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Defuzzification Index for Ranking of Fuzzy Numbers on the Basis of Geometric Mean

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Abstract: The importance of fuzzy numbers to express uncertainty in certain applications, concerned with decision making, is observed in a large number of problems of different kinds. In Decision making problems, the best of available alternatives is chosen to the possible extent. In the process of ordering the alternatives, ranking of fuzzy numbers plays a key role. A large volume of ranking methods, based on different features, have been available in this domain. Owing to the complicated nature of fuzzy numbers, the so far introduced methods suffered setbacks or posed difficulties or showed drawbacks in one context or other. In addition, some methods are lengthy and complicated to apply on concerned problems. In this article, a new ranking procedure based on defuzzification, stemmed from the concepts of geometric mean and height of a fuzzy number, is proposed. Finally, numerical comparisons are made with other existing procedures for testing and validation of proposed method with the support of some standard numerical examples.

Index Terms: Fuzzy sets, fuzzy numbers, ranking of fuzzy numbers, defuzzification.

1. Introduction

Fuzzy sets, proposed by Zadeh [1], play a significant role to represent uncertain (or imprecise or vague) information in real world problems. An integrated knowledge about the uncertainty is very helpful in the problems related to decision making. For this sort of problems, a suitable, meaningful and reliable solution is provided by the theory of fuzzy sets. Fuzzy numbers are classified as one of the kinds of fuzzy sets. In recent years, applications of fuzzy numbers are extensively found in different kinds of problems which are with the need of decision making of choosing better preferences. Thus, naturally it is essential to rank (or order) fuzzy numbers and hence, ranking of fuzzy numbers has significant and meaningful role in the applications of decision making involving uncertainty.

The idea of ranking fuzzy numbers has been consistently discussed and dealt with by many researchers over the years. A considerable good volume of methods of ranking have been brought to this domain over the years since Jain [2, 3] who proposed pioneering works in ordering the preferences of fuzzy numbers with the help of maximizing set. Bass & Kwakernaak [4] applied the general natural ranking of real numbers to fuzzy numbers in their method. Dubios & Prade [5] attempted to rank fuzzy numbers with the help of maximising set thoery. The concept of finding centroid $(\bar{x}, \bar{y})$ to rank fuzzy numbers was firstly applied by Yager [6] where the ranking index is $\bar{x}$. This method considered a fuzzy number larger if it's respective $\bar{x}$ is larger. Fuzzy numbers were not correctly ranked by this method as it showed no difference in results whose $\bar{x}$ are same and $\bar{y}$ are different. Later, Yager [7] introduced a technique to order fuzzy subsets in the unit interval. Chen [8] propounded an index for ordering fuzzy numbers by making use of the properties of maximising and minimising sets.

Based on set difference properties and membership function's area, Choobineh & Li [9] ranked fuzzy numbers. Ranking and defuzzication techniques, relied on area compensation, were introduced by Fortemps & Roubens [10]. For ranking fuzzy numbers, in Cheng's [11] distance method, the distance that lies in between the centroid and (0, 0) was taken into account. The Cheng's technique has some flaws also which gives the same ranking for fuzzy numbers and their corresponding images. Yao & Wu [12] proposed a technique to rank fuzzy numbers making use of decomposition principle and signed distance. It was observed by Chu & Tsao [13] that the negative fuzzy numbers could not be ranked correctly by distance method and hence, instead of distance, area that lie between the centroid and the origin was considered as ranking tool. With this tool, a fuzzy number is identified to be greater if its corresponding area is more. Further, Chu & Tsao's [13] area method was observed to produce counter intuitive rankings. Therefore, Abbasbandy & Asady [14] proposed a sign distance technique. Furthermore, Wang et al. [15] introduced a tool for ordering generalized fuzzy numbers. Using the notion of distance minimization, an index was initiated by Asady & Zendehnam [16]. Chen & Chen [17] ranked generalized fuzzy numbers making use of their different heights and different supports. A magnitude method was brought forward by Abbasbandy & Hajjari [18] basing on the concept of dual spreads at some α - levels to order trapezoidal fuzzy numbers. Giving equal ranking for any two symmetric trapezoidal fuzzy numbers was identified to be a drawback for this method. Revision of distance minimization method is introduced by Asady [19] to get rid of drawbacks in magnitude method.

Having relied on fuzzy distance, an ordering index was introduced by Allahviranloo et al. [20]. A method was brought forward by Chen & Sanguansat [21] to order generalised fuzzy numbers by taking their areas lie on both positive and negative sides and heights. A method that considers the areas on both sides along with centroids of generalised fuzzy numbers, was introduced by Chen et al.[22].

Further, Allahviranloo & Saneifard [23], utilising the concept of centre of gravity of fuzzy numbers, proposed a method. A method that takes into account the angle of reference functions was introduced by Nasseri et al. [24]. Utilizing the values obtained in the form of variances, Rezvani [25] put forward a method. The drawback of this method is that it could not order crisp numbers. Along with this, it could not order fuzzy numbers of equal support with different cores. In Chutia and Chutia [26], ranking is specified by the concepts of value and ambiguity of fuzzy numbers which are calculated with decision level α , taken in the range $0<\alpha<1$, and the height w. This method of ranking fails when the decision level $\alpha=1$ and height $w=1$. Owing to some drawbacks of this method, a modified epsilon-deviation degree method is proposed by Chutia [27]. Bortolan & Degani [28]; Wang & Kerre [29] and Brunelli & Mezei [30] made a thorough review of the available methods, and observed some inappropriate and illogical conditions among them.

On consolidation of afore mentioned literature, it may be observed that the ranking of fuzzy numbers is done by different techniques such as maximising set and minimising set, distance method, sign distance method, distance minimization, α cuts, defuzzification and so forth. However, the researchers keep on introducing new methods to cater academic and industrial needs. After thorough observation of available methods and integrated studies [28, 29, 30], it is felt that a major volume of the existing methods have one or other type of limitations and drawbacks in some aspects or other. Further, most of the existing methods have complicated and lengthy computational procedures. Hence, in this article, a novel defuzzification technique originated from the concepts of geometric mean and height of a fuzzy number is proposed to make an ordering of fuzzy numbers. The preferences of alternatives, that arise due to comparison of given fuzzy numbers, could be clearly differentiated by this method. Further, this method may also clear the ambiguities among the alternatives and the counter intuitive rankings that were showed up by earlier ranking methods. As an addition to this, the computational technique may be considered to be simpler and easier to apply on concerned applications.

This paper is organised as following. The necessary fundamentals related to fuzzy numbers concept are presented in 2nd section. The 3rd section is devoted for clear illustration of the proposed method with necessary data and examples. The 4th section is devoted for necessary discussions and useful comparisons of this work with other author works. With a mentioning of key points in a nut shell, the paper is concluded with 5th section.

2. Related Works

A fuzzy set is characterised by a membership mapping which allocates a grade of membership, ranging between zero and one, to elements within its universe of discourse. This membership value describes whether given element belongs to a set or possesses the property of the set. If the membership grade possessed by an element is 1, then that is in the set. If the membership grade possessed by an element is 0, then that is not in the set. If the membership grade possessed by an element varies in between 0 and 1, then the element is said to belong to the set partially. Hence, a fuzzy set could be uniquely specified and identified with clear mentioning of its membership function.

Definition 2.1: Let L be a universe of discourse. A fuzzy set $\tilde{P}$ is defined by $\tilde{P}=\{l, \mu_{\tilde{P}}(l), l\in L\}$ and $\mu_{\tilde{P}}(l): L\to[0,1]$. Here $\mu_{\tilde{P}}(l)$ is called membership function.

Definition 2.2: Support of a fuzzy set $\tilde{P}$ is defined as $S(\tilde{P})=\{l\in L / \mu_{\tilde{P}}(l)>0\}$.

Definition 2.3: Core of a fuzzy set $\tilde{P}$ is defined as $C(\tilde{P})=\{l\in L / \mu_{\tilde{P}}(l)=1\}$.

Definition 2.4: Height of a fuzzy set $\tilde{P}$ is defined as $H(\tilde{P}) = \max(\mu_{\tilde{P}}(l))$.

Definition 2.5: A fuzzy set $\tilde{P}$ is said to be convex if $\mu_{\tilde{P}}(\lambda l_1 + (1-\lambda)l_2) \geq \min(\mu_{\tilde{P}}(l_1), \mu_{\tilde{P}}(l_2))$ for all $l_1,\ l_2 \in L$ and $\lambda \in [0,1]$.

Definition 2.6: α - cut of a fuzzy set $\tilde{P}$ is defined as a crisp set P^{α} (or a crisp interval) for a particular degree of membership α and mathematically stated as $P^{\alpha} = \{l \in L / \mu_{\tilde{P}}(l) \geq \alpha\}$ where $\alpha \in [0,1]$.

Definition 2.7: A fuzzy set $\tilde{P}$ is said to be normal if there exists a $l \in L$ satisfying $\mu_{\tilde{P}}(l) = 1$.

Definition 2.8: A Fuzzy set, which is both convex and normal, is called Fuzzy Number.

Definition 2.9: A fuzzy set $\tilde{P}$ is said to be Generalized Fuzzy Number if it is convex and need not be normal.

The most commonly used fuzzy numbers are trapezoidal fuzzy number (TrFN) and triangular fuzzy number (TFN) which are respectively defined as follows:

(i) A TrFN is denoted by an ordered quadruple as $\tilde{P} = (p,q,r,s)$ whose membership function $\mu_{\tilde{P}}(l)$ is described as

$$\mu_P(l) = \begin{cases} \dfrac{(l-p)}{(q-p)}, & p \leq l \leq q \\ 1, & \mathrm{q} \leq l \leq r \\ \dfrac{(s-l)}{(s-r)}, & r \leq l \leq s \\ 0, & otherwise \end{cases} \tag{1}$$

where $\mu_P(l)$ satisfies the following properties

1. $\mu_P(l) = 0$, outside the interval $[p,s]$
2. $\mu_P(l)$ is non-decreasing (monotonic increasing) on $[p,q]$ and non-increasing (monotonic decreasing) on $[r,s]$.
3. $\mu_P(l) = 1$ for each $l \in [r,s]$ where $p \leq q \leq r \leq s$ are real numbers.

If $q = r$ then the TrFN becomes TFN and is defined as follows.

(ii) A TFN is denoted by an ordered triple as $P = (p,q,r)$ whose membership function $\mu_P(l)$ is described as

$$\mu_P(l) = \begin{cases} \dfrac{(l-p)}{(q-p)}, & p \leq l \leq q \\ \dfrac{(r-l)}{(r-q)}, & q \leq l \leq r \\ 0, & otherwise \end{cases} \tag{2}$$

(iii) The generalized TrFN and TFN are represented by $P = (p,q,r,s;w)$ and $P = (p,q,r;w)$ respectively, where w indicates the height of the fuzzy number.

Definition 2.10: The image of the fuzzy number $P = \left(l, \mu_P(l)\right)$, denoted by "$-P$", is obtained by multiplying each element of support of the P by '-1' that is $-P = \left(-l, \mu_P(l)\right)$. If each '$l$' in P is negative then the fuzzy number is considered to be negative fuzzy number. The image (or opposite) of a fuzzy number $P = (p,q,r,s)$ can be given by a fuzzy number $-P = (-s,-r,-q,-r)$.

3. Defuzzification Based on Geometric Mean (Proposed Method)

It is carefully observed that some of the ranking methods have been proposed on the basis of either average or weighted average concepts. The geometric mean also could play a key role in distinguishing fuzzy numbers. In the cases, where ranking is done among generalised fuzzy numbers, their heights play a key role. Hence, on the basis of geometric mean and heights of fuzzy numbers, this method is proposed.

Let P be an arbitrary fuzzy number with membership function $\mu_P(l)$, support $S(P)$ and height $H(P)$ then the defuzzified value of P by geometric mean is defined to be

$$D_P = \exp\left[\frac{\int_{S(P)} \mu_P(l)\ln l dl}{\int_{S(P)} \mu_P(l) dl}\right] \bullet H(P) \forall l > 0 \tag{3}$$

Remark:

If $P = (p,q,r)$ be a Triangular fuzzy number then its defuzzified value is

$$D_P = \exp\left[\frac{\int_p^q \left(\frac{l-p}{q-p}\right)\ln l dl + \int_q^r \left(\frac{r-l}{r-s}\right)\ln l dl}{\int_p^q \left(\frac{l-p}{q-p}\right) dl + \int_q^r \left(\frac{r-l}{r-q}\right) dl}\right] \bullet H(P) \forall l > 0 \tag{4}$$

If $P = (p,q,r,s)$ be a Trapezoidal fuzzy number then its defuzzified value is

$$D_P = \exp\left[\frac{\int_p^q \left(\frac{l-p}{q-p}\right)\ln l dl + \int_q^r \ln l dl + \int_r^s \left(\frac{s-l}{s-r}\right)\ln l dl}{\int_p^q \left(\frac{l-p}{q-p}\right) dl + \int_q^r dl + \int_r^s \left(\frac{s-l}{s-r}\right) dl}\right] \bullet H(P) \forall l > 0 \tag{5}$$

3.1. Procedure for ranking of fuzzy numbers:

If $\tilde{P}$, $\tilde{Q}$ are two arbitrary non negative fuzzy numbers, then

Step 1: Compute defuzzified values D_P and D_Q of $\tilde{P}$, $\tilde{Q}$ respectively.

Step 2: (i) If $D_P < D_Q$ then $P \prec Q$

(ii) If $D_Q < D_P$ then $Q \prec P$

(iii) If $D_P = D_Q$ then $P \sim Q$ where '~' is fuzzy equality

If $\tilde{P}$ is a negative fuzzy number, then consider its image "$-\tilde{P}$" for defuzzification and the defuzzified value of $\tilde{P}$ is defined to be $D_P = -D_{-P}$.

Proposition: Let F be a set of fuzzy numbers. The relation $\preceq$, defined for any two fuzzy numbers $\tilde{P}$ and $\tilde{Q}$ in F such that $P \preceq Q$ if and only if $D_P \le D_Q$, is a partial order.

Proof:

Reflexive: Clearly for any fuzzy number $\tilde{P}$, $D_P = D_P$ hence, $P \preceq P$.

Anti-symmetry: Let $P \preceq Q$ and $Q \preceq P$ then $D_P \le D_Q$ and $D_Q \le D_P$ which implies $D_P = D_Q$ follows that $P \sim Q$.

Transitive: Let $P \preceq Q$ and $Q \preceq R$ then it can be observed that $D_P \le D_Q$ and $D_Q \le D_R$ which implies $D_P \le D_R$ follows that $P \preceq R$.

Hence the relation $\preceq$ is partial order.

The proposed method for ranking fuzzy numbers is explained through the following examples where in different types of fuzzy numbers and different cases of support and cores are taken.

Example 1: Let $P=(0.3,0.5,0.7)$ and $Q=(0.3,0.5,0.8,0.9)$ be two given fuzzy numbers. Clearly P is TFN and $\tilde{Q}$ is TrFN. Here P and Q are of different type. Their supports $S(P)=[0.3,0.7]$, $S(Q)=[0.3,0.9]$ and cores $C(P)=0.5$, $C(Q)=[0.5,0.8]$ are different but heights are same. Computed D_P, D_Q are $D_{\tilde{P}}=0.493154$ and $D_Q=0.606089$. Thus, $D_P<D_Q$ and hence, concluded that $P\prec Q$.

Example 2: Let $P=(0.4,0.5,1)$ and $Q=(0.4,0.7,1)$ be two given fuzzy numbers. Cleary, $\tilde{P}$ and $\tilde{Q}$ are of same type. They have same support and same height and different cores. Computed defuzzified values are D_P = 0.620204 and $D_{\tilde{Q}}$ = 0.688952. Thus, it is clear that $D_P<D_Q$ and this implies $P\prec Q$.

If given fuzzy numbers are generalized fuzzy numbers, then their heights play a key role in deciding their order when their support and cores are same, and it is explained in following example.

Example 3: Let $P=(0.1,0.2,0.3;\ 1)$ and $Q=(0.1,0.2,0.3;0.8)$ be two given fuzzy numbers. Their supports are respectively $S(P)=[0.1,\ 0.3]$ and $S(Q)=[0.1,\ 0.3]$, and their cores are $C(P)=0.2$ and $C(Q)=0.2$ respectively. Clearly the supports and cores of the fuzzy numbers P and $\tilde{Q}$ are same however their heights $H(P)=1$ and $H(Q)=0.8$ are different. It is observed that $D_P=0.1957$ and $D_Q=0.1565$ implies $D_Q<D_P$ and hence $Q\prec P$.

4. Numerical Comparison of Proposed Method with Other Methods

Now, the authors compare the proposed method with the important existing methods

Example 4: Consider the following 4 sets of examples from Yao & Wu [12] for numerical comparisons

Set 1: $P=(0.4,0.5,1)$, $Q=(0.4,0.7,1)$, $R=(0.4,0.9,1)$

Set 2: $P=(0.3,0.4,0.7,0.9)$, $Q=(0.3,0.7,0.9)$, $R=(0.5,0.7,0.9)$.

Set 3: $P=(0.3,0.5,0.7)$, $Q=(0.3,0.5,0.8,0.9)$, $R=(0.3,0.5,0.9)$.

Set 4: $P=(0,0.4,0.7,0.8)$, $Q=(0.2,0.5,0.9)$, $R=(0.1,0.6,0.8)$.

In Set 1, the fuzzy numbers have same support and different cores with same height 1. In Set 2, first two fuzzy numbers have same support and the third one has different support. Second and third fuzzy numbers have same core. In Set 3, second and third numbers have same support; first and third number have same core. In Set 4, all the three numbers have different supports and different cores. The shapes of these fuzzy numbers, set wise, are presented in Fig. 1.

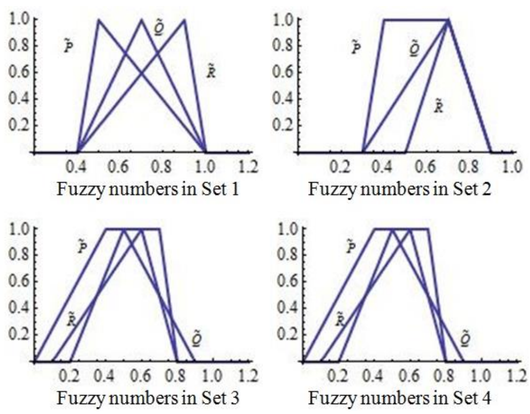

Fig.1. Graphical representation of Fuzzy numbers in Example 4

Using the proposed method, the computed values for the fuzzy numbers in Set1 are D_P = 0.620204, D_Q = 0.688952 and D_R = 0.754388. Thus, the rank order is $P \prec Q \prec R$. Similarly the proposed method is applied on the remaining fuzzy numbers in each sets 2, 3 and 4 and observed the following results.

Set 2: $D_{\tilde{P}} = 0.560915$, $D_{\tilde{Q}} = 0.619998$ and $D_{\tilde{R}} = 0.695175$. Hence the ordering is $P \prec Q \prec R$.

Set 3: $D_{\tilde{P}}$ = 0.493154, $D_{\tilde{Q}}$ = 0.606089 and $D_{\tilde{R}}$ = 0.552951. Hence, $P \prec R \prec Q$.

Set 4: $D_{\tilde{P}}$ = 0.416234, $D_{\tilde{Q}}$ = 0.513094 and $D_{\tilde{R}}$ = 0.473833. Thus, $P \prec Q \prec R$.

The ranking order of the fuzzy numbers for all the sets by proposed method and various methods is presented in Table 1.

Table 1. Comparative results in Example 4.

Methods	**Set 1**	**Set 2**	**Set 3**	**Set 4**
Proposed method	$P \prec Q \prec R$	$P \prec Q \prec R$	$P \prec R \prec Q$	$P \prec R \prec Q$
Weighted Distance method [20]	$P \prec Q \prec R$	$P \prec Q \prec R$	$P \prec R \prec Q$	$P \prec R \prec Q$
Sign Distance method with $p = 1$ [14]	$P \prec Q \prec R$	$P \prec Q \prec R$	$P \prec R \prec Q$	$P \prec Q \sim R$
Sign Distance method with $p = 2$ [14]	$P \prec Q \prec R$	$P \prec Q \prec R$	$P \prec R \prec Q$	$P \prec Q \sim R$
Distance Minimisation method [16]	$P \prec Q \prec R$	$P \prec Q \prec R$	$P \prec R \prec Q$	$P \prec Q \sim R$
Magnitude method [18]	$P \prec Q \prec R$	$P \prec Q \prec R$	$P \prec R \prec Q$	$Q \prec P \prec R$
Choobineh & Li method [9]	$P \prec Q \prec R$	$P \prec Q \prec R$	$P \prec Q \prec R$	$P \prec Q \prec R$
Yager method [7]	$P \prec Q \prec R$	$P \prec Q \prec R$	$P \prec Q \prec R$	$P \prec Q \prec R$
Chen method [8]	$P \prec Q \prec R$	$P \prec Q \prec R$	$P \prec Q \prec R$	$P \prec Q \prec R$
Baldwin & Guild method [31]	$P \prec Q \prec R$	$P \sim Q \prec R$	$P \prec Q \prec R$	$P \prec Q \sim R$
Chu & Tsao method [13]	$P \prec Q \prec R$	$P \prec Q \prec R$	$P \prec R \prec Q$	$P \prec R \prec Q$
Yao & Wu method [12]	$P \prec Q \prec R$	$P \prec Q \prec R$	$P \prec R \prec Q$	$P \prec Q \sim R$
Cheng's Distance method [11]	$P \prec Q \prec R$	$P \prec Q \prec R$	$P \prec R \prec Q$	$P \prec R \prec Q$
Cheng's CV uniform Distribution method [11]	$P \prec R \prec Q$	$P \prec Q \prec R$	$Q \prec R \prec P$	$P \prec R \prec Q$
Cheng's CV proportional Distribution method [11]	$P \prec R \prec Q$	$P \prec Q \prec R$	$Q \prec R \prec P$	$P \prec R \prec Q$

From Table 1, it is observed that, the ranking given by proposed method for set 1 is noticed to be same as it is provided by all the other methods except by Cheng [11] CV methods. The ranking order for set 2, except by Baldwin & Guild [31], but by all other methods is $P \prec Q \prec R$. This ranking order is observed to be in good agreement with the proposed method of this paper. In set 3, the ranking order by the proposed method is consistent with some authors and inconsistent with other authors, which is clearly seen from Table 1. Some methods showed drawbacks by failing to discriminate given fuzzy numbers of set 4 where as proposed method's ranking is remarked to agree with some authors and differ with other authors.

Example 5: Consider the following sets of fuzzy numbers from Chutia & Chutia [26] for sake of comparison of proposed method with the other methods numerically

Set 5: $P = (0.1, 0.3, 0.5)$, $Q = (0.3, 0.5, 0.7)$.

Set 6: $P = (0.1, 0.4, 0.5)$, $Q = (0.2, 0.3, 0.6)$.

Set 7: $P = (0.1, 0.3, 0.5)$, $Q = (0.2, 0.3, 0.4)$.

Set 8: $P = (0.1, 0.3, 0.5; 0.8)$, $Q = (0.1, 0.3, 0.5; 1)$.

Set 9: $P = (-0.5, -0.3, -0.1)$, $Q = (0.1, 0.3, 0.5)$.

Set 10: $P = (0, 0.4, 0.6, 0.8)$, $Q = (0.2, 0.5, 0.9)$, $R = (0.1, 0.6, 0.7, 0.8)$.

The fuzzy numbers in set 8 are generalized TFNs and the fuzzy number P in set 9 is negative TFN. The graphical representation of the each fuzzy number, in set wise, is shown in Fig. 2.

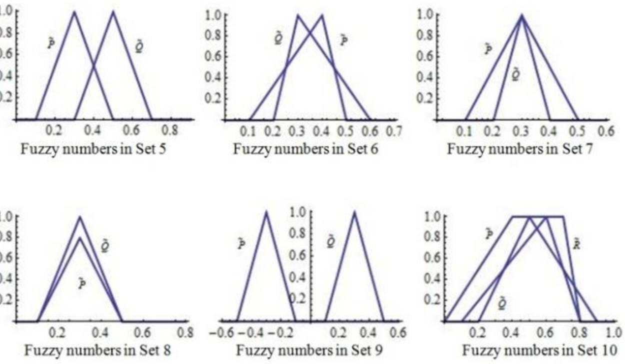

Fig.2. Graphical representation of Fuzzy numbers in Example 5

Using the proposed method, the following are computed.

Set 5: $D_{\tilde{P}}$ = 0.2879, $D_{\tilde{Q}}$ = 0.4931 implies that $P \prec Q$. Set 6: $D_{\tilde{P}}$ = 0.3207, $D_{\tilde{Q}}$ = 0.3570 implies that $P \prec Q$.

Set 7: $D_{\tilde{P}}$ =0.28795, $D_{\tilde{Q}}$ = 0.297171follows $P \prec Q$. Set 8: Clearly the supports of P are Q same, the heights are $H(P)$ = 0.8 and $H(Q)$ = 1

Thus $D_{\tilde{P}}$ =0.23036, $D_{\tilde{Q}}$ = 0.28795, hence obtained that $P \prec Q$.

Set 9: The fuzzy number P is negative TFN, thus the image of P taken for defuzzification is $-P = (0.1, 0.3, 0.5)$, Computed that $D_{-\tilde{P}}$ =0.2880 and by multiplying this with '-1', defuzzified value of P is obtained as $D_{\tilde{P}}$ = -0.2880 and $D_{\tilde{Q}}$ = 0.2880; hence $P \prec Q$.

Set 10: $D_{\tilde{P}}$ = 0.3953, $D_{\tilde{Q}}$ = 0.5131 and $D_{\tilde{R}}$ = 0.4974 and implies that $P \prec R \prec Q$.

The ranking orders of the fuzzy numbers from Set 1 to Set 6 by different methods (taken from the Table1 and 2 of Chutia & Chutia [26]) and the ranking order by proposed method are presented in Table 2.

Table 2. Comparative results of set 5 to set 10 in Example 5

Methods	Set 5	Set 6	Set 7	Set 8	Set 9	Set 10
Proposed Method	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{R} \prec \tilde{Q}$
Chu & Tsao method [13]	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Wang et al method [15]	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Chen & Sanguansat [21]	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Chen & Chen [17]	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Chen et al., [22]	$\tilde{P} \prec \tilde{Q}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Nesseri et al., [24]	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Rezvani [25]	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Yager [6]	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Shureshjani& Darehmiraki [32] $\alpha = 0.1,\ 0.5,\ 0.8$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Chutia & Chutia [26] $\alpha = 0.1,\ 0.5,\ 0.8$	$\tilde{P} \prec \tilde{Q}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$

It may be observed from the Table 2 that: The ranking order $P \prec Q$, for the set 5, obtained by the proposed method, is observed to be in coincidence with all the other methods. It is clear for the set 6 that the different fuzzy numbers could not be discriminated by the ordering methods given by Chu & Tsao [13]; Chen & Sanguansat [21]; Chen & Chen [17]; and Nesseri et al.,[24]. The proposed method's ranking, while coinciding with Wang et al., [15]; Rezvani [25]; and Yager [6] and is in disagreement with Chen et al. [22]; and Chutia & Chutia [26]. The different fuzzy numbers, in the set 7, being symmetric and sharing the same core, have been discriminated by the proposed method in agreement with ranking given by Chen & Chen [17]; and Nesseri et al.[24], though they could not be ranked by some other methods mentioned in Table 2. However, the Rezvani [25] and Chutia & Chutia [26] ranking is found to be in contrast with proposed method. For the set 8, a good consistent nature is observed between the ranking given by the proposed method and other methods, except Yager [6] which failed to rank the given numbers. It is to be understood with human intuition that the ranking result for the set 9 is $P \prec Q$ which is agreed by all other methods except three which failed to rank as mentioned in Table 2. Now for the last set 10, the ranking result $P \prec R \prec Q$, given by the proposed method, is observed to agree with the methods by Rezvani [25] and Yager [6] and disagree with other methods.

Example 6: Consider the following sets of fuzzy numbers from Chutia & Chutia [26] for numerical comparison of proposed method with other methods.

Set 11: $P = (0.3, 0.5, 1)$, $Q = (0.1, 0.6, 0.8)$.

Set 12: $P = (0.1, 0.2, 0.4, 0.5)$, $Q = (1,1,1,1)$.

Set13: $P = (0.1, 0.1, 0.1, 0.1; 0.8)$, $Q = (-0.1, -0.1, -0.1, -0.1; 1)$.

Set 14: $P = (0.3, 0.4, 0.6, 0.7)$, $Q = (0.4, 0.5, 0.6)$.

Set 15: $P = (1,1,1,1; 0.5)$, $Q = (1,1,1,1; 1.0)$.

Set 16: $P = (0.4, 0.5, 1)$, $Q = (0.4, 0.7, 1)$ $R = (0.4, 0.9, 1)$.

The fuzzy number $\tilde{Q}$ in Set 12 is a crisp valued fuzzy number. In Set 13 and 15, the fuzzy number $\tilde{P}$ is generalised fuzzy number and $\tilde{Q}$ is a crisp valued fuzzy number. The graphical representation of the each fuzzy number, in set wise, is shown in Fig. 3.

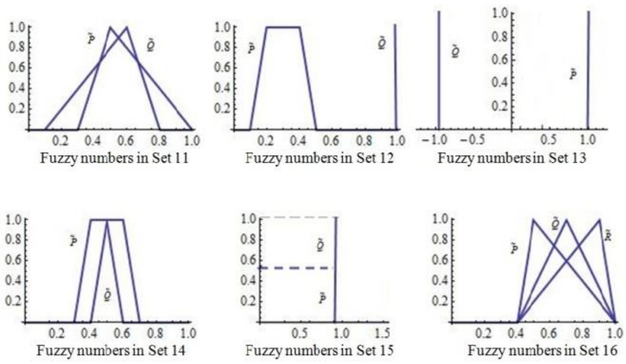

Fig.3. Graphical representation of Fuzzy numbers in Example 6

Using the proposed method, the following are computed

Set 11: $D_{\tilde{P}}$ =0.2880, $D_{\tilde{Q}}$ = 0.4738 and hence $P \prec Q$.

Set 12: $D_{\tilde{P}}$ = 0.2849, $D_{\tilde{Q}}$ = 1 that follows $P \prec Q$.

Set 13: $D_{\tilde{P}}$ =0.8, $D_{\tilde{Q}}$ = -1.0 implies that $Q \prec P$.

Set 14: $D_{\tilde{P}}$ =0.4914, $D_{\tilde{Q}}$ = 0.4983 and hence $P \prec Q$.

Set 15: $D_{\tilde{P}}$ =0.5, $D_{\tilde{Q}}$ = 1.0 which follows $P \prec Q$.

Set 16: $D_{\tilde{P}}$ =0.3207, $D_{\tilde{Q}}$ = 0.3791 and $D_{\tilde{R}}$ = 0.4352 hence got that $P \prec Q \prec R$.

The ranking orders of the fuzzy numbers in Set 11 to Set 16 by different methods (taken from the Table2 and 3 of Chutia & Chutia [26] and the ranking order by proposed method are presented in Table 3.

Table 3. Comparative results of set 11-16 in Example 6

Methods	Set 11	Set 12	Set 13	Set 14	Set 15	Set 16
Proposed Method	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Chu & Tsao [13]	$\tilde{Q} \prec \tilde{P}$	*	*	$\tilde{P} \sim \tilde{Q}$	*	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Wang et al [15]	$\tilde{Q} \prec \tilde{P}$	*	*	$\tilde{Q} \prec \tilde{P}$	*	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Chen & Sanguansat [21]	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Chen & Chen [17]	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Chen et al [22]	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Nesseri et al [24]	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Rezvani [25]	$\tilde{Q} \prec \tilde{P}$	*	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	*	$\tilde{P} \sim \tilde{Q} \sim \tilde{R}$
Yager [6]	$\tilde{Q} \prec \tilde{P}$	*	*	$\tilde{P} \sim \tilde{Q}$	*	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Shureshjani & Darehmiraki [32]						
$\alpha = 0.1$	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
$\alpha = 0.5$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
$\alpha = 0.8$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \sim \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
Chutia & Chutia [26]						
$\alpha = 0.1$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
$\alpha = 0.5$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$
$\alpha = 0.8$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{Q} \prec \tilde{P}$	$\tilde{P} \prec \tilde{Q}$	$\tilde{P} \prec \tilde{Q} \prec \tilde{R}$

*****Method fails to give ranking

From Table 3, the following observations have been made:The ranking result for set 11, given by the proposed method is identified to be in agreement with all the other methods except Shureshjani & Darehmiraki [32] for decision level $\alpha = 0.1$ mentioned in Table 3. Since the fuzzy numbers $\tilde{Q}$ being real, the ranking result $P \prec Q$ for the set 12 is to be understood with intuition. Though discrimination is not given by the methods of Chu & Tsao [13]; Wang et al [15]; Rezvani [25]; and Yager [6], a good agreement for the ranking result is observed among proposed and other methods.

For the fuzzy numbers of set 13, though ranking result $Q \prec P$ is clearly evident, an illogical ranking result is given by Rezvani [25]; and some methods by Chu & Tsao [13]; Wang et al. [15]; and Yager [6] showed difficulty in ordering those fuzzy numbers. However, there is good agreement in ranking order among proposed and other remaining methods. For set 14, a good number of mentioned methods posed a drawback in ordering them. In this context, proposed method ranked them and showed an agreement with two methods and a disagreement with another two methods which is seen from Table 3.

Having no vagueness in $\tilde{P}$, $\tilde{Q}$ for the set 15, as those are real numbers, with human intuition, the ranking is $P \prec Q$. Though this is clear, an illogical result was provided by Chen et al [22]. Some of the methods mentioned in the Table 3 showed a shortcoming as they could not provide ranking for these numbers. A good agreement is found in giving correct ranking $P \prec Q$ among proposed and other remaining methods in the Table 3. For the Set 16, the ranking order $P \prec Q \prec R$ computed by proposed method is completely consistent with all the other methods except by Rezvani [25].

Example 7: In the following fuzzy numbers, the fuzzy number $\tilde{P}$ is neither TFN nor TrFN whereas $\tilde{Q}$ is a TrFN. The membership function of $\tilde{P}$ is nonlinear and that of $\tilde{Q}$ is linear.

$$\mu_P = \begin{cases} 0.1\sqrt{l} & \text{when } l \in [0,9) \\ 0.7l^2\text{-}12.6l\text{+}57 & \text{when } l \in [9,10] \\ 1 & \text{when } l \in [10,11] \\ 12-l & \text{when } l \in (11,12] \\ 0 & \text{otherwise} \end{cases}$$

and

$$\mu_Q = \begin{cases} 0.1l & \text{when } l \in [0,10) \\ 1 & \text{when } l \in [10,11] \\ 12-l & \text{when } l \in (11,12] \\ 0 & \text{otherwise} \end{cases}$$

The graphical representation of these fuzzy numbers is given in the fig. 4.

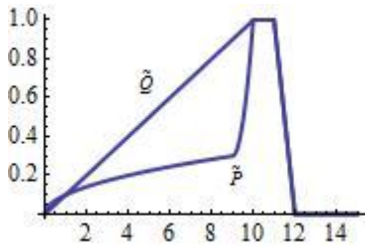

Fig.4. Graphical representation of Fuzzy numbers in Example 7.

The proposed method is applied on these numbers and obtained results are presented in the Table 4.

Table 4. Comparative results of example 7

		P	Q	Results
Proposed Method		7.12344	6.92425	$\tilde{Q} \prec \tilde{P}$
Allahviranloo et al. [20] (Weighted Distance method)		10.39	8.57	$\tilde{Q} \prec \tilde{P}$
Abbasbandy& Asady with [14] (Sign Distance Method)	$p = 1$	21.68	16.5	$\tilde{Q} \prec \tilde{P}$
	$p = 2$	16.13	12.87	$\tilde{Q} \prec \tilde{P}$
Cheng Distance [11]		7.51	7.62	$\tilde{P} \prec \tilde{Q}$
Chu and Tsao [13]		4.11	4.15	$\tilde{P} \prec \tilde{Q}$

It is noted from Table 4, represents ranking orders for example 8 by different authors, that the ranking result by the method, explained in this paper, complied with the result given by two authors and differed with two others.

5. Conclusions

In this paper, a new ranking procedure, based on geometric mean and heigh of a fuzzy number, is introduced and this method is tested with standard typical examples, A consistency in ranking different fuzzy numbers and their related images by this process is worth to be noted. Another key point is that this method is proved to be good enough in ranking negative fuzzy numbers also. Further, this method is observed to be effective in clearing some ambiguities presented by other works. This mehod may not give counter intuitive rankings. Moreover, having an easier and simpler procedure is an added credit to this method. Fuzzy numbers represented with non-linear membership function are also treated by this method efficiently. Comparative examples(or results), which are necessary for authentication of a method, are provided to show the good features of this ranking system.

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