fuzzy square matrix and thereafter to establish some of its properties. In the process, many properties of the trace of fuzzy matrices were illustrated with the help of numerical examples to make the matter clear and simple. With the use of the process, we can look far into the future to stimulate imagination. The future development seems quite promising.

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Mamoni Dhar is an Assistant Professor in the department of Mathematics, Science College, Kokrajhar-783370, Assam, India. She received M.Sc degree from Gauhati University, M.Phil degree from Madurai Kamraj University, B.Ed from Gauhati University and PGDIM from Indira Gandhi National Open University. She had received National Scholarship from Matriculation to Post Graduation. Her research interest is in Fuzzy Set Theory from new perspectives. She has published eleven articles in different national and international journals.

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