Control and Synchronization of Hyperchaotic System based on SDRE method

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Abstract—In this paper, stabilization and synchronization problems of the hyperchaotic system is investigated. For this reason, state dependent Riccati equation (SDRE) is used. First, stabilizer is designed by SDRE method. Then, robust controller is designed that it can stabilize hyperchaotic system with uncertainly. Finally, synchronization problem between two hyperchaotic systems is considered. The optimal controller is designed that it synchronizes two hyperchaotic systems. Numerical simulation results are presented to show the effectiveness of the proposed controllers.

Index Terms—Hyperchaotic system, state dependent Riccati equation (SDRE), optimal control, robust control, stabilization, synchronization.

I. INTRODUCTION

An interesting phenomenon of nonlinear systems is chaos. In recent years, studies of chaos and hyperchaos generation, control and synchronization have attracted. Therefore, various effective methods have been proposed one the past decades to achieve the control and synchronization of chaotic system, such as Robust Control [1], the sliding method control [2], linear and nonlinear feedback control [3], adaptive control [4], active control [5], backstepping control [6] and generalized backstepping method control [7-9], ect. The purpose of the present work lies in the design of a robust optimal control system for the control and synchronization of new hyperchaotic system using the state-dependent Riccati equation (SDRE) method. The State-Dependent Riccati Equation (SDRE) techniques are general design methods that control problems involving nonlinear systems [10-14]. For a nonlinear system a form of linearization is required which is not an approximation but simply a rewriting of the mathematical model in a different form. This form, which is not unique, is then possible to obtain feedback control laws.

The rest of the paper is organized as follows: In section 2, a new hyperchaotic system is described. In section 3, stability conditions in new hyperchaotic system are derived by SDRE method. In section 4, the stability conditions in new hyperchaotic system with uncertainly are derived by robust optimal method. In section 5, synchronization between two new hyperchaotic systems are achieved by SDRE method. Finally, section 6 is provided conclusion of this work.

II. HYPERCHAOTIC SYSTEM

Recently, Dadras and Momeni proposed the new hyperchaotic system [15]. The system is described by:

\[
\begin{align*}
\dot{x}_1 &= ax_1 - x_2x_3 \\
\dot{x}_2 &= x_1x_3 - bx_2 \\
\dot{x}_3 &= cx_1x_2 - dx_3 + gx_1x_4 \\
\dot{x}_4 &= fx_4 - hx_2
\end{align*}
\]  (1)

Here \(x, y, z, w\) are the state variables and \(a, b, c, d, f, g, h\) are the positive constant parameters. System (1) is hyperchaotic when \(a = 8, b = 40, c = 2, d = 14, f = 0.05, g = 5, h = 0.2\). The corresponding phase portraits are depicted in Fig 1 and the state trajectory of the system (1) is displayed in Fig 2.

Fig. 1. phase portraits of the hyperchaotic (7).
Now, system (1) is described by state dependent Riccati equation (SDRE).

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix} =
\begin{bmatrix}
a & -x_3 & 0 & 0 \\
x_3 & -b & 0 & 0 \\
cx_2 & 0 & -d & x_1 \\
0 & -h & 0 & f
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
+ \begin{bmatrix}
1 \\
0 \\
0 \\
1
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix}
\tag{2}
\]

III. STABILIZATION OF HYPERCHAOTIC SYSTEM

In this section, the SDRE method is applied to stabilize hyperchaotic system (2) with now parameters.

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix} =
\begin{bmatrix}
a & -x_3 & 0 & 0 \\
x_3 & -b & 0 & 0 \\
cx_2 & 0 & -d & x_1 \\
0 & -h & 0 & f
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
+ \begin{bmatrix}
1 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix}
\tag{3}
\]

Where $u_1, u_2$ are control function to be determined for achieving minimize cost function (4).

\[
J = \int_0^\infty \left( x^T(t)Qx(t) + u^T(t)Ru(t) \right) dt
\tag{4}
\]

Where

\[
Q = I_{4\times4}, R = I_{2\times2}
\tag{5}
\]

A state dependent Riccati equation (SDRE) is the solved at each point $X$ along the trajectory to obtain a nonlinear feedback controller of the form (6), where $P(t)$ is the solution of the SDRE.

\[
u(t) = -k(t)x(t)
\tag{6}
\]

\[
k(t) = R^{-1}B^T(t)P(t)
\tag{7}
\]

\[
\dot{P}(t) + P(t)(A(x(t)) + aI) + (A^T(x(t)) + aI)P(t) + Q - P(t)B(t)R^{-1}B^T(t)P(t) = 0
\tag{8}
\]

Where scaler $\alpha$ is a design parameter [17]. We choose $\alpha = 20$. As can be seen, the SDRE method produces a stabilizing solution. The time response of $x_1, x_2, x_3, x_4$ states for system (3) is shown in Fig 3. The time response of control inputs $u_1, u_2$ is shown in Fig 4.
System (9) is rewrote as follows:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix} =
\begin{bmatrix}
a + \theta_1 x_3 & 0 & 0 & 0 \\
-x_3 & -b & 0 & 0 \\
c x_2 & 0 & -d g x_1 & 0 \\
0 & -h & 0 & f + \theta_2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix} +
\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u_1 +
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_2
\]

\[B\phi(\theta)\]  

(12)

We must find the control signals (6) that it stabilizes the system (12) with uncertainty \(\theta_i, i = 1, 2\). This problem is solved by optimal control of system (3) that it minimizes cost function (13) [16].

\[J = \int_0^\infty (x^T(t)Fx(t) + x^T(t)x(t) + u^T(t)Ru(t))dt\]  

(13)

Where \(F\) matrix is

\[\phi^T(\theta)\phi(\theta) \leq F\]  

(14)

\[F = \phi^T(\theta)\phi(\theta) = \begin{bmatrix} \theta_1^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0\end{bmatrix} = \begin{bmatrix} 90000 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0\end{bmatrix}\]  

(15)

Again, the Riccati equation is solved and the optimal feedback control (6) is obtained. The time response of \(x_1, x_2, x_3, x_4\) states for system (9) is shown in Fig 5. The time response of control inputs \(u_1, u_2\) is shown in Fig 6.
V. SYNCHRONIZATION OF HYPERCHAOTIC SYSTEM

In this section, the SDRE method is applied to synchronize two hyperchaotic systems. Suppose the drive system takes the following form

\[\begin{align*}
\dot{x}_1 &= ax_1 - y_1 z_1 \\
\dot{y}_1 &= x_1 z_1 - by_1 \\
\dot{z}_1 &= cx_1 y_1 - dz_1 + gx_1 w_1 \\
\dot{w}_1 &= f w_1 - hy_1
\end{align*}\]  

(16)

And the response system is given as follows

\[\begin{align*}
\dot{x}_2 &= ax_2 - y_2 z_2 + u_1(t) \\
\dot{y}_2 &= x_2 z_2 - by_2 + u_2(t) \\
\dot{z}_2 &= cx_2 y_2 - dz_2 + gx_2 w_2 + u_4(t) \\
\dot{w}_2 &= f w_2 - hy_2 + u_4(t)
\end{align*}\]  

(17)

Where \(u_1, u_2, u_3, u_4\) are control inputs. Define state errors between system (16) and (17) as follows

\[\begin{align*}
e_x &= x_2 - x_1 \\
e_y &= y_2 - y_1 \\
e_z &= z_2 - z_1 \\
e_w &= w_2 - w_1
\end{align*}\]  

(18)

We obtain the following error dynamical system by subtracting the drive system (16) from the response system (17)

\[\begin{align*}
\dot{e}_x &= ae_x - e_y e_z + z_1 e_y + y_1 e_x + u_1(t) \\
\dot{e}_y &= e_x e_y - be_y - x_1 e_y - z_1 e_x + u_2(t) \\
\dot{e}_z &= ce_x e_y - de_x + ge_x e_w - cx_1 e_y - cy_1 e_x - g x_1 e_w \\
&\quad - gw_1 e_x + u_3(t) \\
\dot{e}_w &= f e_w - he_y + u_4(t)
\end{align*}\]  

(19)

Error dynamical system (19) is rewrote as follows

\[\begin{bmatrix}
\dot{e}_x \\
\dot{e}_y \\
\dot{e}_z \\
\dot{e}_w
\end{bmatrix} = A \begin{bmatrix}
e_x \\
e_y \\
e_z \\
e_w
\end{bmatrix} + \Delta A + B \begin{bmatrix}
u_1 \\
u_2 \\
u_3 \\
u_4
\end{bmatrix}\]  

(20)

Where

\[
A = \begin{bmatrix}
a & -e_x & 0 & 0 \\
e_x & b & 0 & 0 \\
-c e_y & -d e_x & c & 0 \\
0 & -h & 0 & f
\end{bmatrix},
\Delta A = \begin{bmatrix}
a & -e_x & 0 & 0 \\
e_x & b & 0 & 0 \\
-c e_y & -d e_x & c & 0 \\
0 & -h & 0 & f
\end{bmatrix},
B = \begin{bmatrix}
1000 \\
0.100 \\
0.010 \\
0.001
\end{bmatrix}
\]  

(21)

We can rewrite \(\Delta A\) matrix as follows
\[ \Delta A = \begin{bmatrix} 0 & z_1 & y_1 & 0 \\ -z_1 & 0 & -x_1 & 0 \\ -c y_1 - g W_1 & -c x_1 & 0 & -g x_1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1000 \\ 0100 \\ 0010 \\ 0001 \end{bmatrix} \]

Where \( X = [x, y, z, w]^T \). Substituting (22) into (20), the error dynamics is obtained.

\[
\begin{align*}
\begin{bmatrix}
\dot{e}_x \\
\dot{e}_y \\
\dot{e}_z \\
\dot{e}_w
\end{bmatrix} = A 
\begin{bmatrix}
e_x \\
e_y \\
e_z \\
e_w
\end{bmatrix} + B 
\begin{bmatrix}
u_1 \\
u_2 \\
u_3 \\
u_4
\end{bmatrix} + B \phi(X)
\end{align*}
\] (23)

We obtain optimal feedback control (6) that it minimizes the cost function (13) [16]. The \( F \) matrix is obtained.

\[
F = \phi(X)\phi(X) =
\begin{bmatrix}
y_1^2 + z_1^2 & -x_1 y_1 & -c x_1 z_1 & 0 \\
x_1^2 z_1 & x_1^2 + z_1^2 & c y_1 z_1 + g z_1 w_1 & 0 \\
-c x_1 z_1 c y_1 + g z_1 w_1 (c y_1 + g w_1)^2 + (c x_1)^2 + (g x_1)^2 & 0 & 0 & 0
\end{bmatrix}
\] (24)

Again, the Riccati equation is solved and the optimal feedback control (6) is obtained. Synchronization errors \( e_x, e_y, e_z, e_w \) in hyperchaotic system are shown in Fig 7. The time response of \( x, y, z, w \) states for drive system (16) and response system (17) is shown in Fig 8. The time response of the control inputs \( u_1, u_2, u_3, u_4 \) for the synchronization of hyperchaotic system is shown in Fig 9.
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VI. CONCLUSION

In this paper, stabilization and synchronization problem of the new hyperchaotic system was investigated. For this reason, state dependent Riccati equation (SDRE) was used. This method was applied to new hyperchaotic system in three ways. Stabilized system with know parameters, stabilized system with unknow parameters, synchronized system. In every way, simulations proved abilities of method.

REFERENCES


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