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Comparison of Circular and Linear Orthogonal Polarization Bases in Electromagnetic Field Parameters Measurement

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Abstract: This article considers the peculiarities of using circular orthogonal polarization basis for measuring the parameters of an electromagnetic wave. In particular, the angle of inclination of the major axis of the polarization ellipse and the ellipticity coefficient are among measuring parameters. The main expressions for calculation of field parameters in circular and linear orthogonal polarization basis are developed and analyzed. The advantages of using the ring as a measuring antenna in comparison with symmetrical vibrators of the turnstile antenna are substantiated. The expressions obtained in the article for calculating the measurement errors of polarization parameters in a linear orthogonal polarization basis illustrate the multifactorial dependence of the measurement accuracy on the angular and amplitude parameters. In contrast to the linear polarization basis, in case of circular basis, the inclination angle of the polarization ellipse axis can be found by direct measurements of the phase shift, and the accuracy of measuring the ellipticity coefficient is affected only by the error of measuring the ratio of voltage amplitudes, which are proportional to the modules of the field strength vectors of the left and right directions of the circular polarization rotation. This provides better potential accuracy of measurement for the electromagnetic wave parameters when using circular polarization antennas and, correspondingly, more reasonable analysis in the circular orthogonal polarization basis.

Index Terms: Circular Orthogonal Polarization Basis, Linear Orthogonal Polarization Basis, Ellipticity Coefficient, Inclination Angle of Polarization Ellipse Major Axis**,** Measurement Errors of Polarization Characteristics.

1. Introduction

In many sophisticated applications of polarimetric radar [1], for example, for monitoring the electromagnetic environment [2], remote sensing of the atmosphere [3] and the earth [4], object recognition [5] and others [6], the problems appear in measuring parameters of the electromagnetic field with appropriate signal processing. The measurement accuracy of the electromagnetic field parameters by the antenna system is affected not only by the chosen measurement technique, but also by the structural elements that make up the system, including the elements of the antenna array. Therefore, their selection and accuracy of production are of great importance.

It is believed that in antenna arrays (AR) for radio monitoring systems it is advisable to use rather simple antenna elements [7]. Depending on the frequency range, in which the measuring antenna system operates, these elements can be made either in wired or printed form. Elements of the array can be more complex types of antennas. For example, log-periodic antenna [8] can be used as an element with linear polarization, which allows to significantly expand the operating frequency range. For arrays with circular polarization, it is possible to use helical or quadrifilar antennas [9], which will increase the accuracy of determining the polarization parameters of the field under study.

In the ranges of decimeter and longer waves, it is desirable to reduce the dimensions of the passive vibrator when using it as an element of the AA. To reduce the dimensions of the linear half-wave vibrator, different shoulder designs are used: shoulders in the form of a meander, shoulders with a load in the form of spirals, etc. But at the same time, other characteristics of the AA elements do not change for the better. For example, the polarization properties (characteristics) of such vibrators become more complicated. In the microwave range, antenna dimensions also remain very important, especially in today's miniaturized applications. In this aspect, ring elements have certain advantages over linear symmetrical or asymmetrical vibrators.

Despite the presence of a developed mathematical apparatus of technical electrodynamics theory, a comparative analysis of the use of circular and linear orthogonal polarization bases in these problems of measuring the parameters of electromagnetic field has not yet been sufficiently performed. In this paper, the authors tried to fill this gap and, on this basis, justify the expediency of using ring elements.

2. Literature Review and Problem Statement

The literature describes a fairly large number of different implementations of individual ring antenna elements or AA with ring elements. The article [10] considers a simple slotted microstrip antenna with a capacitive load for the implementation of radiation with circular polarization. Articles [11, 12] present a slotted ring array that can switch between the S- and C-bands or between the S, C and X bands, respectively. The C-band mode uses a diagonal feed method to excite a 2×2 array. This method can provide vertical or horizontal polarization with the highest level of decoupling. Article [13] presents a dual-band annular slot antenna array for L and C bands. The study [14] presents the design of a broadband single-layer ring antenna with a low level of cross-polarization.

Ring elements can be combined into the AA to solve various types of problems, including the problem of measuring the parameters of an electromagnetic field. The principles of constructing the AA are widely reflected in the literature [15-18].

When measuring the parameters of the electromagnetic field, linear polarization antennas are most often used. This method of studying the field is used even in the case when, in the further analysis of the measurement results, it is necessary to proceed to the representation of electromagnetic waves in circular orthogonal polarization basis.

Meanwhile, there are antennas that, directly due to the peculiarities of their design, divide a wave of any polarization into two waves in circular orthogonal polarization basis. Such antennas include the ring, the length of which is approximately equal to the wavelength of the field under study.

The use of the ring as measuring antenna or as an element of measuring antenna array has some advantages over symmetrical vibrators of turnstile antenna. The main ones are:

1. The ring with small transverse dimension has wider operating frequency range than the thin vibrator.
2. The ring has smaller dimensions than the turnstile antenna of half-wave vibrators at the same operating frequencies.
3. The results of measurements of the field parameters in circular orthogonal polarization basis, in comparison with the results obtained by linear polarization antennas, require simpler transformations when calculating the field parameters.
4. The accuracy of the measured field parameters, as follows from the previous paragraph, according to the circular polarization antennas can be significantly higher than the results obtained from the data of linear polarization antennas. It should also be taken into account that when processing data obtained by linear polarization antennas, functional dependencies are used, which are very sensitive (sometimes critically sensitive) to the errors of individual quantities, the values of which are used to calculate the field parameters.

Despite these advantages of ring antennas, their use is quite limited. This is due to several reasons. One of the most important reasons is the difficulty of providing the necessary power to the ring antenna or power extraction while maintaining a uniform amplitude current distribution along the ring (traveling wave mode). This is the basic prerequisite for axial emission or reception of electromagnetic waves with a high level of elliptic coefficient (axial ratio). Thus, there is a need to ensure strict symmetry of the uniform amplitude distribution of the current relative to the center of the ring with a simultaneous linear phase shift of the current along the conductor of the ring element.

All these points characterizing the advantages of the circular orthogonal polarization basis and the circular polarization ring antenna can be proved with a certain level of rigor using the analytical representation of an arbitrarily polarized electromagnetic wave in circular and linear polarization orthogonal bases.

3. Theoretical Foundations

Consider the wave surface (wave front) and find analytical expressions that describe the dependence of the electric field intensity vector on time. Since the electric field intensity vector E is on the plane of the wave front, its value for any moment of time can be described either in polar or in rectangular coordinate systems. The origin of the coordinate system (the pole of the polar coordinate system or the origin of the linear coordinate system) is placed at the point of intersection of the wave propagation direction with the wave front plane.

It is obvious that in the polar coordinate system it is expedient to consider the wave expanded in circular orthogonal polarization basis. In the general case, electromagnetic waves in circular orthogonal polarization basis consist of two circularly polarized waves in which the electric field intensity vectors do not change their amplitudes, but rotate with an angular velocity ω (angular frequency) around the pole in opposite directions. Analytically, this is written as follows

$$\left.\begin{aligned} e_R(t) &= (E_R, \omega t); \\ e_L(t) &= (E_L, -\omega t + \varphi_0), \end{aligned}\right\},$$

where $e(t)$ is instantaneous position of the electric field intensity vector on the coordinate plane at the moment of time t; E is amplitude value of electric field intensity; R and L are indices that determine whether certain values belong to the wave of the right or left rotation direction of the vector E respectively; φ_0 is initial (at t = 0) position of the vector $e_L(t=0)$. In this case, it is assumed that the vector $e_R(t=0)$ coincides with the polar axis.

Using an exponential function of an imaginary argument, which in the polar coordinate system is a moving point with the speed ω along a circle with unit radius, two coordinate dependences (radial coordinate E and polar angle $\Phi_R = \omega t$ or $\Phi_L = -\omega t + \varphi_0$) are combined in one formula:

$$\left.\begin{aligned} e_R(t) &= E_R e^{i\omega t}; \\ e_L(t) &= E_L e^{-i(\omega t - \varphi_0)}. \end{aligned}\right\} \tag{1}$$

The image of the above functions is shown in Fig. 1. As can be seen from Fig. 1, the polar axis OM and the major axis of the ellipse AB generally diverge at a certain angle depending on the angles Φ_R and Φ_L. If the polar axis is shifted so that it coincides with the major axis of the ellipse, then the initial angles are zero (Fig. 2). The value of the angle γ, by which it is necessary to shift the polar axis, is found as follows.

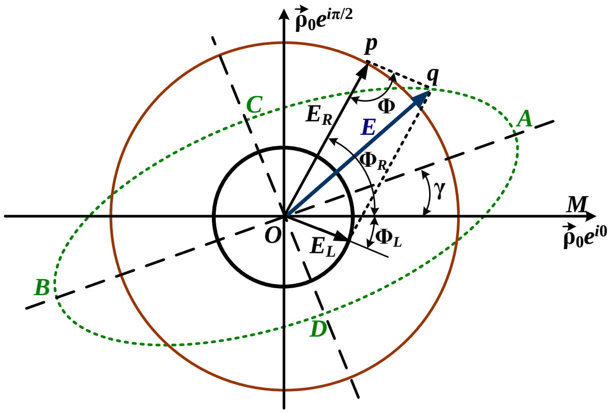

Fig. 1. Graphical representation of functions according to formula (2).

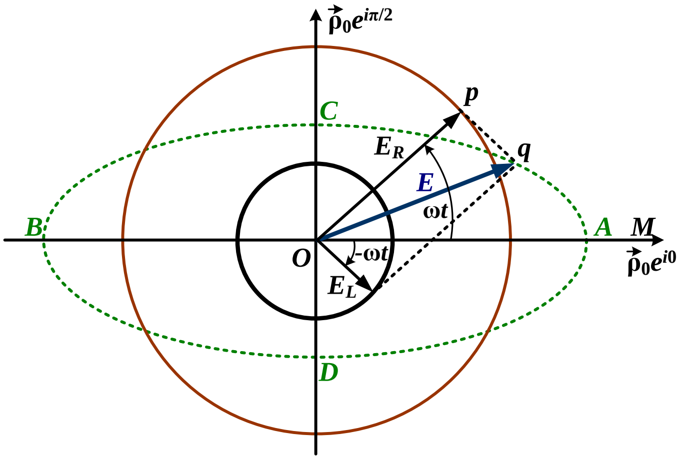

Fig. 2. Graphic representation of functions according to formula (2) when the polar axis *OM* is aligned with the major axis of the ellipse *AB*.

Let's change the moment of reference ($t = 0$) by the time interval τ so that the angles $\Phi_R = \omega t$ and $\Phi_L = -\omega t + \varphi_0$ (Fig. 1) are the same in their values (modules). That is, the dependence of the polar angles on time is equal to

$$\left.\begin{aligned}\Phi'_R &= \omega(t-\tau) = \omega t - \omega\tau = \omega t - \gamma;\\ \Phi'_L &= -\omega(t-\tau) + \varphi_0 = -\omega t + \varphi_0 + \omega\tau = -\omega t + \gamma.\end{aligned}\right\} \qquad (2)$$

Therefore, from equations (2) it is found that

$$\gamma = \frac{\varphi_0}{2} \ \text{i} \ \omega\tau = \gamma = \frac{\varphi_0}{2}. \qquad (3)$$

If the polar axis is shifted by an angle γ, then the phase factors in expressions (1) in the new polar coordinate system and the new initial time reference take the following form:

$$\left.\begin{aligned}e_R(t) &= E_R e^{i\omega t};\\ e_L(t) &= E_L e^{-i\omega t}.\end{aligned}\right\} \qquad (4)$$

In the previous coordinate system (the general case in Fig. 1), the instantaneous value of the amplitude of the field intensity vector is determined from the triangle *Opq*. As follows from the construction of the triangle, the angle at the vertex *p* is equal to $\Phi = \pi - (\Phi_R - \Phi_L) = \pi - (2\omega t - \varphi_0)$.

Therefore,

$$E = \sqrt{E_R^2 + E_L^2 + 2E_R E_L \cos(2\omega t - \varphi_0)}. \qquad (5)$$

In the new polar coordinate system (Fig. 2):

$$E = \sqrt{E_R^2 + E_L^2 + 2E_R E_L \cos(2\omega t)}. \qquad (6)$$

In this polar system (Fig. 2), it is not difficult to describe in canonical form a closed curve along which the end of the field intensity vector with amplitude (6) moves. It is even easier to do this in a rectangular coordinate system, in which the axis 0*x* coincides with the polar axis, and the axis 0*y* coincides with the perpendicular to the polar axis.

Then from expressions (4), using the connection of the exponential function with trigonometric functions and taking into account that the transition from the polar system to the rectangular coordinate system occurs due to the fixation of the polar angle in two planes ($\vec{\rho}_0 e^{i0} = \vec{x}_0$ and $\vec{\rho}_0 e^{i\pi/2} = i\vec{\rho}_0 = \vec{y}_0$), it is possible to obtain for any moment of time the coordinates of the point, where the end of the vector *E* is located, in the next form

$$\left.\begin{aligned} e_x(t) &= E_R\cos\omega t + E_L\cos\omega t; \\ e_y(t) &= E_R\sin\omega t - E_L\sin\omega t. \end{aligned}\right\} \tag{7}$$

Considering the left side of expressions (7) as the coordinate x and y at an arbitrary point on the curve, one can make sure that the end of the field intensity vector moves along an ellipse-shaped trajectory, the major axis of which is $a = E_R + E_L$, and the minor axis is $b = E_R - E_L$. That is, the coordinates x and y satisfy the canonical equation of an ellipse in a rectangular coordinate system:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\,.$$

In general, the axes of the ellipse do not coincide with the polar axis of the coordinate system. Therefore, the polarization ellipse is characterized by the inclination angle γ of the major axis to the polar axis of the polar coordinate system, or by the same inclination angle γ to one of the axes of the rectangular coordinate system. In circular orthogonal polarization basis, this angle is determined by formula (3).

The second important parameter of electromagnetic wave polarization is the ellipticity coefficient. In circular orthogonal basis, it is calculated very simply, namely

$$K_e = \frac{E_R - E_L}{E_R + E_L}\,. \tag{8}$$

that is, the ratio of the minor half-axis of the ellipse b to the major half-axis a.

To expand the field intensity vector, the amplitude of which in the polar basis is determined by formula (5), in the linear orthogonal basis, formula (1) is used. More often, the radiation field is considered in a spherical coordinate system.

The wave front coincides with the surface r = const and the rectangular coordinate system on this surface is characterized by the variables θ and φ. The orts of this system are $\vec{\theta}_0 = \vec{\rho}_0 e^{i0}$ and $\vec{\varphi}_0 = \vec{\rho}_0 e^{i\pi/2}$. That is, the axis 0θ of the rectangular coordinate system coincides with the polar axis, and the axis 0φ is perpendicular to the polar axis. Then the projections of the quantities $e_R(t)$ and $e_L(t)$ that rotate around the pole O on the axis of the rectangular system are

$$\left.\begin{aligned} e_\theta(t) &= E_R\cos\omega t + E_L\cos(\omega t - \varphi_0); \\ e_\varphi(t) &= E_R\sin\omega t - E_L\sin(\omega t - \varphi_0). \end{aligned}\right\} \tag{9}$$

Therefore, the circle with unit radius in the polar coordinate system is described in the rectangular coordinate system by two trigonometric functions with the same time dependence as the position of the radius of the unit circle in the polar coordinate system.

Using simple trigonometric transformations, the following relations are obtained:

$$\left.\begin{aligned} e_\theta(t) &= E_\theta\cos(\omega t - \alpha); \\ e_\varphi(t) &= E_\varphi\cos(\omega t - \beta), \end{aligned}\right\} \tag{10}$$

where $E_\theta = \sqrt{E_R^2 + E_L^2 + 2E_R E_L\cos\varphi_0}$; $E_\varphi = \sqrt{E_R^2 + E_L^2 - 2E_R E_L\cos\varphi_0}$; $\cos\alpha = (E_R + E_L\cos\varphi_0)/E_\theta$; $\sin\alpha = E_L\sin\varphi_0/E_\theta$; $\cos\beta = E_L\sin\varphi_0/E_\varphi$; $\sin\beta = (E_R - E_L\cos\varphi_0)/E_\varphi$.

Most often, two components of the vector E in the rectangular coordinate system are written as follows

$$\left.\begin{aligned} e_\theta(t) &= E_\theta\cos\omega t; \\ e_\varphi(t) &= E_\varphi\cos(\omega t + \psi), \end{aligned}\right\} \tag{11}$$

where ψ is phase shift of the wave $e_\varphi(t)$ relative to the wave $e_\theta(t)$.

Comparing equations (10) and (11), one can see that oscillations (10) are identical to oscillations (11) if the time reference is shifted by the interval $\tau = \alpha/\omega$. With this change in the beginning of time, the following equation is obtained

$$\psi = -(\beta - \alpha) = \alpha - \beta \quad \text{or} \quad \psi = \arccos\alpha - \arccos\beta . \tag{12}$$

As an alternative to the formula (12), one can also consider the connection between the trigonometric functions of the angle ψ and the trigonometric functions α and β, which are defined by equations (10), that is

$$\cos\psi = \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta;$$
$$\sin\psi = \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta.$$

Substituting the values of cosines and sines, fairly simple relations are obtained:

$$\left.\begin{aligned} \cos\psi &= \frac{2E_R E_L \sin\varphi_0}{E_\theta E_\varphi}; \\ \sin\psi &= -\frac{E_R^2 - E_L^2}{E_\theta E_\varphi}, \end{aligned}\right\}$$

where $E_\theta E_\varphi = \sqrt{E_R^4 + E_L^4 - 2E_R^2 E_L^2 \cos 2\varphi_0}$.

The radiation field of the ring antenna, provided that the ring is flowed around by traveling current wave ($\dot{I}(\varphi) = I_A e^{-ik\alpha\varphi}$,where I_A is the amplitude of the supply current, $k = 2\pi/\lambda$ is the wave number, φ is the polar angle), has been studied in sufficient detail [19]. Therefore, it is possible to write the result of the radiation field analysis

$$\left.\begin{aligned} \dot{E}_\theta &= A_0 F_\theta(\theta); \\ \dot{E}_\varphi &= iA_0 F_\varphi(\theta), \end{aligned}\right\} \tag{13}$$

where $A_0 = 30\pi I_A/r$; $F_\theta(\theta) = 2\text{ctg}\theta J_1(\sin\theta)$ and $F_\varphi(\theta) = J_0(\sin\theta) - J_2(\sin\theta)$ are normalized directivity; $J_0(\sin\theta)$, $J_1(\sin\theta)$, $J_2(\sin\theta)$ are Bessel functions of the first kind of zero, first and second orders.

As follows from expressions (9), the radiation field in the analysis is considered in linear orthogonal polarization basis.

If the antenna is used to create the circular polarization field, or to measure the field parameters in circular orthogonal polarization basis, then the observation point or the source of the studied radiation must be located on the ring antenna axis. Then $F_\theta(\theta = 0) = F_\varphi(\theta = 0) = 1$ and expressions (13) take the form

$$\left.\begin{aligned} \dot{E}_\theta &= \frac{30\pi I_A}{r}; \\ \dot{E}_\varphi &= i\,\frac{30\pi I_A}{r}. \end{aligned}\right\} \tag{14}$$

Formulas (14) can be used to determine the effective length of the ring antenna. As is known, in the radiation mode of plane polarized electromagnetic wave, the effective length of the antenna is calculated as follows.

$$l_{eff} = \frac{rE_{\max}}{30kI_A}, \tag{15}$$

where $E_{\max}$ is the maximum value of the electric field intensity at a distance r from the emitter when it is supplied with current I_A .

Since the ring antenna simultaneously radiates two linearly polarized waves, the planes of polarization of which are mutually perpendicular, it is obvious that the effective length of the ring antenna can be represented as a vector quantity, namely

$$\mathbf{l}_{eff} = \vec{\theta}_0 l_{eff}^{\theta} + \vec{\varphi}_0 l_{eff}^{\varphi} .$$

The effective length (15) is introduced as an antenna parameter under the condition of the emitted (or received) wave of linear polarization. Formulas (14) just determine the maximum intensity of ring antenna radiation field in mutually perpendicular planes of the spherical coordinate system (θ = const and φ = const) of plane polarized waves. Substituting the value of E_θ and E_φ into formula (15), it is possible to get that

$$\mathbf{l}_{eff}=\frac{\lambda}{2}\left(\vec{\theta}_0+\vec{\varphi}_0\right). \tag{16}$$

At the output of the ring antenna in the receive mode, a voltage is generated, which is scalar value. That is, the voltages $\dot{U}_\theta$ and $\dot{U}_\varphi$ are combined into one output voltage, the value of which can be calculated as follows. Let us represent the field intensities (1) acting on the ring antenna in linear orthogonal polarization basis:

$$\left.\begin{aligned}\mathbf{e}_R(t)&=\vec{\theta}_0E_R\cos\omega t+\vec{\varphi}_0E_R\sin\omega t;\\ \mathbf{e}_L(t)&=\vec{\theta}_0E_L\cos(\omega t-\varphi_0)-\vec{\varphi}_0E_L\sin(\omega t-\varphi_0).\end{aligned}\right\} \tag{17}$$

The scalar product of the left and right parts of equations (16) and (17) determines the electromotive forces (EMF) at the outputs of the ring antenna, which are formed by devices for extracting electrical quantities, depending on the rotation direction of the electric field intensity vectors:

$$\left.\begin{aligned}e_R(t)&=\mathbf{e}_R(t)\mathbf{l}_{eff}=E_R\frac{\lambda}{2}(\cos\omega t+\sin\omega t);\\ e_L(t)&=\mathbf{e}_L(t)\mathbf{l}_{eff}=E_L\frac{\lambda}{2}(\cos(\omega t-\varphi_0)-\sin(\omega t-\varphi_0)).\end{aligned}\right\}$$

Summing up the trigonometric functions, the final result is

$$\left.\begin{aligned}e_R(t)&=\frac{\sqrt{2}}{2}\lambda E_R\cos\left(\omega t-\frac{\pi}{4}\right);\\ e_L(t)&=\frac{\sqrt{2}}{2}\lambda E_L\cos\left(\omega t+\frac{\pi}{4}-\varphi_0\right).\end{aligned}\right\} \tag{18}$$

From formula (18) it follows that the effective length of the ring antenna is equal to

$$l_{eff}=\frac{\sqrt{2}}{2}\lambda$$

and does not depend on the rotation direction of the electric field intensity vector.

Waves emitted by a turnstile antenna can have circular polarization. If the turnstile antenna is built on half-wave vibrators, then its effective length is equal to

$$\mathbf{l}_{eff}^{t.a}=\frac{\lambda}{\pi}\left(\vec{\theta}_0+\vec{\varphi}_0\right) \quad \text{or} \quad l_{eff}^{t.a}=\frac{\sqrt{2}}{\pi}\lambda .$$

The ratio of the effective length of the ring antenna to the effective length of the turnstile antenna is

$$l_{eff}/l_{eff}^{t.a}=\frac{\pi}{2}=1{,}57 ,$$

that is, the effective length of the ring antenna is greater than the effective length of the turnstile antenna.

The overall dimensions of the turnstile antenna are equal to $\lambda/2$ (the dimension of the symmetrical half-wave vibrator). The diameter of the annular axial radiation antenna is λ/π. Consequently, the overall dimensions of the ring antenna are one and a half times less than the dimensions of the turnstile antenna.

The output voltages of the ring antenna, formed at matched loads, are half that of the EMF (18), that is,

$$\left.\begin{aligned} u_R(t) &= \frac{1}{2\sqrt{2}}\lambda E_R \cos\left(\omega t - \frac{\pi}{4}\right); \\ u_L(t) &= \frac{1}{2\sqrt{2}}\lambda E_L \cos\left(\omega t + \frac{\pi}{4} - \varphi_0\right). \end{aligned}\right\} \tag{19}$$

Their amplitudes

$$\left.\begin{aligned} U_R &= \frac{1}{2\sqrt{2}}\lambda E_R; \\ U_L &= \frac{1}{2\sqrt{2}}\lambda E_L. \end{aligned}\right\}$$

make it possible to determine the ellipticity coefficient:

$$K_e = \frac{U_R - U_L}{U_R + U_L}, \tag{20}$$

which completely coincides with expression (8).

The phase shift of the voltages (19), which is quite easy to measure, is equal to

$$\varphi_\Delta = \arg[U_R(t)] - \arg[U_L(t)] = \varphi_0 - \frac{\pi}{2}.$$

Therefore, the inclination angle of the major axis of the polarization ellipse to the polar axis, or in the rectangular coordinate system to the axis 0θ, according to expression (3), is calculated as

$$\gamma = \frac{\varphi_\Delta + 0{,}5\pi}{2}. \tag{21}$$

For comparison, the formulas are given for calculating K_e and γ when using the turnstile antenna. From expressions (11), which describe the field of arbitrarily polarized electromagnetic waves in linear orthogonal polarization basis, the voltages on the matched loads of symmetrical vibrators are found:

$$\left.\begin{aligned} u_\theta(t) &= U_\theta \cos\omega t; \\ u_\varphi(t) &= U_\varphi \cos(\omega t + \psi), \end{aligned}\right\}$$

where $U_\theta = \dfrac{\lambda}{2\pi}E_\theta$ and $U_\varphi = \dfrac{\lambda}{2\pi}E_\varphi$.

It is obvious that the voltages U_θ and U_φ the phase shift ψ are measured. The inclination angle of the polarization ellipse is defined as [19]

$$\mathrm{tg}2\gamma = \frac{2m\cos\psi}{m^2 - 1}, \tag{22}$$

and the coefficient of ellipticity as [19]

$$K_e = \pm\sqrt{\frac{m\sin^2\gamma - \sin 2\gamma\cos\psi + (1/m)\cos^2\gamma}{m\cos^2\gamma + \sin 2\gamma\cos\psi + (1/m)\sin^2\gamma}}, \tag{23}$$

where $m = E_\theta/E_\varphi = U_\theta/U_\varphi$; the choice of the sign "plus" or "minus" is determined by the rotation direction of the vector E, or by the sign of the phase shift ψ.

4. Calculation of Measurement Errors of Polarization Characteristics

From the above formulas (21) and (22), (20) and (23) it can be seen that the calculation of the inclination angle γ of the polarization ellipse in linear orthogonal polarization basis is not much more complicated than in circular basis. But the ellipticity coefficient (23) has a much more complex dependence on the measured quantities than in formula (20).

Significant differences between formulas (20) and (23) are observed if the accuracy of calculating the polarization parameters is considered. In circular orthogonal polarization basis, the error of the inclination angle of the polarization ellipse is determined from formula (21) and is equal to

$$\Delta\gamma = \Delta\varphi_{\Delta}/2, \tag{24}$$

that is, it completely depends only on the accuracy of the devices for measuring the phase displacement φ_{Δ} of voltages $u_R(t)$ and $u_L(t)$.

In linear polarization basis from formula (22) it follows that the error in calculating the angle γ is defined as

$$\Delta\gamma = K_m^{\gamma}\Delta m + K_{\psi}^{\gamma}\Delta\psi, \tag{25}$$

where is the sensitivity coefficient of the angle value to the measurement error of the amplitude ratio U_{θ} and U_{φ} is

$$K_m^{\gamma} = \frac{(m^2+1)\cos\psi}{m^4 + 2m^2\cos 2\psi + 1}; \tag{26}$$

the sensitivity coefficient to the measurement error of voltage phase displacement $u_{\theta}(t)$ and $u_{\varphi}(t)$ is

$$K_{\psi}^{\gamma} = \frac{m(m^2-1)\sin\psi}{m^4 + 2m^2\cos 2\psi + 1}; \tag{27}$$

voltage ratio measurement error is

$$\Delta m = 2m\delta u, \tag{28}$$

where δu is relative error of voltage measurement; $\Delta\psi$ is phase shift measurement error.

The error in calculating the ellipticity coefficient in circular orthogonal polarization basis by formula (20) is equal to

$$\Delta K_e = \frac{4U_R U_L}{(U_R+U_L)^2}\delta U \quad \text{or} \quad \Delta K_e = \frac{4n}{(1+n)^2}\delta U, \tag{29}$$

where $n = U_L/U_R$ is the ratio of voltage amplitudes proportional to the modules of the field intensity vectors of the left and right rotation directions; δU is the relative error of voltage measurement.

The dependence of the error in calculating the ellipticity coefficient in circular orthogonal polarization basis on the ratio of voltages amplitudes $n = U_L/U_R$ at the fixed value of the relative error in measuring voltage δU is shown graphically in Fig. 3.

As can be seen from the graphs, the maximum absolute value of the error (29) is observed at $n = 1$, or at $K_e = 0$. That is, when the polarization coefficient of linearly polarized wave is measured. In this case, the error is equal to the error ΔK_e in measuring the electric field intensity.

From expression (23) the calculation error of the ellipticity coefficient in linear polarization basis is obtained:

$$\Delta K_e = K_m^{e}\Delta m + K_{\gamma}^{e}\Delta\gamma + K_{\psi}^{e}\Delta\psi. \tag{30}$$

In formula (30), the sensitivity coefficients are determined as follows:
sensitivity to error in measuring the voltage ratio Δm, which is calculated by formula (28), is:

$$K_m^e = \frac{K_e}{2Q}\left[\left(1-\frac{1}{m^2}\right)\sin 2\gamma \cos\psi - \frac{2}{m}\cos 2\gamma\right]; \tag{31}$$

sensitivity to measurement errors of the angle γ is

$$K_\gamma^e = \frac{K_e}{2Q}\left(m+\frac{1}{m}\right)\left[\left(m-\frac{1}{m}\right)\sin 2\gamma - 2\cos 2\gamma \cos\psi\right]; \tag{32}$$

sensitivity to phase shift measurement errors is

$$K_\psi^e = \frac{K_e}{2Q}\left(m+\frac{1}{m}\right)\sin 2\gamma \sin\psi \, . \tag{33}$$

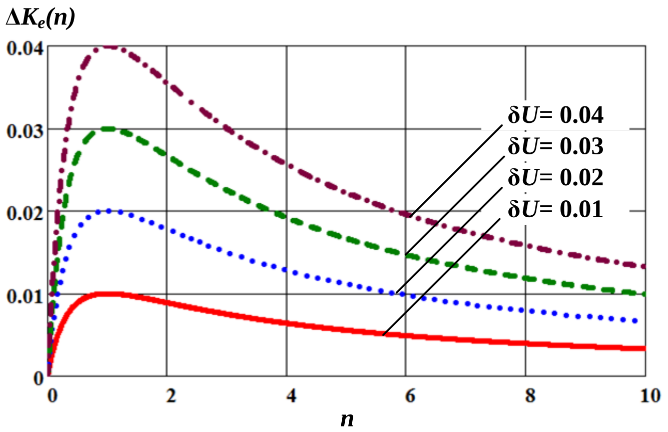

Fig. 3. Dependences of the error in calculating the ellipticity coefficient in circular orthogonal polarization basis on the ratio of voltage amplitudes $n = U_L / U_R$ at the fixed value of the relative error in measuring voltage (field intensity).

In expressions (31)- (33) the value of the coefficient Q is equal to

$$Q = 1 + \left(m-\frac{1}{m}\right)^2 \cos^2\gamma \sin^2\gamma - \left(m-\frac{1}{m}\right)\cos 2\gamma \sin 2\gamma \cos\psi - \sin^2 2\gamma \cos^2\psi \, .$$

Using expressions (23), (31)-(33), dependences of the sensitivity coefficients (Fig. 4-9) included in the error formula for calculating the ellipticity coefficient in linear polarization basis (30) were plotted on the amplitude (*m*) and angular (γ, ψ) parameters.

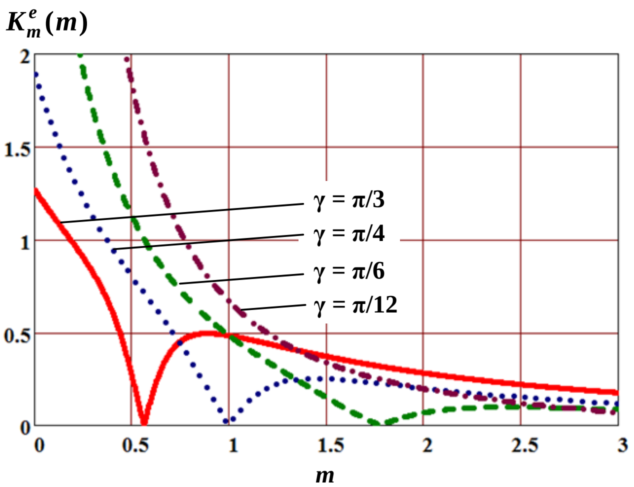

Fig. 4. Dependence of the sensitivity coefficient to the error of the voltage ratio Δm on the ratio of the voltage amplitudes $m = U_\theta / U_\varphi$ at fixed values of the inclination angle γ of the polarization ellipse and the constant value of the phase shift $\psi = \pi/10$.

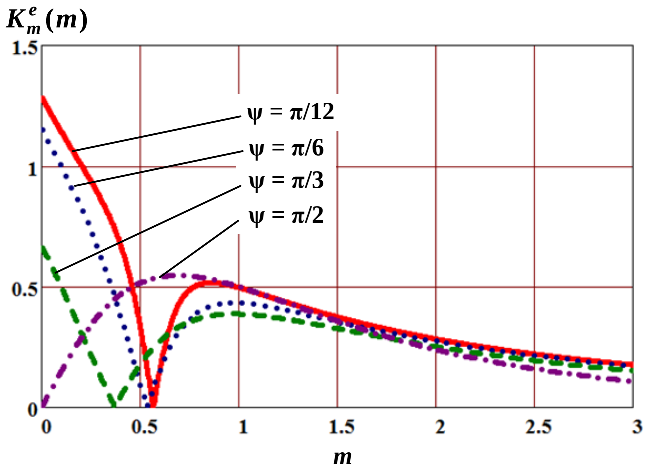

Fig. 5. Dependence of the sensitivity coefficient to the error of the voltage ratio Δm on the ratio of the voltage amplitudes $m = U_\theta / U_\varphi$ at fixed values of the phase shift ψ and the constant value of the inclination angle γ = π/3.

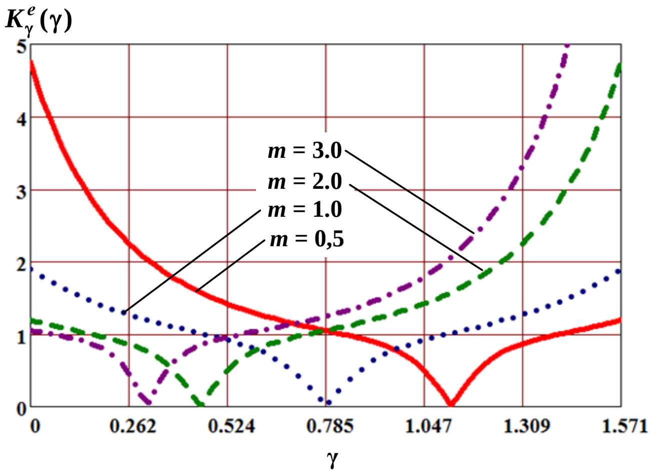

Fig. 6. Dependence of the sensitivity coefficient to the error of the inclination angle of the polarization ellipse Δγ on the inclination angle γ of the ellipse at fixed values of the voltage amplitudes ratio m and the constant value of the phase shift ψ = π / 10.

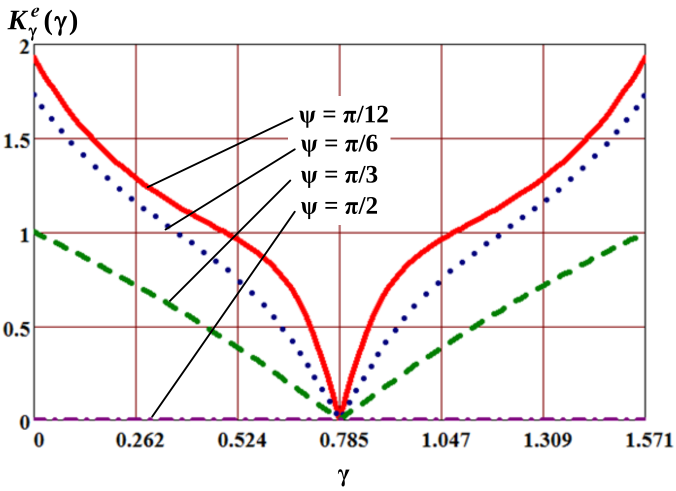

Fig. 7. Dependence of the sensitivity coefficient to the error of the inclination angle of the polarization ellipse Δγ on the inclination angle γ of the ellipse at fixed values of the phase shift ψ and the constant value of the voltage amplitudes ratio $m = 1$.

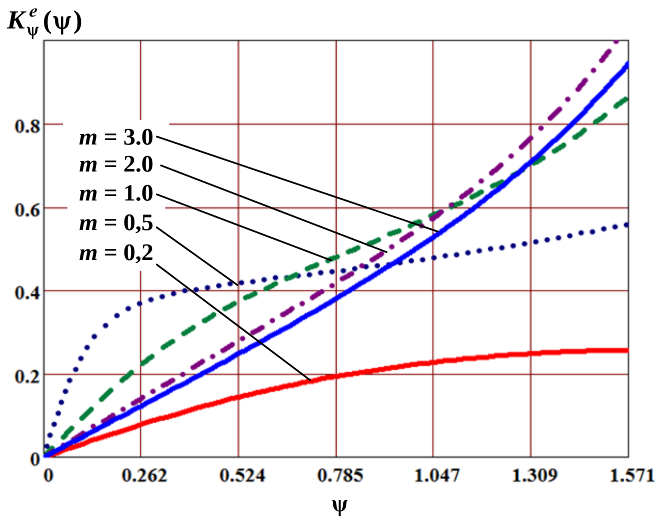

Fig. 8. Dependence of the sensitivity coefficient to the error of the phase shift Δψ on the phase shift ψ at fixed values of the voltage amplitudes ratio *m* and the constant value of the inclination angle of the polarization ellipse γ = π / 3.

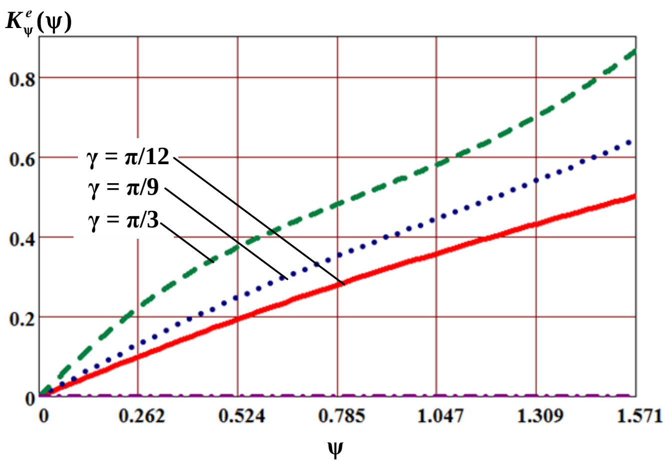

Fig. 9. Dependence of the sensitivity coefficient to the error of the phase shift Δψ on the phase shift ψ at fixed values of the inclination angle of the polarization ellipse γ and the constant value of the voltage amplitudes ratio *m* = 1

From Fig. 4-9 it can be seen that the dependences of the sensitivity coefficients K_m^e and K_γ^e have singular points. These are zero values, that is, at these points it is impossible to calculate the individual components of the measurement error of the ellipticity coefficient. As for the sensitivity coefficient to the error in determining the phase shift (Fig. 8 and 9), this feature should be noted. The sensitivity coefficient K_ψ^e within the range of angle change from 0 to 90 °is always less than unity, which reduces the influence of the error Δψ on the total error in the calculation of the ellipticity coefficient.

The obtained expressions for calculating errors (25) and (30) illustrate the multifactor dependence of the accuracy of indirect measurements of the polarization parameters in linear orthogonal polarization basis. At the same time, from expressions (24) and (29) it follows that in circular orthogonal polarization basis, the inclination angle γ is found as a result of direct measurement of the phase shift. And the accuracy of measuring the ellipticity coefficient is affected by only one error source, namely, the measurement error of the voltages ratio. In this case, the sensitivity coefficient is in a rather narrow range of values (from zero to one).

Direct comparison of the errors in measuring the ellipticity coefficient in different polarization bases can be carried out by reducing the number of independent parameters on the right side of equation (23). For this, the axes of the polarization ellipse are compatible with the axes of the rectangular coordinate system (Fig. 2). In this case, γ = 0. The expression for calculating the ellipticity coefficient in the linear orthogonal polarization basis (23) takes the next form

$$K_e = \pm 1/m\,.$$

Expressions for sensitivity coefficients (31), (32) and (33) are significantly simplified:

$$\left.\begin{aligned} K_m^e &= \frac{1}{m^2}; \\ K_\gamma^e &= \frac{1}{m}(m + 1/m)\cos\psi; \\ K_\psi^e &= 0. \end{aligned}\right\}$$

Therefore, the absolute error of ellipticity measurement according to expression (30) is equal to

$$\Delta K_e = K_m^e \Delta m + K_\gamma^e \Delta\gamma \,. \tag{34}$$

It is obvious that Δγ is the inaccuracy of alignment of the ellipse axes with the axes of the coordinate system. Using the error values Δ*m* and Δγ of expressions (25), (26), (27) and (28), formula (34) can be represented as a function of two parameters.

$$\Delta K_e = \left[\frac{1}{m} + \frac{(m^2+1)\cos\psi}{m^4 + 2m^2\cos 2\psi + 1}\right] 2m\delta u + \frac{m(m^2+1)}{m^4 + 2m^2\cos 2\psi + 1}\Delta\psi. \ \Delta K_e = K_m^e \Delta m + K_\gamma^e \Delta\gamma \,.$$

By fixing the values of ψ, δ*u* and Δψ, it is possible to construct graphs families (Fig. 10).

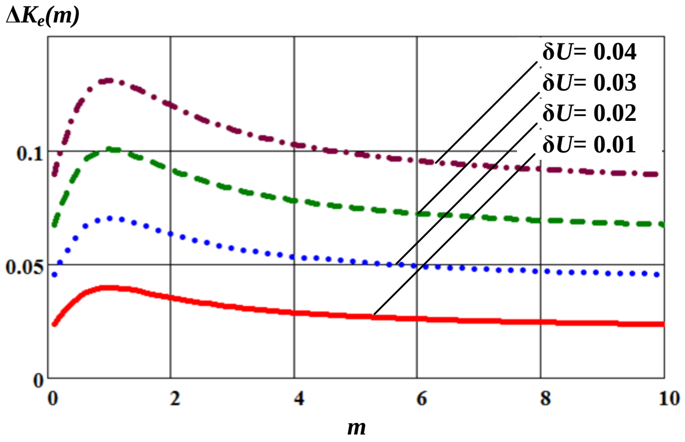

Fig. 10. Dependence of the error in calculating the ellipticity coefficient in linear orthogonal polarization basis on the voltage amplitudes ratio $m = U_\theta / U_\varphi$ at the fixed values of the relative error of voltage measurement

The graphs in Fig. 10 is convenient to compare with the graphs of the error in calculating the ellipticity coefficient in circular orthogonal polarization basis, shown in Fig. 3. As can be seen from this approximate comparison, the measurement errors of the ellipticity coefficient are an order of magnitude greater when using linear orthogonal polarization basis.

Therefore, in a theoretical sense, the advantages of circular polarization antenna in measuring the field polarization parameters over linear polarization antennas are quite significant. But in the practical implementation of these advantages, well-known problems arise, which consist in creating means for isolating electrical processes associated with the rotation directions of the electric field intensity vectors.

For example, a helical antenna creates a field with circular polarization of one direction. Therefore, to measure the polarization parameters, it is necessary to take two helical antennas with different directions of helix rotation. But at the same time, the phase centers of the helices are separated. If the helices are implemented on one cylinder, the problem of mutual influences arises.

A simple ring antenna makes it possible to get rid of these two disadvantages [20]. Since the conductors of the ring are in the same plane, it equally receives or radiates waves with the right rotation direction and waves with the left rotation direction.

5. Conclusion and Future Work

The comparative analysis has shown the essential difference in case of using two popular polarization bases for measurement of polarization parameters.

The expressions that have been obtained in this work for calculating the measurement errors of polarization parameters in the linear orthogonal polarization basis illustrate a multifactor dependence. This is the dependence of the error in calculating the ellipticity coefficient on the sensitivity coefficients to the error in the inclination angle of the polarization ellipse, the error in the phase shift, the error in the voltage amplitudes ratio, which are proportional to the modules of the field intensity vectors of the left and right directions of rotation.

In case of the circular orthogonal polarization basis, the inclination angle can be found as a result of direct measurement of the phase shift, and only the error in measuring the voltage amplitudes ratio affects the accuracy of measuring the ellipticity coefficient.

A number of advantages when using the ring as measuring antenna, or as an element of measuring antenna array, were discovered and proved in comparison with the use of the turnstile antenna made of symmetrical vibrators.

One of the advantages is that the ring antenna with a small transverse dimension has wider operating frequency range compared to a thin vibrator and smaller overall dimensions. Also, the effective length of the ring antenna is 1.57 times greater than the effective length of the turnstile antenna.

The accuracy when measuring the polarization parameters of the field with circular polarization antennas is potentially higher than when measuring with linear polarization antennas. This is due to a simplified procedure for calculating these parameters in a circular polarization basis.

In future works, based on the developed here, it is planned to carry out mathematical modeling and simulation in more detail and investigate experimentally the process of measuring the parameters of an electromagnetic field using the mathematical apparatus of circular orthogonal polarization basis.

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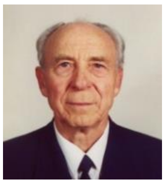

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