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New Condition of Stabilization of Uncertain Continuous Takagi-Sugeno Fuzzy System based on Fuzzy Lyapunov Function

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Abstract— **This paper deals with the stabilization of Takagi-Sugeno fuzzy models. Using non-quadratic Lyapunov function, new sufficient stabilization criteria with PDC controller are established in terms of Linear Matrix Inequality. Finally, a stabilization condition for uncertain system is given.**

Index Terms— **Takagi-Sugeno fuzzy system, uncertain system, Linear Matrix Inequalities LMIs, Fuzzy Lyapunov Function, Parallel Distributed Compensation PDC.**

I. INTRODUCTION

Fuzzy control systems have experienced a big growth of industrial applications in the recent decades, because of their reliability and effectiveness. Many researches are investigated on the Takagi-Sugeno models [1-2] which can combine the flexible fuzzy logic theory and rigorous mathematical theory into a unified framework. Thus, it becomes a powerful tool in approximating a complex nonlinear system.

Two classes of Lyapunov functions are used to analysis these systems: quadratic Lyapunov functions and non-quadratic Lyapunov ones which are less conservative than first class. Many researches are investigated with non-quadratic Lyapunov functions [1-9].

In this paper, a new stability conditions for Takagi Sugeno uncertain fuzzy models based on the use of fuzzy Lyapunov function are presented. This criterion is expressed in terms of Linear Matrix Inequalities (LMIs) which can be efficiently solved by using various convex optimization algorithms [10]. The presented method is less conservative than existing results.

The organization of the paper is as follows. In section 2, we present the system description and problem formulation and we give some preliminaries which are needed to derive results. Section 3 will be concerned to stability analysis for T-S fuzzy systems. Section 4 concerns the proposed approach to stabilize a T-S fuzzy system with Parallel Distributed Compensation (PDC). Next, a new stabilization condition for uncertain system is given. Finally section 6 makes conclusion.

Notation: Throughout this paper, a real symmetric matrix $S>0$ denotes S being a positive definite matrix. The superscript ‘‘T’’ is used for the transpose of a matrix.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider an uncertain T-S fuzzy continuous model for a nonlinear system as follows:

$$\begin{aligned}&IF\ z_1(t)\ is\ M_{i1}\ and\ \ldots and\ z_p(t)\ is\ M_{ip}\\&THEN\quad \dot{x}(t)=(A_i+\Delta A_i)x(t)+(B_i+\Delta B_i)u(t)\\&\qquad i=1,\ldots,r\end{aligned}\tag{1}$$

where $M_{ij}\ (i=1,2,\ldots,r,\ j=1,2,\ldots,p)$ is the fuzzy set and r is the number of model rules; $x(t)\in\Re^n$ is the state vector, $u(t)\in\Re^m$ is the input vector, $A_i\in\Re^{n\times n}$, $B_i\in\Re^{n\times m}$, and $z_1(t),\ldots,z_p(t)$ are known premise variables. ΔA_i and ΔB_i are time-varying matrices representing parametric uncertainties in the plant model. These uncertainties are admissibly norm-bounded and structured.

The final outputs of the fuzzy systems are:

$$\dot{x}(t)=\sum_{i=1}^{r}h_i(z(t))\{(A_i+\Delta A_i)x(t)+(B_i+\Delta B_i)u(t)\}\tag{2}$$

where

$$z(t)=\left[z_1(t)z_2(t)\ldots z_p(t)\right]$$

$$h_i(z(t))=w_i(z(t))\Big/\sum_{i=1}^{r}w_i(z(t)),$$

$$w_i(z(t))=\prod_{j=1}^{p}M_{ij}(z_j(t))\quad \text{for all t.}$$

The term $M_{i1}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{i1}

Since $\begin{cases} \sum_{i=1}^{r} w_i\left(z\left(t\right)\right) \succ 0 \\ w_i\left(z\left(t\right)\right) \geq 0, \qquad i = 1,2,\ldots,r \end{cases}$

we have $\begin{cases} \sum_{i=1}^{r} h_i\left(z\left(t\right)\right) = 1 \\ h_i\left(z\left(t\right)\right) \geq 0, \qquad i = 1,2,\ldots,r \end{cases}$ for all t.

The time derivative of premise membership functions is given by:

$$\dot{h}_i\left(z\left(t\right)\right) = \frac{\partial h_i}{\partial z\left(t\right)} \cdot \frac{\partial z\left(t\right)}{\partial x\left(t\right)} \cdot \frac{dx\left(t\right)}{dt} = \sum_{l=1}^{s} \upsilon_{il}\xi_{il} \times \frac{dx\left(t\right)}{dt} \quad (3)$$

We have the following property:

$$\sum_{k=1}^{r} \dot{h}_k\left(z\left(t\right)\right) = 0 \quad (4)$$

The PDC fuzzy controller is represented by

$$u\left(t\right) = -\frac{\sum_{i=1}^{r} w_i\left(z\left(t\right)\right) F_i x\left(t\right)}{\sum_{i=1}^{r} w_i\left(z\left(t\right)\right)} = -\sum_{i=1}^{r} h_i\left(z\left(t\right)\right) F_i x\left(t\right) \quad (5)$$

The fuzzy controller design is to determine the local feedback gains F_i in the consequent parts.

The open-loop system is given by the equation (6)

$$\dot{x}\left(t\right) = \sum_{i=1}^{r} h_i\left(z\left(t\right)\right)\left(A_i + \Delta A_i\right) x\left(t\right) \quad (6)$$

By substituting (5) into (2), the closed-loop fuzzy system can be represented as:

$$\dot{x}\left(t\right) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(z\left(t\right)\right) h_j\left(z\left(t\right)\right)\left\{A_{\Delta i} - B_{\Delta i} F_j\right\} x\left(t\right) \quad (7)$$

where $A_{\Delta i} = A_i + \Delta A_i$ *and* $B_{\Delta i} = B_i + \Delta B_i$

Assumption 1

The time derivative of the premises membership function is upper bounded such that $\left|\dot{h}_k\right| \leq \phi_k$, for $k = 1,\ldots,r$, where, $\phi_k, k = 1,\ldots,r$ are given positive constants.

Assumption 2

The matrices denote the uncertainties in the system and take the form of

$$\left[\Delta A_i \quad \Delta B_i\right] = DF\left(t\right)\left[E_{A_i} \quad E_{B_i}\right]$$

where D, E_{A_i} and E_{B_i} are known constant matrices and $F\left(t\right)$ is an unknown matrix function satisfying :

$$F^T\left(t\right)F\left(t\right) \leq I, \forall t$$

where I is an appropriately dimensioned identity matrix.

Lemma 1 (Boyd et al. Schur complement [6])

Given constant matrices Ω_1, Ω_2 and Ω_3 with appropriate dimensions, where $\Omega_1 = \Omega_1^T$ and $\Omega_2 = \Omega_2^T$, then

$$\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 \prec 0$$

if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ * & -\Omega_2 \end{bmatrix} \prec 0 \text{ or } \begin{bmatrix} -\Omega_2 & \Omega_3 \\ * & \Omega_1 \end{bmatrix} \prec 0$$

Lemma 2 (Peterson and Hollot [8])

Let $Q = Q^T, H, E$ and $F\left(t\right)$ satisfying $F^T\left(t\right)F\left(t\right) \leq I$ are appropriately dimensional matrices then the following inequality

$$Q + HF\left(t\right)E + E^T F^T\left(t\right)H^T \prec 0$$

is true, if and only if the following inequality holds for any $\varepsilon \succ 0$

$$Q + \varepsilon^{-1}HH^T + \varepsilon E^T E \prec 0$$

III. BASIC STABILITY CONDITIONS

Consider the open-loop system (8).

$$\dot{x}\left(t\right) = \sum_{i=1}^{r} h_i\left(z\left(t\right)\right) A_i x\left(t\right) \quad (8)$$

This section gives a new condition for stability of the unforced T-S fuzzy system by using the Lyapunov theory.

Theorem 1 [11]

Under assumption 1 and for $0 \leq \varepsilon \leq 1$, the Takagi Sugeno fuzzy system (8) is stable if there exist positive definite symmetric matrices $P_k, k = 1,2,\ldots,r$, matrix $R = R^T$ such that the following LMIs hold.

$$P_k + R \succ 0, \quad k \in \left\{1,\ldots,r\right\} \quad (9)$$

$$P_j + \mu R \succ 0, \quad j \in \left\{1,\ldots,r\right\} \quad (10)$$

$$\begin{aligned} P_\phi + \frac{1}{2}\Big\{ & A_i^T\left(P_j + \mu R\right) + \left(P_j + \mu R\right)A_i \\ & + A_j^T\left(P_i + \mu R\right) + \left(P_i + \mu R\right)A_j \Big\} \prec 0, \quad i \leq j \end{aligned} \quad (11)$$

where $i, j = 1,2,\ldots,r$ and $P_\phi = \sum_{k=1}^{r} \phi_k\left(P_k + R\right)$ and $\mu = 1 - \varepsilon$

IV. STABILIZATION WITH PDC CONTROLLER

Consider the closed-loop system without uncertainties which can be rewritten as

$$\dot{x}(t)=\sum_{i=1}^{r}h_i(z(t))h_i(z(t))G_{ii}x(t) \\ +2\sum_{i=1}^{r}\sum_{i\prec j}h_i(z(t))h_j(z(t))\left\{\frac{G_{ij}+G_{ji}}{2}\right\}x(t), \tag{12}$$

where

$G_{ij}=A_i-B_iF_j$ and $G_{ii}=A_i-B_iF_i$.

In this section we define a fuzzy Lyapunov function and then consider stability conditions.

Theorem 2

Under assumption 1 and for given $0\le\varepsilon\le 1$, the Takagi-Sugeno system (12) is stable if there exist positive definite symmetric matrices $P_k, k=1,2,\ldots,r$, and R, matrices $F_1,\ldots,F_r$ such that the following LMIs hols.

$$P_k+R\succ 0,\quad k\in\{1,\ldots,r\} \tag{13}$$

$$P_j+\mu R\ge 0,\quad j=1,2,\ldots,r \tag{14}$$

$$P_\phi+\left\{G_{ii}^T(P_k+\mu R)+(P_k+\mu R)G_{ii}\right\}\prec 0, \\ i,k\in\{1,\ldots,r\} \tag{15}$$

$$\left\{\frac{G_{ij}+G_{ji}}{2}\right\}^T(P_k+\mu R)+(P_k+\mu R)\left\{\frac{G_{ij}+G_{ji}}{2}\right\}\prec 0, \\ \text{for } i,j,k=1,2,\ldots,r \text{ such that } i\prec j \tag{16}$$

where

$G_{ij}=A_i-B_iF_j, G_{ii}=A_i-B_iF_i$, $\mu=1-\varepsilon$,

and $P_\phi=\sum_{k=1}^{r}\phi_k(P_k+R)$

Proof

Let consider the Lyapunov function in the following form:

$$V(x(t))=\sum_{k=1}^{r}h_k(z(t))\cdot V_k(x(t)) \tag{17}$$

with

$$V_k(x(t))=x^T(t)(P_k+\mu R)x(t),\qquad k=1,2,\ldots,r$$

where

$$P_k=P_k^T, R=R^T,\ 0\le\varepsilon\le 1, \mu=1-\varepsilon, \\ \text{and } (P_k+\mu R)\ge 0,\quad k=1,2,\ldots,r .$$

The time derivative of $V(x(t))$ with respect to t along the trajectory of the system (12) is given by:

$$\dot{V}(x(t))=\sum_{k=1}^{r}\dot{h}_k(z(t))V_k(x(t))+\sum_{k=1}^{r}h_k(z(t))\dot{V}_k(x(t)) \tag{18}$$

The equation (18) can be rewritten as,

$$\dot{V}(x(t))=x^T(t)\left(\sum_{k=1}^{r}\dot{h}_k(z(t))(P_k+\mu R)\right)x(t) \\ +\dot{x}^T(t)\left(\sum_{k=1}^{r}h_k(z(t))(P_k+\mu R)\right)x(t) \\ +x^T(t)\left(\sum_{k=1}^{r}h_k(z(t))(P_k+\mu R)\right)\dot{x}(t) \tag{19}$$

By substituting (12) into (19), we obtain,

$$\dot{V}(x(t))=\Upsilon_1(x,z)+\Upsilon_2(x,z)+\Upsilon_3(x,z) \tag{20}$$

where

$$\Upsilon_1(x,z)=x^T(t)\left(\sum_{k=1}^{r}\dot{h}_k(z(t))\cdot(P_k+\mu R)\right)x(t) \tag{21}$$

$$\Upsilon_2(x,z)=x^T(t)\sum_{k=1}^{r}\sum_{i=1}^{r}h_k(z(t))h_i^2(z(t)) \\ \times\left\{G_{ii}^T(P_k+\mu R)+(P_k+\mu R)G_{ii}\right\}x(t) \tag{22}$$

$$\Upsilon_3(x,z)=x(t)^T\sum_{k=1}^{r}\sum_{i=1}^{r}\sum_{i\prec j}h_k(z(t))h_i(z(t))h_j(z(t)) \\ \times\left\{\left(\frac{G_{ij}+G_{ji}}{2}\right)^T(P_k+\mu R)+(P_k+\mu R)\left(\frac{G_{ij}+G_{ji}}{2}\right)\right\}\Bigg]x(t) \tag{23}$$

Then, based on assumption 1, an upper bound of $\Upsilon_1(x,z)$ obtained as:

$$\Upsilon_1(x,z)\le\sum_{k=1}^{r}\phi_k\cdot x(t)^T(P_k+\mu R)x(t) \tag{24}$$

Based on (4), it follows that $\sum_{k=1}^{r}\dot{h}_k(z(t))\varepsilon R=\bar{R}=0$ where R is any symmetric matrix of proper dimension.

Adding $\bar{R}$ to (24), then

$$\Upsilon_1(x,z)\le\sum_{k=1}^{r}\phi_k\cdot x(t)^T(P_k+R)x(t) \tag{25}$$

Then,

$$\dot{V}(x(t))\le\sum_{k=1}^{r}\phi_k x^T(t)(P_k+R)x(t) \\ +\Upsilon_2(x,z)+\Upsilon_3(x,z)$$

If (15) and (16) holds, the time derivative of the fuzzy Lyapunov function is negative. Consequently, we have

$$\dot{V}\left(x(t)\right) \leq x^T(t)\Bigg(\sum_{k=1}^{r}\sum_{i=1}^{r} h_k\left(z(t)\right)h_i^2\left(z(t)\right) \times\left\{\left(G_{ii}^T\left(P_k+\mu R\right)+\left(P_k+\mu R\right)G_{ii}\right)\right\} + \sum_{k=1}^{r}\phi_k\left(\left(P_k+R\right)\right) + \sum_{k=1}^{r}\sum_{i=1}^{r}\sum_{i\prec j} h_k\left(z(t)\right)h_i\left(z(t)\right)h_j\left(z(t)\right) \times\left(\left\{\frac{G_{ij}+G_{ji}}{2}\right\}^T\left(P_k+\mu R\right)+\left(P_k+\mu R\right)\left\{\frac{G_{ij}+G_{ji}}{2}\right\}\right)\Bigg)x(t) \prec 0$$

and the closed loop fuzzy system (12) is stable. This is complete the proof.

V. Robust Stability Condition with PDC Controller

Consider the closed-loop system (7). A sufficient robust stability condition is given follow.

Theorem 3

Under assumption 1, and assumption 2 and for given $0 \leq \varepsilon \leq 1$, the Takagi-Sugeno system (7) is stable if there exist positive definite symmetric matrices $P_k, k=1,2,\ldots,r$, and R, matrices $F_1,\ldots,F_r$ such that the following LMIs hols.

$$P_k + R \succ 0, \quad k \in \{1,\ldots,r\} \qquad (26)$$

$$P_j + \mu R \geq 0, \quad j=1,2,\ldots,r \qquad (27)$$

$$\begin{bmatrix} \Phi_1 & (P_k+\mu R)D_{ai} & (P_k+\mu R)D_{bi} \\ * & -\lambda I & 0 \\ * & * & -\lambda I \end{bmatrix} \prec 0 \qquad (28)$$

$i,k \in \{1,\ldots,r\}$

with

$$\Phi_1 = P_\phi + G_{ii}^T\left(P_k+\mu R\right)+\left(P_k+\mu R\right)G_{ii} + \lambda\left(P_k+\mu R\right)\left[E_{ai}^T E_{ai} + \left(E_{bi}F_i\right)^T E_{bi}F_i\right]$$

$$\begin{bmatrix} \Phi_2 & (P_k+\mu R)\left(D_{ai}+D_{aj}\right) & (P_k+\mu R)\left(D_{bi}+D_{bj}\right) \\ * & -\lambda I & 0 \\ * & * & -\lambda I \end{bmatrix} \prec 0$$

for $i,j,k=1,2,\ldots,r$ such that $i \prec j$

(29)

with

$$\Phi_2 = \left(\frac{G_{ij}+G_{ji}}{2}\right)^T\left(P_k+\mu R\right)+\left(P_k+\mu R\right)\left(\frac{G_{ij}+G_{ji}}{2}\right) + \lambda\left(P_k+\mu R\right)\Big[\left(E_{ai}+E_{aj}\right)^T\left(E_{ai}+E_{aj}\right) + \left(E_{bi}F_j+E_{bj}F_i\right)^T\left(E_{bi}F_j+E_{bj}F_i\right)^T\Big]$$

where $G_{ij} = A_i - B_iF_j$, $G_{ii} = A_i - B_iF_i$, $\mu = 1-\varepsilon$, and $P_\phi = \sum_{k=1}^{r}\phi_k\left(P_k+R\right)$

Proof

Let consider the Lyapunov function in the following form:

$$V\left(x(t)\right) = \sum_{k=1}^{r} h_k\left(z(t)\right)\cdot V_k\left(x(t)\right) \qquad (30)$$

with

$$V_k\left(x(t)\right) = x^T(t)\left(P_k+\mu R\right)x(t), \quad k=1,2,\ldots,r$$

where

$$P_k = P_k^T, R = R^T, \ 0 \leq \varepsilon \leq 1, \mu = 1-\varepsilon, \text{ and } \left(P_k+\mu R\right) \geq 0, \quad k=1,2,\ldots,r .$$

The time derivative of $V\left(x(t)\right)$ with respect to t along the trajectory of the system (12) is given by:

$$\dot{V}\left(x(t)\right) = \sum_{k=1}^{r}\dot{h}_k\left(z(t)\right)V_k\left(x(t)\right) + \sum_{k=1}^{r} h_k\left(z(t)\right)\dot{V}_k\left(x(t)\right) \qquad (31)$$

The equation (31) can be rewritten as,

$$\dot{V}\left(x(t)\right) = x^T(t)\left(\sum_{k=1}^{r}\dot{h}_k\left(z(t)\right)\left(P_k+\mu R\right)\right)x(t) + \dot{x}^T(t)\left(\sum_{k=1}^{r} h_k\left(z(t)\right)\left(P_k+\mu R\right)\right)x(t) + x^T(t)\left(\sum_{k=1}^{r} h_k\left(z(t)\right)\left(P_k+\mu R\right)\right)\dot{x}(t) \qquad (32)$$

By substituting (7) into (32), we obtain,

$$\dot{V}\left(x(t)\right) = \Upsilon_1(x,z) + \Upsilon_2(x,z) + \Upsilon_3(x,z) \qquad (33)$$

where

$$\Upsilon_1(x,z) = x^T(t)\left(\sum_{k=1}^{r}\dot{h}_k\left(z(t)\right)\cdot\left(P_k+\mu R\right)\right)x(t) \qquad (34)$$

$$\Upsilon_2(x,z) = x^T(t)\sum_{k=1}^{r}\sum_{i=1}^{r} h_k\left(z(t)\right)h_i^2\left(z(t)\right) \times\left\{G_{ii}^T\left(P_k+\mu R\right)+\left(P_k+\mu R\right)G_{ii}\right\}x(t) + x^T(t)\sum_{k=1}^{r}\sum_{i=1}^{r} h_k\left(z(t)\right)h_i^2\left(z(t)\right) \times\left\{\left(\begin{bmatrix} D_{ai} & D_{bi} \end{bmatrix}\begin{bmatrix} \Delta_{ai} & 0 \\ 0 & \Delta_{bi} \end{bmatrix}\begin{bmatrix} E_{ai} \\ -E_{bi}F_i \end{bmatrix}\right)^T\left(P_k+\mu R\right) + \left(P_k+\mu R\right)\left(\begin{bmatrix} D_{ai} & D_{bi} \end{bmatrix}\begin{bmatrix} \Delta_{ai} & 0 \\ 0 & \Delta_{bi} \end{bmatrix}\begin{bmatrix} E_{ai} \\ -E_{bi}F_i \end{bmatrix}\right)\right\}x(t) \qquad (35)$$

$$\Upsilon_3(x,z)=x(t)^T\sum_{k=1}^{r}\sum_{i=1}^{r}\sum_{i\prec j}h_k(z(t))h_i(z(t))h_j(z(t))$$
$$\times\left\{\left(\frac{G_{ij}+G_{ji}}{2}\right)^T(P_k+\mu R)+(P_k+\mu R)\left(\frac{G_{ij}+G_{ji}}{2}\right)\right\}x(t)$$
$$+x(t)^T\sum_{k=1}^{r}\sum_{i=1}^{r}\sum_{i\prec j}h_k(z(t))h_i(z(t))h_j(z(t))$$
$$\times\left\{\left(\begin{bmatrix}D_{ai} & D_{bi}\end{bmatrix}\begin{bmatrix}\Delta_{ai} & 0\\ 0 & \Delta_{bi}\end{bmatrix}\begin{bmatrix}E_{ai}\\ -E_{bi}F_j\end{bmatrix}\right)^T(P_k+\mu R)\right.$$
$$\left.+(P_k+\mu R)\left(\begin{bmatrix}D_{ai} & D_{bi}\end{bmatrix}\begin{bmatrix}\Delta_{ai} & 0\\ 0 & \Delta_{bi}\end{bmatrix}\begin{bmatrix}E_{ai}\\ -E_{bi}F_j\end{bmatrix}\right)\right\}x(t)$$
$$+x(t)^T\sum_{k=1}^{r}\sum_{i=1}^{r}\sum_{i\prec j}h_k(z(t))h_i(z(t))h_j(z(t))$$
$$\times\left\{\left(\begin{bmatrix}D_{aj} & D_{bj}\end{bmatrix}\begin{bmatrix}\Delta_{aj} & 0\\ 0 & \Delta_{bj}\end{bmatrix}\begin{bmatrix}E_{aj}\\ -E_{bj}F_i\end{bmatrix}\right)^T(P_k+\mu R)\right.$$
$$\left.+(P_k+\mu R)\left(\begin{bmatrix}D_{aj} & D_{bj}\end{bmatrix}\begin{bmatrix}\Delta_{aj} & 0\\ 0 & \Delta_{bj}\end{bmatrix}\begin{bmatrix}E_{aj}\\ -E_{bj}F_i\end{bmatrix}\right)\right\}x(t) \tag{36}$$

Then, based on assumption 1, an upper bound of $\Upsilon_1(x,z)$ obtained as:

$$\Upsilon_1(x,z)\le\sum_{k=1}^{r}\phi_k\cdot x(t)^T(P_k+\mu R)x(t) \tag{37}$$

Based on (4), it follows that $\sum_{k=1}^{r}\dot{h}_k(z(t))\varepsilon R=\bar{R}=0$ where R is any symmetric matrix of proper dimension.

Adding $\bar{R}$ to (34), then

$$\Upsilon_1(x,z)\le\sum_{k=1}^{r}\phi_k\cdot x(t)^T(P_k+R)x(t) \tag{38}$$

Then,

$$\dot{V}(x(t))\le\sum_{k=1}^{r}\phi_k x^T(t)(P_k+R)x(t)+\Upsilon_2(x,z)+\Upsilon_3(x,z)$$

If

$$\sum_{k=1}^{r}\phi_k(P_k+R)+G_{ii}^T(P_k+\mu R)+(P_k+\mu R)G_{ii}$$
$$+\left\{\left(\begin{bmatrix}D_{ai} & D_{bi}\end{bmatrix}\begin{bmatrix}\Delta_{ai} & 0\\ 0 & \Delta_{bi}\end{bmatrix}\begin{bmatrix}E_{ai}\\ -E_{bi}F_i\end{bmatrix}\right)^T(P_k+\mu R)\right.$$
$$\left.+(P_k+\mu R)\left(\begin{bmatrix}D_{ai} & D_{bi}\end{bmatrix}\begin{bmatrix}\Delta_{ai} & 0\\ 0 & \Delta_{bi}\end{bmatrix}\begin{bmatrix}E_{ai}\\ -E_{bi}F_i\end{bmatrix}\right)\right\}\prec 0$$

Then, based on Lemma 2, an upper bound of $\Upsilon_1(x,z)$ obtained as:

$$\sum_{k=1}^{r}\phi_k(P_k+R)+G_{ii}^T(P_k+\mu R)+(P_k+\mu R)G_{ii}$$
$$+\lambda^{-1}(P_k+\mu R)\begin{bmatrix}D_{ai} & D_{bi}\end{bmatrix}\begin{bmatrix}D_{ai}^T\\ D_{bi}^T\end{bmatrix}$$
$$+\lambda\begin{bmatrix}E_{ai}^T & -(E_{bi}F_i)^T\end{bmatrix}\begin{bmatrix}E_{ai}\\ -E_{bi}F_i\end{bmatrix}(P_k+\mu R)\prec 0$$

by Schur complement, we obtain,

$$\begin{bmatrix}\Phi_1 & (P_k+\mu R)D_{ai} & (P_k+\mu R)D_{bi}\\ * & -\lambda I & 0\\ * & * & -\lambda I\end{bmatrix}\prec 0$$

with

$$\Phi_1=P_\phi+G_{ii}^T(P_k+\mu R)+(P_k+\mu R)G_{ii}$$
$$+\lambda(P_k+\mu R)\left[E_{ai}^TE_{ai}+(E_{bi}F_i)^TE_{bi}F_i\right]$$

$$\left\{\left(\frac{G_{ij}+G_{ji}}{2}\right)^T(P_k+\mu R)+(P_k+\mu R)\left(\frac{G_{ij}+G_{ji}}{2}\right)\right\}$$
$$+\left\{\left(\begin{bmatrix}D_{ai}+D_{aj} & D_{bi}+D_{bj}\end{bmatrix}\begin{bmatrix}\Delta_{ai}+\Delta_{aj} & 0\\ 0 & \Delta_{bi}+\Delta_{bj}\end{bmatrix}\right.\right.$$
$$\left.\times\begin{bmatrix}E_{ai}+E_{aj}\\ -E_{bi}F_j-E_{bj}F_i\end{bmatrix}\right)^T(P_k+\mu R)+(P_k+\mu R)$$
$$\times\left(\begin{bmatrix}D_{ai}+D_{aj} & D_{bi}+D_{bj}\end{bmatrix}\begin{bmatrix}\Delta_{ai}+\Delta_{aj} & 0\\ 0 & \Delta_{bi}+\Delta_{bj}\end{bmatrix}\right.$$
$$\left.\left.\times\begin{bmatrix}E_{ai}+E_{aj}\\ -E_{bi}F_j-E_{bj}F_i\end{bmatrix}\right)\right\}$$
$$\prec 0$$

Then, based on Lemma 2, an upper bound of $\Upsilon_1(x,z)$ obtained as:

$$\left(\frac{G_{ij}+G_{ji}}{2}\right)^T(P_k+\mu R)+(P_k+\mu R)\left(\frac{G_{ij}+G_{ji}}{2}\right)$$
$$+\lambda^{-1}(P_k+\mu R)\begin{bmatrix}D_{ai}+D_{aj} & D_{bi}+D_{bj}\end{bmatrix}\begin{bmatrix}D_{ai}^T+D^T{}_{aj}\\ D_{bi}^T+D^T{}_{bj}\end{bmatrix}$$
$$+\lambda\left[(E_{ai}+E_{aj})^T\quad(-E_{bi}F_j-E_{bj}F_i)^T\right]$$
$$\times\begin{bmatrix}E_{ai}+E_{aj}\\ -E_{bi}F_j-E_{bj}F_i\end{bmatrix}(P_k+\mu R)\prec 0$$

by Schur complement, we obtain,

$$\begin{bmatrix}\Phi_2 & (P_k+\mu R)(D_{ai}+D_{aj}) & (P_k+\mu R)(D_{bi}+D_{bj})\\ * & -\lambda I & 0\\ * & * & -\lambda I\end{bmatrix}\prec 0$$

with

$$\Phi_2 = \left(\frac{G_{ij}+G_{ji}}{2}\right)^T (P_k+\mu R) + (P_k+\mu R)\left(\frac{G_{ij}+G_{ji}}{2}\right) + \lambda(P_k+\mu R)\Big[(E_{ai}+E_{aj})^T(E_{ai}+E_{aj}) + (E_{bi}F_j+E_{bj}F_i)^T(E_{bi}F_j+E_{bj}F_i)^T\Big]$$

If (28) and (29) holds, the time derivative of the fuzzy Lyapunov function is negative. Consequently, we have $\dot V(x(t)) \prec 0$ and the closed loop fuzzy system (7) is stable. This is complete the proof.

VI. NUMERICAL EXAMPLE

Consider the following T-S fuzzy system:

$$\dot x(t) = \sum_{i=1}^{r} h_i(z(t)) A_i x(t) \qquad (39)$$

with: $r = 2$

the premise functions are given by:

$$h_1(x_1(t)) = \frac{1+\sin x_1(t)}{2}\ ; \quad h_2(x_1(t)) = \frac{1-\sin x_1(t)}{2}\ ;$$

$$A_1 = \begin{bmatrix} -5 & -4 \\ -1 & -2 \end{bmatrix}; \quad A_2 = \begin{bmatrix} -2 & -4 \\ 20 & -2 \end{bmatrix};$$

It is assumed that $|x_1(t)| \le \frac{\pi}{2}$. For $\xi_{11} = 0, \xi_{12} = 0.5$, $\xi_{21} = -0.5$, and $\xi_{22} = 0$, we obtain

$$P_1 = \begin{bmatrix} 37.7864 & 26.8058 \\ 26.8058 & 36.2722 \end{bmatrix}; P_2 = \begin{bmatrix} 98.5559 & 28.7577 \\ 28.7577 & 22.9286 \end{bmatrix};$$

$$R = \begin{bmatrix} -1.2760 & -2.2632 \\ -2.2632 & -0.6389 \end{bmatrix}$$

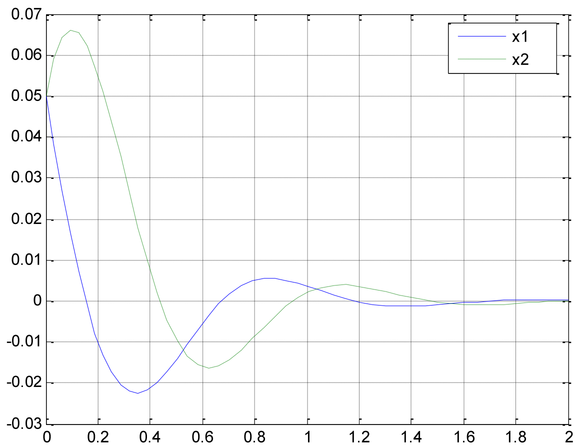

Figure 1. State variables

Figure 3 shows the evolution of the state variables. As can be seen, the conservatism reduction leads to very interesting results regarding fast convergence of this Takagi-Sugeno fuzzy system.

VII. CONCLUSION

This paper provided a new condition for the stability and stabilization of Takagi-Sugeno fuzzy systems in terms of a combination of the LMI approach and the use of non-quadratic Lyapunov function as Fuzzy Lyapunov function.

In addition, a new condition of stability of uncertain system is given for Takagi-Sugeno fuzzy systems by the use of proposed fuzzy Lyapunov function.

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