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An Analysis of the Atlantic Ocean Wave Via Random Cosine and Sine Alternate Wavy ARIMA Functions

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Abstract: In this research, alternate random wave sine and cosine for discrete time-varying processes via Autoregressive Integrated Moving Average (ARIMA) in a deterministic manner were developed. The mean and variance of the cosine and sine periodical time-varying wavy functions were derived such that Maclaurin series via full Taylor series expansion was used to rewrite the mean and variance functions. Wavy buoys of sea temperature, significant wave height, and mean wave direction of Belmullet Inner (Berth B) and Belmullet Outer (Berth A) of the Atlantic Ocean based on the west coastal of Ireland were subjected to the random sine and cosine wave functions of ARIMA. Cosine-ARIMA (1, 1, 3) and cosine-ARIMA (0, 1, 1) were the sea temperature inner and outer oceanic climate wave buoys of Berth B and A with time-periods of 8437.5 and 8035.714 respectively. Cosine-ARIMA (5, 1, 0) gave minimum performance for peak direction of inner and outer oceanic climate wave buoys of both Berth B and A, but with different time-periods of 168750 and 56250 respectively. Lastly, cosine-ARIMA (2, 1, 2) and sine-ARIMA (0, 1, 5) put in the ideal generalization for wave height of Berth B and A with the same associated wave time-periods of 56250, that is, it takes 56250 seconds to complete one swaying cycle.

Index Terms: ARIMA, Cosine random wave, Maclaurin series, Taylor series, Sine random wave.

1. Introduction

In physical sciences, a wave is a propagating dynamic disturbance of one or more quantities. When waves are periodic, the quantities oscillate repeatedly about an equilibrium value at some frequency. The traveling wave moves the entire waveform in one direction and a standing wave as a pair of superimposed periodic waves travels in opposite directions [1]. Waves in the ocean can travel thousands of kilometers before reaching land. These waves can sometimes be carried by the wind that occurs on the free surface of the bodies of water resulting from the wind blowing over a fluid surface [2]. The wind waves have a certain amount of randomness, which differs in height, duration, and shape with limited predictability. The interdependence between the flow quantities is so phenomenal in terms of the surface water movements, flow velocities, and water pressure [3]. A sine wave is a type of a continuous wave and a smooth periodic function, which is a mathematical curve, defined in terms of the sine trigonometric function [4]. The sine wave is a significant component in mathematical physics due to its phenomenon in retaining its wave shape when added to another sine wave of the same frequency and arbitrary, and magnitude [5]. A cosine wave is a waveform that has a shape identical to a sine wave unless that each point on the cosine wave occurs exactly a quarter cycle earlier than the point on the sine wave. In comparison, both the frequencies are the same for sine and cosine wave, but the cosine function leads the sine function by 90 degrees of phase [6].

Auto-regressive Integrated Moving Average is a model that is used to measure events that happen over a period. We use the model to understand past data or predict future data in a series. It is mostly use when a metric is recorded in regular intervals, from fractions of a second to daily, weekly or monthly periods [7]. If the time series contains a pure sine, cosine, or complex-valued exponential process, the predictable component is treated as a non-zero-mean but a seasonal component in the ARIMA framework so that it is eliminated by the seasonal difference [8].

Have known that Autoregressive Moving Average (ARMA) is the ideal process of any variant time series related datasets, irrespective of the differencing or integrated that might occur in the long. The major research objective in this article is the assignment of cosine and sine functions to the ARIMA or ARMA process in order to cater for the cosine and sine wavy of the Atlantic Ocean wave datasets. The introduction of cosine and sine into the ARIMA process will make it possible to solve the problem applying Atlantic Ocean wave to ARIMA and its variants without considering that the generalization of wavy (trigonometry traits) embedded in its. In extension to ARIMA cosine and sine, there have been numerous articles on Atlantic Ocean wave in application to ARIMA with applications with the limitation of ignoring the trigonometry wave attached to the dataset(s) generalization. In this article, we properly address how a cosine and sine-time varying processes can contain an ARIMA process and apply it to Belmullet Inner and Belmullet Outer of the Atlantic Ocean wave.

2. Literature Review

Ref. [9] employed the ensemble of random Gaussian Laplace eigenfunctions on 3D arithmetic random waves and studied the distribution of their nodal surface area. In their study, they have shown that the nodal area variance follows an asymptotic and thus, the expected area is proportional to the square root of the eigenvalue (energy) of the eigenfunction. Their result has shown that the resulting asymptotic formula is closely related to the angular distribution and correlations of the lattice points lying on spheres. Ref. [10] generated waves with JONSWAP spectrum for various steepness, height and constant period experimenting results describing random, uni-directional, long crested, water waves over non-uniform bathymetry confirming the formation of stable coherent wave packages traveling with almost uniform group velocity. They applied sets of statistical procedures to the experimental data, including the space and time variation of kurtosis, skewness, BFI, Fourier, and moving Fourier spectra, and probability distribution of wave heights. In their result, the Stable wave packages formed out of the random field and traveling over shoals, valleys and slopes were compared with exact solutions of the NLS equation resulting in good matches. According to [11] the Lipschitz-Killing Curvatures for the excursion set of toral Gaussian eigenfunctions are dominated, in the high-frequency regime, by a single chaotic component. In their study, they have written the latter as a simple explicit function of the threshold parameter times the centered norm of these random fields; as a result, these geometric functional are fully correlated in the high-energy limit. Therefore, the derived formulae show a clear analogy with related results on the round unit sphere and suggest the existence of a general formula for geometric functional of random eigenfunctions on Riemannian manifolds. Ref. [12] considered Berry's random planar wave model of 1977 for a positive Laplace eigenvalue $E > 0$, both in the real and complex case, and prove limit theorems for the nodal statistics associated with a smooth compact domain, in the high energy limit $(E \to \infty)$ their findings can be naturally reformulated in terms of the nodal statistics of a single random wave restricted to a compact domain diverging to the whole plane. As a good byproduct of their analysis, they rigorously confirm the asymptotic behavior for the variances of the nodal length and of the number of nodal intersections of isotropic random waves, as derived in Berry (2002).

Ref. [13] developed a mathematical model to analyze the hydrodynamics of a novel oscillating water column (OWC) in a hybrid wind-wave energy system. They studied the effects of both skirt and internal radius dimensions on the power extraction efficiency for monochromatic and random waves. Then they applied the matching method of eigenfunctions to solve boundary value problem since the shape of the device is complex. Moreover, within the framework of their linearized theory, they model the turbine damping effects assuming the airflow proportional to the air chamber pressure. That led the velocity potential to be decomposed into radiation and diffraction problems. We show that the internal cylinder affects the values of the sloshing eigenfrequencies since the skirt has strong effects on the global behavior. They used laboratory data to validate the analytical model and showed a good agreement between analytical and experimental results.

Ref. [14] proposed rogue waves as a statistical theory and tested experimental data collected against in a long water tank where random waves with different degrees of nonlinearity are mechanically generated and free to propagate along the flume. So strong evidence is given that the rogue waves observed in the tank are hydrodynamic instantons and these hydrodynamic instantons are complex spatiotemporal wave field configurations, which can be defined using the mathematical framework of large deviation theory and calculated via tailored numerical methods [15]. Their results have shown that, instantons describe equally well rogue waves in weakly nonlinear conditions or in strongly nonlinear conditions creating the way for the development of a unified explanation to rogue wave formation. According to [16], in wind turbines, some offshore structures near the coast are crucial for supplying energy as a material or changing the natural resources into energy. Due to the loads of the structure by its function and waves, wind, the current seawater level, and ice, the issues to overcome about the structures is mechanical fatigue. In their study, under random processes, the wave loads, and fatigue damage are reviewed. Ref. [17] demonstrated a floating buoy-based triboelectric nanogenerator (FB-TENG) to effectively harvest the vibrational energy with full satisfaction and the floating buoy-based triboelectric nanogenerator consisted of a power generation unit, which was packed in an acrylic case, and a height-adjustable support, which was attached to a floating buoy. The data processing of random wave field was developed because of inverse scattering method by [18] in which the soliton component obscured in a random wave field was determined and a corresponding distribution function of number of solitons on their amplitudes was constructed. The approach they developed was illustrated by means of artificially generated quasi-random wave field and it was to the real data interpretation of wind waves generated in the laboratory wind tank.

In this research, random wave for sine and cosine would be developed via Autoregressive Integrated Moving Average (ARIMA) time series processes. The proposed cosine and sine-time varying processes via ARIMA relevance will make it viable for swaying cycle of the Atlantic Ocean wave to be captured and evaluated via frequencies and time-period of completing a wave oscillation. The means and variances of the random sine and cosine time-varying processes would the estimated and simply via the Taylor series using Maclaurin series for simplicity. The random wave for sine and cosine via ARIMA process would be subjected to wave measured buoys of sea temperature measured in Celsius, Significant Wave Height measured in meter, and mean wave direction measured in degree for Belmullet Inner (Berth B) and Belmullet Outer (Berth A) of the Atlantic Ocean based on the west coastal of Ireland.

3. Methods

3.1. Random Cosine Wave

In accordance with the sine cosine algorithm for any time-varying process proposed by [19], a cosine time-varying process with ARIMA process would be defined as:

$$X_t = \cos\left[2\pi\left(\frac{t}{12}+\Theta\right)\right] \quad \forall\ t = 0, \pm 1, \pm 2, \pm 3, \cdots \tag{1}$$

Where, $\Theta = I + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \cdots + \alpha_p X_{t-p} + e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \cdots + \theta_q e_{t-q}$

Otherwise, $\Theta = I + \sum_{i=1}^{p} \alpha_i X_{t-i} + \sum_{j=0}^{q} \theta_i e_{t-i}$; $\theta_0 = 1$ is called the ARIMA (p,d,q) process. Estimating the coefficients "α" and "θ" for a given (p,d,q) is what ARIMA does when it learns from the training data in a time series.

- "p" connotes he number of lag observations included in the model, also called the lag order (deals with window of X_t)
- "d" connotes the number of times that the raw observations are differenced, also called the degree of differencing (deals with order of differencing of X_t)
- "q" connotes the size of the moving average window, also called the order of moving average (deals with residuals).

The cosine time-varying process will appear lofty deterministic since X_t might repeat itself in a discrete time-varying monovular manner, every time units to give a cosine curve. Maximum cannot be ascertained at point $t = 0$, but to be ascertained randomly for the distinct time period in a sequence of ARIMA process of Θ . The distinct period in a sequence of the ARIMA process of Θ that can be misconstrued as fraction of a total cycle implemented by time $t = 0$. Statistical properties of the process follow as:

$$E(X_t) = \cos\left[2\pi\left(\frac{t}{12}+\Theta\right)\right] \tag{2}$$

$$= \int_0^1 \cos\left[2\pi\left(\frac{t}{12}+\Theta\right)\right]\partial\phi \tag{3}$$

$$= \frac{\sin}{2\pi}\left[2\pi\left(\frac{t}{12}+\phi\right)\right]_{\phi=0}^{1} \tag{4}$$

$$= \frac{\left[\sin\left(\frac{2\pi t}{12}+2\pi\right)-\sin\left(\frac{2\pi t}{12}\right)\right]}{2\pi} \tag{5}$$

Also,

$$\gamma_{t,s} = E\left\{\cos\left[\left(\frac{t}{12}+\Theta\right)2\pi\right]+\cos\left[\left(\frac{s}{12}+\Theta\right)2\pi\right]\right\} \tag{6}$$

$$= \int_0^1 \cos\left[\left(\frac{t}{12}+\phi\right)2\pi\right]\cos\left[\left(\frac{s}{12}+\phi\right)2\pi\right]\partial\phi \tag{7}$$

$$= \frac{1}{2}\int_0^1 \left\{\cos\left[\frac{t-s}{12}\right]2\pi+\cos\left[\frac{t+s}{12}\right]2\pi\right\}\partial\phi \tag{8}$$

$$= \frac{1}{2}\left\{\cos\left[\left(\frac{t-s}{12}\right)2\pi\right]+\frac{\sin}{4\pi}\left[\left(\frac{t+s}{12}+2\phi\right)2\pi\right]\right\}_{\phi=0}^{1} \tag{9}$$

$$= \frac{1}{2}\cos\left[\left(\frac{|t-s|2\pi}{12}\right)\right] \tag{10}$$

Recall from Taylor Polynomial of degree "n" for cosine smooth function

$f\in C^{\infty}$, $f:\Re\to\Re$, of "f" at x_0 defined as $T_{\infty(x)}=\sum_{0}^{\infty}\frac{f^{(k)}(x_0)}{k!}\left(x-x_0\right)^k$, where $T_{\infty(x)}$ is called analytic. It is to be noted that when $x=x_0$, then $T_{\infty(x)}$ is referred to as Maclaurin Series. Consider the function

$$f(x)=\cos(x)\in C^{\infty}$$

Therefore, the full Taylor series expansion of $f(x)$ at $x=x_0$ is given by

$$T_{\infty(x)}=\sum_{x=0}^{\infty}\frac{f^{(k)}(x_0)}{k!}\left(x-x_0\right)^k \tag{11}$$

$$=1+x-\frac{1}{2!}x^2-\frac{1}{3!}x^3+\frac{1}{4!}x^4+\frac{1}{5!}x^5-\cdots \tag{12}$$

$$=1-\frac{1}{2!}x^2+\frac{1}{4!}x^4\mp\cdots \tag{13}$$

$$=\sum_{k=0}^{\infty}\left(-1\right)^k\frac{1}{(2k)!}x^{2k}=\cos(x) \tag{14}$$

So,

$$\gamma_{t,s}=\sum_{k=0}^{\infty}\left(-1\right)^k\frac{1}{(2k)!}\left[\frac{|t-s|2\pi}{12}\right]^{2k} \tag{15}$$

Variance of the process,

$$Var(X_t)=\cos\left[2\pi\left(\frac{t}{12}+\Theta\right)\right] \tag{16}$$

$$\gamma_{t,s}=\sum_{k=0}^{\infty}\left(-1\right)^k\frac{1}{(2k)!}\left[2\pi\left(\frac{t}{12}+\Theta\right)\right] \tag{17}$$

Therefore, the process is stationary with autocorrelation function.

$$p_j = \cos\left[2\pi\left(\frac{k}{12}\right)\right] \quad \forall \ k = 0, \pm 1, \pm 2, \pm 3, \cdots \tag{18}$$

$$\rho_k = \sum_{k=0}^{\infty}(-1)^k \frac{1}{(2k)!}\left[2\pi\left(\frac{k}{12}\right)\right] \quad \forall \ k = 0, \pm 1, \pm 2, \pm 3, \cdots \tag{19}$$

This example suggests that it will be difficult to assess whether or not stationarity is a reasonable assumption for a given time series based on the time sequence plot of the observed data.

3.2. Random Sine Wave

Considering a similar sine time-varying process as:

$$X_t = \sin\left[2\pi\left(\frac{t}{12}+\Theta\right)\right] \quad \forall \ t = 0, \pm 1, \pm 2, \pm 3, \cdots \tag{20}$$

The statistical properties of the process follow as:

$$E(X_t) = \sin\left[2\pi\left(\frac{t}{12}+\Theta\right)\right] \tag{21}$$

$$= \frac{\left[-\cos\left(\frac{2\pi t}{12}+2\pi\right)+\cos\left(\frac{2\pi t}{12}\right)\right]}{2\pi} \tag{22}$$

Thus, this is also zero since the sine must agree. So, $\mu_t = 0 \ \ \forall \ "t"$.
Also,

$$\varphi_{t,s} = \int_0^1 \sin\left[2\pi\left(\frac{t}{12}+\phi\right)\right]\sin\left[2\pi\left(\frac{s}{12}+\phi\right)\right]\partial\phi \tag{23}$$

$$= \frac{1}{2}\sin\left[2\pi\left(\frac{t-s}{12}\right)\right] \tag{24}$$

Such that,

$$T_{\infty(x)} = \sum_{x=0}^{\infty}\frac{f^{(j)}(x_0)}{j!}\left(x-x_0\right)^j = \sum_{j=0}^{\infty}(-1)^j \frac{1}{(2j+1)!}x^{2j+1} = \sin(x) \tag{25}$$

So that,

$$\varphi_{t,s} = \sum_{j=0}^{\infty}(-1)^j \frac{1}{(2j+1)!}\left[\frac{|t-s|2\pi}{12}\right]^{2j+1} \tag{26}$$

The variance of the process is:

$$Var(X_t) = \sin\left[2\pi\left(\frac{t}{12}+\Theta\right)\right] \tag{27}$$

$$\varphi_{t,s} = \sum_{j=0}^{\infty}(-1)^j \frac{1}{(2j+1)!}\left[2\pi\left(\frac{t}{12}+\Theta\right)\right] \tag{28}$$

Thus, the process is also stationary with autocorrelation function,

$$\tau_j = \sin\left[2\pi\left(\frac{j}{12}\right)\right] \;\forall\; j = 0, \pm 1, \pm 2, \pm 3, \cdots \tag{29}$$

$$\tau_j = \sum_{j=0}^{\infty} (-1)^j \frac{1}{(2j+1)!}\left[2\pi\left(\frac{j}{12}\right)\right] \tag{30}$$

This example also suggests that it will be difficult to assess whether or not stationarity is a reasonable assumption for a given time series based on the time sequence plot of the observed data. The proposed research methodology will help to facilitate the achievement of the research objective of estimating the appropriate order of Autoregressive, Moving Average with the ideal differencing for an alternate cosine and sine embedded ARIMA time-varying process. To be able to estimate via the research methodology are the needed coefficients of the AR, MA, residuals, model performance, frequency and the associated time-period of swaying oscillation.

4. Numerical Analysis

In this research, the real-time wave performance of wave buoys: Belmullet Inner (Berth B) and Belmullet Outer (Berth A) of the Atlantic Ocean based on the west coastal of Ireland will be used via the random sine and cosine wave of ARIMA. The inner and outer oceanic climate wave buoys for (Berth B) and (Berth A) to be used in this research will be from (2012:5) to (2022:4) for both. The historical oceanic climate wave buoys that were observed by Atlantic Marine Energy Test Site (AMETS) in collaboration of Sustainable Energy Authority of Ireland (SEAI) consist of some observed wave measurements. The gathered wave measured buoys considered include — the sea temperature measured in Celsius, Significant Wave Height measured in meter, and mean wave direction measured in degree. It is to be noted that the mentioned measured buoys reply solely on the latitude (degrees' north) and longitude (degrees' east). The test site is an integral component of Ireland's Ocean Energy Strategy that was developed in accordance with the national Offshore Renewable Energy Development Plan (OREDP). The primary aim is to provide an assessment of the monthly recorded of full-scale offshore wave characteristics and resource variability at the two deployment berths via random wave sine and cosine of ARIMA process with or without the presence of drift.

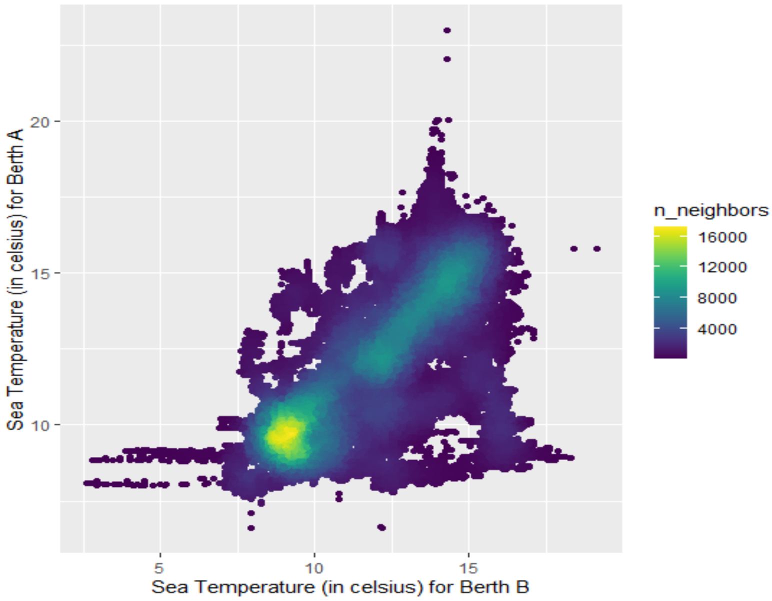

Fig.1. Cross scatters with density plot of Sea Temperature between Berth A and B.

From figure 1, the density of data points of the wave of sea temperature for the cross-sectional Berths of A and B lie between 7 and 10 degrees Celsius. Whereas it was between 250 and 300 degrees Celsius for the mean wave direction for the two berths of A and B as seen in figure 2. The significant wave heights for the two berths were centered around 100m to 400m thereabout as seen in figure 3. It is to be noted that also that each point of color green is in line with the number of neighboring points, such that the distance threshold considered two points as smoothing bandwidth (neighbors). This connotes those cases of overlapping started from the colored region above of each of the characterized measurement of the two Berths studied.

From the superposition of the sine and cosine of sea temperature for Berth B in figure 4, the wave spring of the helix was noted to have been done with a sine and cosine at 360^0 degrees in space. Meaning the sea temperature for Berth B wave for the average months of years studied do complete the wave circular and longitudinal crest that occurs at $\sin\left(2\pi\left(\frac{t}{12}+360^0\right)\right)$, meaning the wave are close to sine wave. Same circular and longitudinal crest occurs for the sea temperature for Berth A wave, but indicated not to be a completed one, probably at 180^0 because the helices did not form a circular one at the lower bottom and as well at the apex for a right-handed and left-handed (wave frequency) superposition respectively. From figures 6 and 7 above for sine and cosine helix curve wave of mean direction for Berth B, it was obviously seen that the cosine and sine wave for the two Berths started by a circular with a bisected three lines before a typical close scanty sine and cosine waves experienced in the middle, but very close to trochoidal shape. At the highest level of waves of the mean direction of the Berths, there was a dense rounded wave in both Berths. This means that a longitudinal acoustic waves. From figure 8 and 9 for sine and cosine helix curve wave of height for Berths A and B, it was obvious that their helix is an handedness one because it is not from the perspective of a right-handed helix that cannot be turned to look like a left-handed one unless it is viewed in a mirror.

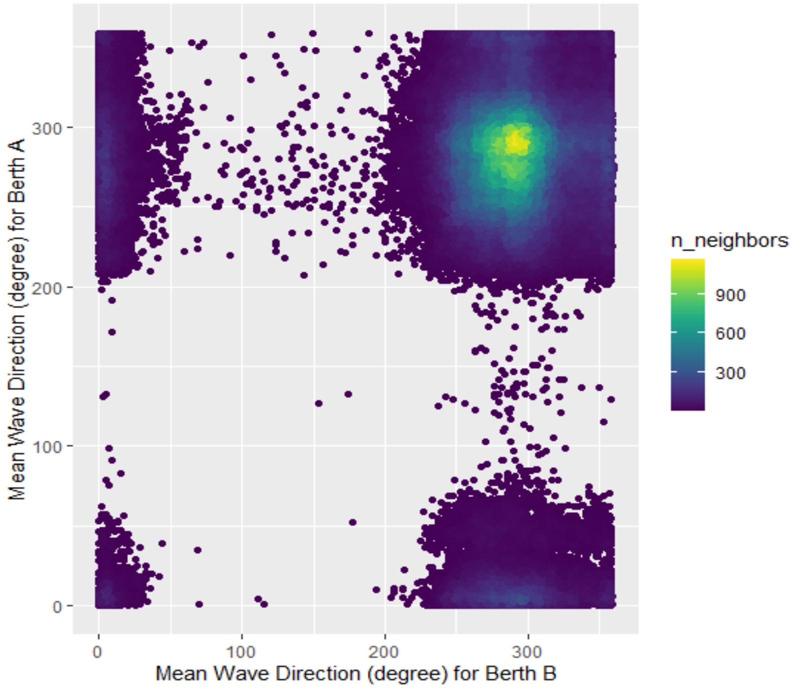

Fig.2. Cross scatters with density plot of Mean Wave Direction between Berth A and B.

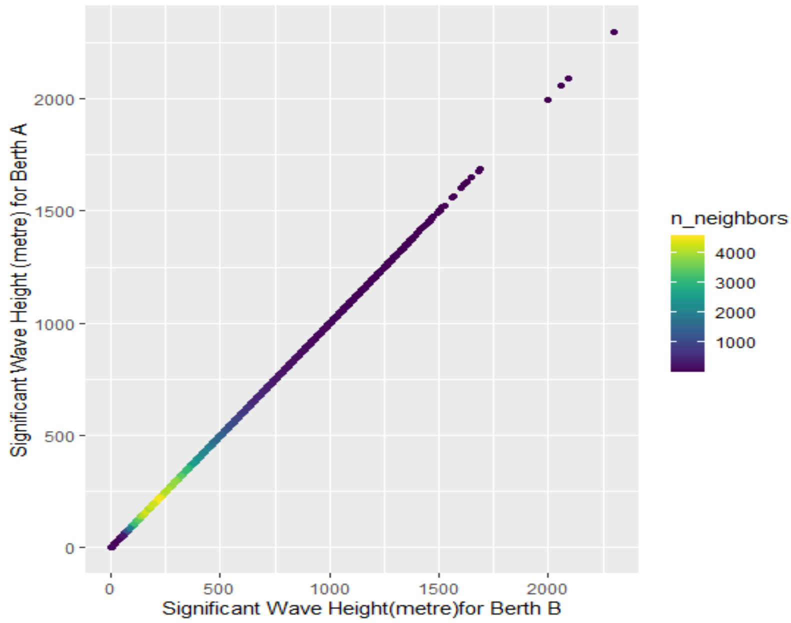

Fig.3. Cross scatters with density plot of Wave Height between Berth A and B.

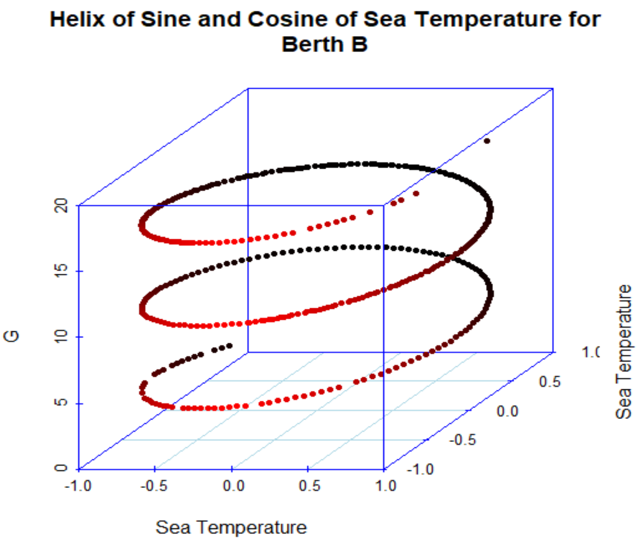

Fig.4. Sine and Cosine Helix Curve Wave of the Sea Temperature of Berth B.

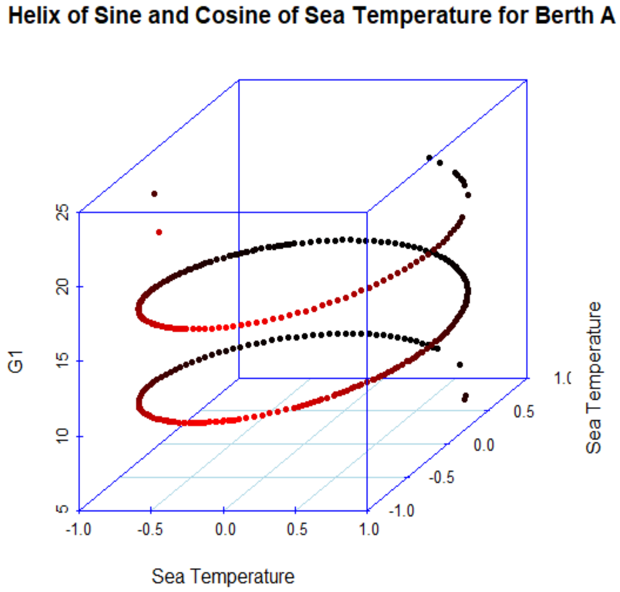

Fig.5. Sine and Cosine Helix Curve Wave of the Sea Temperature of Berth A.

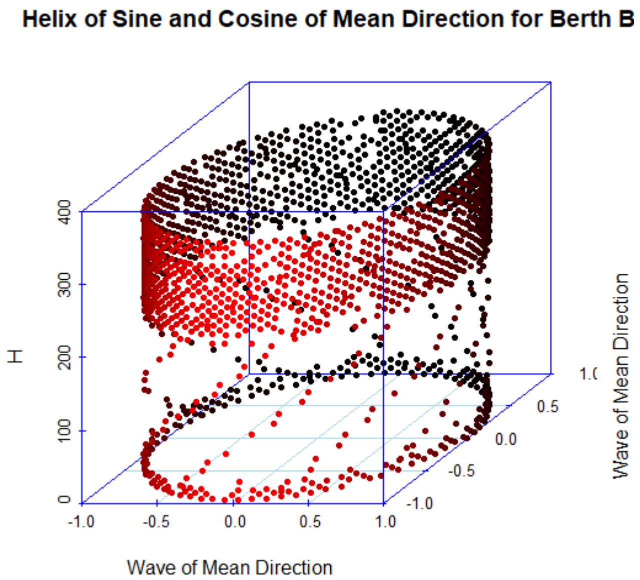

Fig.6. Sine and Cosine Helix Curve Wave of Mean Direction for Berth B

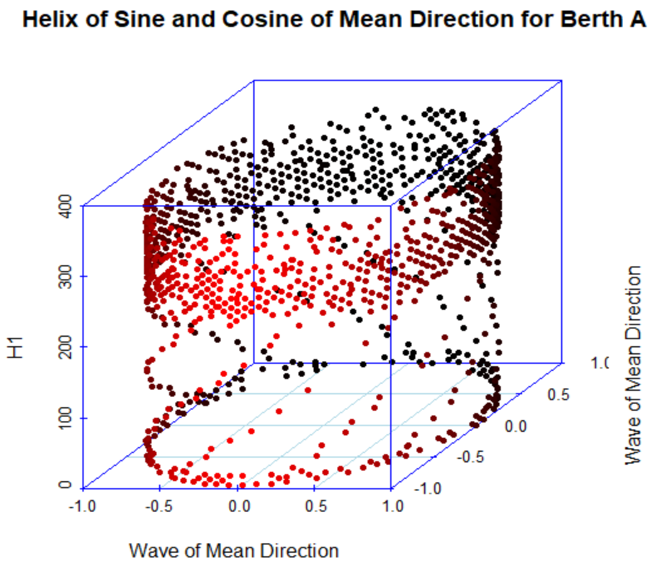

Fig.7. Sine and Cosine Helix Curve Wave of Mean Direction of Berth A.

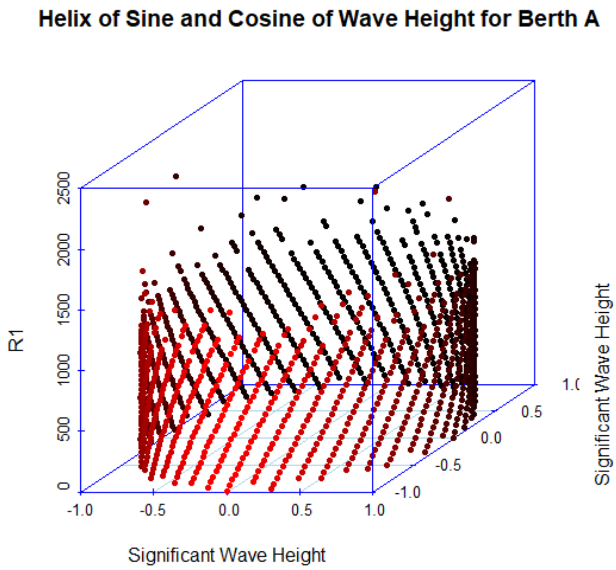

Fig.8. Sine and Cosine Helix Curve Wave of Height of Berth A.

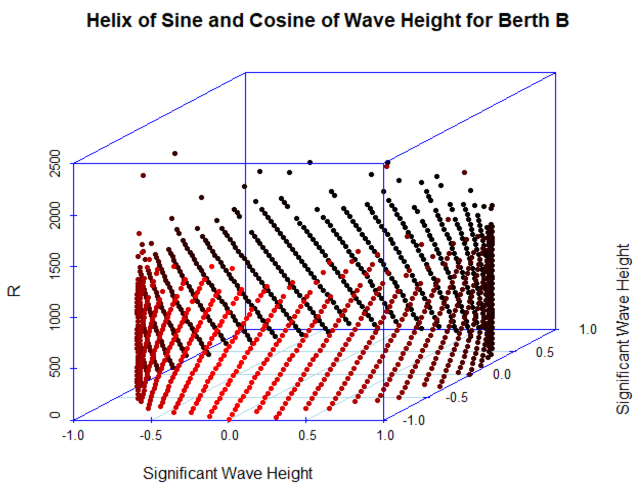

Fig.9. Sine and Cosine Helix Curve Wave of Height of Berth B.

Table 1. Coefficients of the Random Wave of Sine and Cosine via Autoregressive Integrated Moving Average (ARIMA)

ARIMA Model	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)	MA(1)	MA(2)	MA(3)	MA(4)	σ^2	Drift
Sea Temp. B. (Sin) ARIMA(2,0,4)	0.3088 (0.3921)	0.5641 (0.3604)	----	----	----	-0.3487 (0.3923)	-0.6167 (0.3762)	0.0013 (0.0099)	0.0159 (0.0069)	0.0007	----
Sea Temp. B. (Cos.) ARIMA(1,1,3)	0.9101 (0.0023)	----	----	----	----	-0.9495 (0.0033)	-0.0208 (0.0033)	0.0040 (0.0026)	----	0.0009	----
Sea Temp. A. (Sin) ARIMA(2,1,2) with drift	0.0536 (0.0563)	-0.1371 (0.0474)	----	----	----	-0.0406 (0.0560)	0.1819 (0.0468)	----	----	0.0006	0.000 (0.0183)
Sea Temp. A. (Cosine) ARIMA(0,1,1)	----	----	----	----	----	0.0560 (0.0024)	----	----	----	0.0007	----
Peak Direction B. Sine ARIMA(2,1,2)	0.4534 (0.0120)	-0.0506 (0.0027)	----	----	----	-1.5002 (0.0119)	0.5081 (0.0117)	----	----	0.6454	----
Peak Direction B. Cosine ARIMA(5,1,0)	-0.8440 (0.0024)	-0.6854 (0.0030)	-0.5365 (0.003)	-0.376 (0.0030)	-0.222 (0.002)	----	----	----	----	0.5493	----
Peak Direction A. Sine ARIMA(1,1,1) with drift	-0.0074 (0.003)	----	----	----	----	-0.9890 (0.0004)	----	----	----	0.4617	0.0000 (0.000)
Peak Direction A. Cosine ARIMA(5,1,0)	-0.8292 (0.0024)	-0.6639 (0.0030)	-0.5020 (0.003)	-0.3405 (0.0030)	-0.1741 (0.0024)	----	- ---	----	----	0.4998	----
Wave Height.B Sine ARIMA(5,1,0)	-0.8343 (0.0024)	-0.6703 (0.0030)	-0.5084 (0.003)	-0.3392 (0.0030)	-0.1739 (0.002)	----	----	----	----	0.5033	----
Wave Height. B. Cosine ARIMA(2,1,2)	0.4363 (0.0311)	-0.0170 (0.0027)	----	----	----	-1.4409 (0.0310)	0.4512 (0.0305)	----	----	----	----
Wave Height. A. Sine ARIMA(0,1,5)	----	----	----	----	----	-1.0035 (0.0024)	0.0019 (0.0035)	-0.0010 (0.003)	0.0047 (0.0033) MA= 0.0156 (0.003)	0.504	----
Wave Height. A. Cosine ARIMA(0,1,5)	0.4412 (0.0295)	-0.0181 (0.0027)	----	----	----	-1.4475 (0.029)	0.4575 (0.0289)	----	----	----	----

Cosine Autoregressive Integrated Moving Average (ARIMA) wave function truly represented the sea temperature inner and outer oceanic climate wave buoys of Berth B with an ideal minimum model performance of residual mean=-0.0042, AIC= AICc=-700531.7, BIC=-700481.5 with associated wave time-period of 8437.5 and frequencies of 0.0217 and 0.0359 at every 20 and 9 minutes respectively. The fair wavy generalization is ARIMA (1, 1, 3) with 0.9101 (0.0023) , -0.9495 (0.0033), -0.0208 (0.0033), and 0.0040 (0.0026) as the AR and MA coefficients for the integrated wave series. Similarly, cosine-ARIMA (0, 1, 1) (Integrated MA (1) equivalently) ideally gave a true time frequency series of the sea temperature inner and outer oceanic climate wave buoys of Berth A with minimum model performance of residual mean=-0.0108, AIC= AICc= -751347.9, BIC= -751327.9 with associated wave time-period of 8035.714 and frequencies of 4477.148 and 0.0119 at every 21 and 3 minutes respectively at MA (1) = 0.0560 (0.0024).

As regards the peak direction of the wavy function, cosine-ARIMA (5, 1, 0) (Integrated AR(5) equivalently) ideally gave a true time frequency series of the peak direction of the oceanic climate wave buoys of Berth B with minimum model performance of residual mean=-0.0110, AIC= AICc= 377599.2 , BIC= 377659.5 with associated wave time-period of 168750 and frequencies of 0.0147 and 0.0080 at every 1 and 2 minutes respectively with its AR coefficients of -0.8440 (0.0024), -0.6854 (0.0030), -0.5365 (0.003), -0.376 (0.0030) and -0.222 (0.002). Similarly, cosine-ARIMA (5, 1, 0) with model nonesuch of residual mean=-0.0134, AIC= AICc= 361662.6, BIC= 361722.8 with associated wave time-period of 56250 and frequencies of 0.0119 and 0.0160 at every 3 and 4 minutes respectively with its AR coefficients of -0.8292 (0.0024), -0.6639 (0.0030), -0.5020 (0.003), -0.3405 (0.0030), and -0.1741 (0.0024) for the first five lags respectively is for peak direction for Berth A. The time-period of 168750 and 56250 Wave height for Berth B produced cosine-ARIMA (2, 1, 2) with residual mean=0.0011, AIC= AICc= AIC=362550.1, BIC= 362600.3 with associated wave time period of 56250 and frequencies of 0.0120 and 0.0159 at every 3 and 4 minutes respectively with its AR coefficients of 0.4363 (0.0311), -0.0170 (0.0027); MA coefficients of -1.4409 (0.0310), 0.4512(0.0305). Differently, wave height for Berth A gave sine-ARIMA (0, 1, 5) with residual mean=0.0011, AIC= AICc= AIC=363089.3, BIC= 363149.5 with associated wave time-period of 56250 and frequencies of 0.0119 and 0.0159 at every 3 and 4 minutes respectively such that the integrated MA (5) coefficients are -1.0035 (0.0024), 0.0019 (0.0035), -0.0010 (0.003), 0.0047 (0.0033), 0.0156 (0.003)

Table 2. Coefficients of the Model Performance of the Random Wave of Sine and Cosine via Autoregressive Integrated Moving Average (ARIMA)

ARIMA Model		Frequency	Specification	Model Performance	Time Period	Residual Mean
Sea Temp. B. (Sine) ARIMA(2,0,4)	3483	0.0206	0.0305	AIC=-735082.1 AICc=-735082.1	48.44961	-0.0042
	3529	0.0209	0.0267	BIC=-735011.8		
Sea Temp. B. (Cos.) ARIMA(1,1,3)	20	0.0217	9975.200	AIC=-700531.7 AICc=-700531.7	8437.5	-0.0042
	9	0.0359	2599.939	BIC=-700481.5		
Sea Temp. A. (Sine) ARIMA(2,1,2) with drift	11	0.0439	39764.29	AIC=-775943.9 AICc=-775943.9	15340.91	-0.0015
	1	0.0147	36313.10	BIC=-775883.7		
Sea Temp. A. (Cosine) ARIMA(0,1,1)	21	4477.148	7903.786	AIC=-751347.9 AICc=-751347.9	8035.714	0.0108
	3	0.0119	5160.450	BIC=-751327.9		
Peak Direction B. Sine ARIMA(2,1,2)	1	0.0147	38935.45	AIC=404799.1 AICc=404799.1	84375	0.0441
	2	0.0080	33917.93	BIC=404849.2		
Peak Direction B. Cosine ARIMA(5,1,0)	1	0.0147	5638.762	AIC=377599.2 AICc=377599.2	168750	-0.0110
	2	0.0080	4887.920	BIC=377659.5		
Peak Direction A. Sine ARIMA(1,1,1)	3	0.0119	18598.35	AIC=348288.5 AICc=348288.5	56250	0.0144
	4	0.0160	15986.53	BIC=348328.7		
Peak Direction A. Cosine ARIMA(5,1,0) with drift	3	0.0119	2712.511	AIC=361662.6 AICc=361662.6	56250	0.0134
	4	0.0160	2345.127	BIC=361722.8		
Wave Height.B Sine ARIMA(5,1,0)	3	0.0120	4653.238	AIC=362845.2 AICc=362845.2	56250	0.0187
	4	0.0159	4181.610	BIC=362905.4		
Wave Height. B. Cosine ARIMA(2,1,2)	3	0.0120	47785.69	AIC=362550.1 AICc=362550.1	56250	0.0011
	4	0.0159	41125.30	BIC=362600.3		
Wave Height. A. Sine ARIMA(0,1,5)	3	0.0119	47607.48	AIC=363089.3 AICc=363089.3	56250	0.0011
	4	0.0159	41304.10	BIC=363149.5		
Wave Height. A. Cosine ARIMA(2,1,2)	3	0.0119	47777.53	AIC=363337.4 AICc=363337.4	56250	-0.0009
	4	0.0159	40453.20	BIC=363387.6		

5. Conclusion

Novel alternate random cosine and sine time-varying wave processes were developed via an Autoregressive Integrated Moving Average (ARIMA) for a deterministic time series or sequences in a discrete time-varying monovular manner. The mean and variance of the cosine and sine periodical time-varying wavy functions were derived such that Maclaurin series via full Taylor series expansion was used to rewrite the mean and variance functions. Wave measured buoys of sea temperature, significant wave height, and mean wave direction of Belmullet Inner (Berth B) and Belmullet

Outer (Berth A) of the Atlantic Ocean based on the west coastal of Ireland were subjected to the random sine and cosine wave functions of ARIMA. It can be inferred that cosine random wave function of ARIMA ideally represent the sea temperature inner and outer oceanic climate wave buoys of Berth A and B; peak direction of Berth A and B and wave height for Berth B, while sine random wave function of ARIMA gave the smoothing generalization for wave height for Berth A. In conclusion, cosine random wave function should always be used for the wavy Belmullet Inner and Belmullet Outer of the Atlantic Ocean Sea temperature and peak direction, while sine and cosine should be adopted for Belmullet Inner and Belmullet Outer respectively. In extension to the study, the alternate cosine and sine processes can be extended to Generalized Autoregressive Conditional Heteroscedasticity (GARCH) time-varying series.

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