Neutrosophic Crisp Open Set and Neutrosophic Crisp Continuity via Neutrosophic Crisp Ideals

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Abstract—The focus of this paper is to propose a new notion of neutrosophic crisp sets via neutrosophic crisp ideals and to study some basic operations and results in neutrosophic crisp topological spaces. Also, neutrosophic crisp L-openness and neutrosophic crisp L-continuity are considered as a generalizations for a crisp and fuzzy concepts. Relationships between the above new neutrosophic crisp notions and the other relevant classes are investigated. Finally, we define and study two different types of neutrosophic crisp functions.

Index Terms—Neutrosophic Crisp Set; Neutrosophic Crisp Ideals; Neutrosophic Crisp L-open Sets; Neutrosophic Crisp L-Continuity; Neutrosophic Sets.

I. INTRODUCTION

The fuzzy set was introduced by Zadeh [20] in 1965, where each element had a degree of membership. In 1983 the intuitionistic fuzzy set was introduced by K. Atanassov [1, 2, 3] as a generalization of fuzzy set, where besides the degree of membership and the degree of non-membership of each element. Salama et al [11] defined intuitionistic fuzzy ideal and neutrosophic ideal for a set and generalized the concept of fuzzy ideal concepts, first initiated by Sarkar [19]. Smarandache [16, 17, 18] defined the notion of neutrosophic sets, which is a generalization of Zadeh’s fuzzy set and Atanassov’s intuitionistic fuzzy set. Neutrosophic sets have been investigated by Salama et al. [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. In this paper is to introduce and study some new neutrosophic crisp notions via neutrosophic crisp ideals. Also, neutrosophic crisp L-openness and neutrosophic crisp L-continuity are considered. Relationships between the above new neutrosophic crisp notions and the other relevant classes are investigated. Recently, we define and study two different types of neutrosophic crisp functions.

The paper unfolds as follows. The next section briefly introduces some definitions related to neutrosophic set theory and some terminologies of neutrosophic crisp set and neutrosophic crisp ideal. Section 3 presents neutrosophic crisp L-open and neutrosophic crisp L-closed sets. Section 4 presents neutrosophic crisp L-continuous functions. Conclusions appear in the last section.

II. PRELIMINARIES

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [16, 17, 18], and Salama et al. [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

2.1 Definitions [9].

1) Let X be a non-empty fixed set. A neutrosophic crisp set (NCS for short) A is an object having the form A = {A₁,A₂,A₃} where A₁,A₂ and A₃ are subsets of X satisfying A₁ ∩ A₂ = φ, A₁ ∩ A₃ = φ and A₂ ∩ A₃ = φ.

2) Let A = {A₁,A₂,A₃}, be a neutrosophic crisp set on a set X. Then p = (|p₁|,|p₂|,|p₃|). p₁ ≠ p₂ ≠ p₃ ∈ X is called a neutrosophic crisp point. A neutrosophic crisp point (NCP for short) p = (|p₁|,|p₂|,|p₃|), is said to be belong to a neutrosophic crisp set A = {A₁,A₂,A₃}, of X, denoted by p ∈ A, if may be defined by two types

i) Type 1: {p₁} ⊆ A₁, {p₂} ⊆ A₂ and {p₃} ⊆ A₃.

ii) Type 2: {p₁} ⊆ A₁, {p₂} ⊆ A₂ and {p₃} ⊆ A₃.

3) Let X be non-empty set, and L a non-empty family of NCSs. We call L a neutrosophic crisp ideal (NCL for short) on X if

   i. A ∈ L and B ⊆ A ⇒ B ∈ L [heredity].
   ii. A ∈ L and B ∈ L ⇒ A ∨ B ∈ L [Finite additivity].

A neutrosophic crisp ideal L is called a σ-neutrosophic crisp ideal if |A j | j∈N ⊆ L , implies

   ∪ A j ∈ L (countable additivity).

The smallest and largest neutrosophic crisp ideals on a
non-empty set $X$ are $\{\phi_Y\}$ and the NSs on $X$. Also, $NCL_I$, $NCL_A$ are denoting the neutrosophic crisp ideals (NCL for short) of neutrosophic crisp subsets having finite and countable support of $X$ respectively. Moreover, if $A$ is a nonempty NS in $X$, then $\{B \in NCS : B \subseteq A\}$ is an NCL on $X$. This is called the principal NCL of all NCSs, denoted by $NCL\{A\}$.

2.1 Proposition [9]

Let $\{L_j : j \in J\}$ be any non-empty family of neutrosophic crisp ideals on a set $X$. Then $\bigcap_{j \in J} L_j$ and $\bigcup_{j \in J} L_j$ are neutrosophic crisp ideals on $X$, where

$$\bigcap_{j \in J} L_j = \left\{ \bigcap_{j \in J} A_{i_1}, \bigcap_{j \in J} A_{j_2}, \bigcap_{j \in J} A_{j_3} \right\}$$

or

$$\bigcup_{j \in J} L_j = \left\{ \bigcup_{j \in J} A_{i_1}, \bigcup_{j \in J} A_{j_2}, \bigcup_{j \in J} A_{j_3} \right\}$$

and

$$\bigcap_{j \in J} L_j = \left\{ \bigcap_{j \in J} A_{i_1}, \bigcap_{j \in J} A_{j_2}, \bigcap_{j \in J} A_{j_3} \right\}$$

or

$$\bigcup_{j \in J} L_j = \left\{ \bigcup_{j \in J} A_{i_1}, \bigcup_{j \in J} A_{j_2}, \bigcup_{j \in J} A_{j_3} \right\}$$

2.2 Proposition [9]

A neutrosophic crisp set $A = \{A_1, A_2, A_3\}$ in the neutrosophic crisp ideal $L$ on $X$ is a base of L iff every member of $L$ is contained in $A$.

2.1 Theorem [9]

Let $A = \{A_1, A_2, A_3\}$, and $B = \{B_1, B_2, B_3\}$, be neutrosophic crisp subsets of $X$. Then $A \subseteq B$ iff $p \in A$ implies $p \in B$ for any neutrosophic crisp point $p$ in $X$.

2.2 Theorem [9]

Let $A = \{A_1, A_2, A_3\}$, be a neutrosophic crisp subset of $X$. Then $A = \bigcup \{p : p \in A\}$.

2.3 Proposition [9]

Let $\{A_j : j \in J\}$ is a family of NCSs in $X$. Then

(a) $p = \{p_1, p_2, p_3\} \in \bigcap_{j \in J} A_j$ iff $p \in A_j$ for each $j \in J$.

(b) $p \in \bigcup_{j \in J} A_j$ iff $\exists j \in J$ such that $p \in A_j$.

2.4 Proposition [9]

Let $A = \{A_1, A_2, A_3\}$ and $B = \{B_1, B_2, B_3\}$ be two neutrosophic crisp sets in $X$. Then

a) $A \subseteq B$ iff for each $p$ we have $p \in A$ $\Rightarrow$ $p \in B$ for each $p$ we have $p \in A$ $\Rightarrow$ $p \in B$.

b) $A = B$ iff for each $p$ we have $p \in A$ $\Rightarrow$ $p \in B$ and for each $p$ we have $p \in A$ $\Rightarrow$ $p \in B$.
neutrosophic crisp topology generated by $NCA^*(L)$ in [9] we will be denoted by $NC^*$.

2.5 Theorem [9]

Let $(X, \tau)$ be a NCTS and $L_1, L_2$ be two neutrosophic crisp ideals on $X$. Then for any neutrosophic crisp sets $A, B$ of $X$ the following statements are verified

i) $A \subseteq B \Rightarrow NCA^*(L, \tau) \subseteq NCB^*(L, \tau)$,

ii) $L_1 \subseteq L_2 \Rightarrow NCA^*(L_2, \tau) \subseteq NCA^*(L_1, \tau)$,

iii) $NCA^* = NCCI(A^*) \subseteq NCCI(A)$,

iv) $NCA^* \subseteq NC^*$,

v) $NC(A \cup B)^* = NCA^* \cup NCB^*$,

vi) $NC(A \cap B)^* (L) \subseteq NCA^*(L) \cap NCB^*(L)$

vii) $\ell \in L \Rightarrow NC(A \cup \ell)^* = NCA^*$

viii) $NCA^*(L, \tau)$ be a neutrosophic crisp closed set.

2.6 Theorem [9]

Let $NC\tau_1, NC\tau_2$ be two neutrosophic crisp topologies on $X$. Then for any neutrosophic crisp ideal $L$ on $X$, $NC\tau_1 \subseteq NC\tau_2$ implies $NCA^*(L, NC\tau_2) \subseteq NCA^*(NC\tau_1)$, for every $A \in L$ then $NC\tau_1^* \subseteq NC\tau_2^*$. A basis $NC\beta(L, \tau)$ for $NC\tau^*(L)$ can be described as follows:

$NC\beta(L, \tau) = \{A - B : A \in NC\tau, B \in NC\}$.

Then we have the following theorem.

2.7 Theorem [9]

$NC\beta(L, \tau) = \{A - B : A \in \tau, B \in L\}$ forms a basis for the generated NCTS of the NCT $(X, \tau)$ with neutrosophic crisp ideal $L$ on $X$.

2.8 Theorem [9]

Let $NC\tau_1, NC\tau_2$ be two neutrosophic crisp topologies on $X$. Then for any topological neutrosophic crisp ideal $L$ on $X$, $NC\tau_1 \subseteq NC\tau_2$ implies $NC\tau^*_1 \subseteq NC\tau^*_2$.

2.9 Theorem [9]

Let $(X, \tau)$ be a NCTS and $L_1, L_2$ be two neutrosophic crisp ideals on $X$. Then for any neutrosophic crisp set $A$ in $X$, we have

i) $NCA^*(L_1 \cup L_2, \tau) = NCA^*[L_1, NCA^*(L_2)] \cup NCA^*[L_2, NCA^*(L_1)]$

ii) $NC\tau^*(L_1 \cup L_2) = \left( NCA^*(L_1) \cup NCA^*(L_2) \right)(L_1)$

2.1 Corollary [9]

Let $(X, \tau)$ be a NCTS with topological neutrosophic crisp ideal $L$ on $X$. Then

i) $NCA^*(L, \tau) = NCA^*(L, \tau)$ and $NC\tau^*(L) = NC(NC\tau^*(L))^*(L)$

ii) $NC\tau^*(L_1 \cup L_2) = \left( NCA^*(L_1) \cup NCA^*(L_2) \right)(L_1)$.

III. NEUTROSOPHIC CRISP L-OPEN AND NEUTROSOPHIC CRISP L-CLOSED SETS

Definition 3.1

Given $(X, \tau)$ be a NCTS with neutrosophic crisp ideal $L$ on $X$, and $A$ is called a neutrosophic crisp $L$-open set if there exists $\zeta \in \tau$ such that $A \subseteq \zeta \subseteq NC^*$.

We will denote the family of all neutrosophic crisp $L$-open sets by $NCLO(X)$.

Theorem 3.1

Let $(X, \tau)$ be a NCTS with neutrosophic crisp ideal $L$, then $A \in NCLO(X)$ if $A \subseteq NCint(NC^*)$.

Proof

Assume that $A \in NCLO(X)$ then by Definition 3.1 there exists $\zeta \in \tau$ such that $A \subseteq \zeta \subseteq NC^*$. But $NCint(NC^*) \subseteq NC^*$, put $\zeta = NCint(NC^*)$. Hence $A \subseteq NCint(NC^*)$. Conversely $A \subseteq NCint(NC^*)$ then there exists $\zeta = NCint(NC^*) \in \tau$. Hence $A \in NCLO(X)$.

Remark 3.1

For a NCTS $(X, \tau)$ with neutrosophic crisp ideal $L$ and $A$ be a neutrosophic crisp set on $X$, the following holds:

If $A \in NCLO(X)$ then $NCint(A) \subseteq NC^*$.

Theorem 3.2

Given $(X, \tau)$ be a NCTS with neutrosophic crisp ideal $L$ on $X$ and $A, B$ are neutrosophic crisp sets such that $A \in NCLO(X), B \in \tau$ then $A \cap B \in NCLO(X)$.

Proof

From the assumption $A \cap B \subseteq NCint(NC^*) \cap B = NCint(NC^* \cap B)$, we have $A \cap B \subseteq NCint(A \cap B^*)$ and this complete the proof.
Corollary 3.1

If \( \{ A_j \}_{j \in J} \) is a neutrosophic crisp L-open set in NCTS \( (X, \tau) \) with neutrosophic crisp ideal L. Then \( \bigcup \{ A_j \}_{j \in J} \) is neutrosophic crisp L-open sets.

Corollary 3.2

For a NCTS \( (X, \tau) \) with neutrosophic crisp ideal L, and neutrosophic crisp set A on X and \( A \in \text{NCLO}(X) \), then \( \text{NCA}^* = \text{NC}(\text{NCint}(\text{NC}^*(A)))^* \) and \( \text{NCcl}(A) = \text{NCint}(\text{NCA}^*) \).

Proof: It’s clear.

Definition 3.2

Given a NCTS \( (X, \tau) \) with neutrosophic crisp ideal L on X and neutrosophic crisp set A then neutrosophic crisp open set and some known neutrosophic crisp openness.

Theorem 3.3

Given a NCTS \( (X, \tau) \) with neutrosophic crisp ideal L and A is a neutrosophic crisp set on X, then

(i) Neutrosophic crisp \( \tau^* \) – closed (or \( \text{NC}^* \) - closed) if \( \text{NCA}^* \leq A \)
(ii) Neutrosophic crisp \( L \)-dense – in – itself (or \( \text{NC}^* \) – dense – in – itself) if \( A \subseteq \text{NCA}^* \).
(iii) Neutrosophic crisp \( * \) – perfect if A is \( \text{NC}^* \) – closed and \( \text{NC}^* \) – dense – in – itself.

Proof: Follows directly from the neutrosophic crisp closure operator \( \text{NCcl}^* \) for a neutrosophic crisp topology \( \tau^*(L) \) (\( \text{NC}^* \) for short).

Remark 3.2

One can deduce that

(i) Every \( \text{NC}^* \)–dense – in – itself is neutrosophic crisp dense set.
(ii) Every neutrosophic crisp closed (resp. neutrosophic crisp open) set is \( \text{N}^* \)–closed (resp. \( \text{N}^* \)–open).
(iii) Every neutrosophic crisp \( L \)-open set is \( \text{NC}^* \) – dense – in – itself.

Corollary 3.3

Given a NCTS \( (X, \tau) \) with neutrosophic crisp ideal L on X and \( A \in \tau \) then we have:

(i) If A is \( \text{NC}^* \) –closed then \( A^* \subseteq \text{NCint}(A) \) \( \subseteq \text{NCcl}^* (A) \).
(ii) If A is \( \text{NC}^* \) –dense – in – itself then \( \text{Nint}(A) \subseteq \text{NCA}^* \).
(iii) If A is \( \text{NC}^* \) –perfect then \( \text{NCint}(A) = \text{NCcl}(A) = \text{NC}^* \).

Proof: Obvious.

we give the relationship between neutrosophic crisp \( L \)-open set and some known neutrosophic crisp openness.

Theorem 3.4

Given a NCTS \( (X, \tau) \) with neutrosophic crisp ideal L and neutrosophic crisp set A on X then the following holds:

(i) If A is both neutrosophic crisp \( L \) – open and \( \text{NCA}^* \) – perfect then A is neutrosophic crisp open.
(ii) If A is both neutrosophic crisp open and \( \text{NC}^* \) – dense–in – itself then A is neutrosophic crisp \( L \)-open.

Proof. Follows from the definitions.

Corollary 3.4

For a neutrosophic crisp subset A of a NCTS \( (X, \tau) \) with neutrosophic crisp ideal L on X, we have:

(i) If A is \( \text{NCA}^* \)–closed and \( \text{NL} \)–open then \( \text{NCint}(A) = \text{NCint}(\text{NCA}^*) \).
(ii) If A is \( \text{NC}^* \)–perfect and \( \text{NL} \)–open then \( A = \text{NC int}(\text{NCA}^*) \).

Remark 3.3

One can deduce that the intersection of two neutrosophic crisp \( L \)-open sets is neutrosophic crisp \( L \)-open.

Corollary 3.5

Given \((X, \tau)\) be a NCTS with neutrosophic crisp ideal L and neutrosophic crisp set A on X. The following hold: If \( L = \{N^*\} \), then \( \text{NCA}^*(L) = \phi_N \) and hence A is neutrosophic crisp \( L \)-open iff \( A = \phi_N \).

Proof: It’s clear.

Definition 3.5

Given a NCTS \( (X, \tau) \) with neutrosophic crisp ideal L and neutrosophic crisp set A then neutrosophic crisp ideal interior of A is defined as largest neutrosophic crisp \( L \)–open set contained in A, denoted by \( \text{NCL} \cap \text{NCint}(A) \).
Theorem 3.5

If \((X, \tau)\) is a NCTS with neutrosophic crisp ideal \(L\) and neutrosophic crisp set \(A\) then

(i) \(A \wedge \text{Nint} (\text{NCA}^\ast)\) is neutrosophic crisp \(L\)-open set.

(ii) \(NL\text{–Nint} (A) = 0_N\text{iff } \text{Nint} (\text{NCA}^\ast) = 0_N\).

Proof.

(i) Since \(\text{NCint} \text{NCA}^\ast = \text{NCA}^\ast \cap \text{NCint} (\text{NCA}^\ast)\), then \(\text{NCint} \text{NCA}^\ast \subseteq \text{NCint} (\text{NCA}^\ast)^\ast\). Thus \(A \cap \text{NCint} (\text{NCA}^\ast)^\ast \subseteq \text{NCint} (\text{NCA}^\ast)^\ast\). Hence \(A \cap \text{NCint} (\text{NCA}^\ast) \subseteq \text{NCint} (\text{NCA}^\ast)^\ast\).

(ii) Let \(\text{NC} \cap \text{NCint}(A) = \phi_N\), then \(A \cap A^\ast = \phi_N\), implies \(\text{NCcl} (A \cap \text{NCint}(\text{NCA}^\ast)) = \phi_N\) and so \(A \cap \text{Nint} A^\ast = \phi_N\). Conversely assume that \(\text{NCint} \text{NCA}^\ast = \phi_N\), then \(A \cap \text{NCint} (\text{NCA}^\ast) = \phi_N\). Hence \(\text{NC} \cap \text{NCint} (A) = \phi_N\).

Theorem 3.6

If \((X, \tau)\) be a NCTS with neutrosophic crisp ideal \(L\) and \(A\) is a neutrosophic crisp set on \(X\), then \(\text{NC} \cap \text{NCint}(A) = A \cap \text{NCint} (\text{NCA}^\ast)\).

Proof. The first implication follows from Theorem 3.4, that is \(A \cap \text{NCA}^\ast \subseteq \text{NC} \cap \text{NCint}(A)\) (1)

For the reverse inclusion, if \(\xi \in \text{NCLO}(X)\) and \(\xi \subseteq A\) then \(\text{NC}^\ast \subseteq \text{NCA}^\ast\) and hence \(\text{NCint} (\text{NC}^\ast) \subseteq \text{NCint} (\text{NCA}^\ast)\). This implies \(\xi = \xi \cap \text{NCint} (\text{NC}^\ast) \subseteq A \cap \text{NCA}^\ast\).

Thus \(\text{NC} \cap \text{NCint}(A) \subseteq A \cap \text{NCint} (\text{NCA}^\ast)\) (2)

From (1) and (2) we have the result.

Corollary 3.6

For a NCTS \((X, \tau)\) with neutrosophic crisp ideal \(L\) and neutrosophic crisp set \(A\) on \(X\) then the following holds:

(i) If \(A\) is \(\text{NC}^\ast\)-closed then \(\text{NL} \cap \text{Nint} (A) \subseteq A\).

(ii) If \(A\) is \(\text{NC}^\ast\)-dense \(\text{in} A\) then \(\text{NL} \cap \text{Nint} (A) \subseteq A^\ast\).

(iii) If \(A\) is \(\text{NC}^\ast\)-perfect set then \(\text{NC} \cap \text{NCint} (A) \subseteq \text{NCA}^\ast\).

Definition 3.6

Given \((X, \tau)\) be a NCTS with neutrosophic crisp ideal \(L\) and \(\xi\) be a neutrosophic crisp set on \(X\), \(\xi\) is called neutrosophic crisp \(L\)-closed set if its complement is neutrosophic crisp \(L\)-open set. We will denote the family of neutrosophic crisp \(L\)-closed sets by \(\text{NLCC}(X)\).

Theorem 3.7

Given \((X, \tau)\) be a NCTS with neutrosophic crisp ideal \(L\) and \(\xi\) be a neutrosophic crisp set on \(X\), \(\xi\) is called neutrosophic crisp \(L\)-closed set, then \(\text{NC} (\text{NCint} \xi)^\ast \subseteq \xi\).

Proof. It’s clear.

Theorem 3.8

Let \((X, \tau)\) be a NCTS with neutrosophic crisp ideal \(L\) on \(X\) and \(\xi\) be a neutrosophic crisp set on \(X\) such that \(\text{NC} (\text{NCint} \xi)^\ast = \text{NCint} \xi^\ast\) then \(\xi \in \text{NLCC}(X)\) if \(\text{NC} \cap \text{NCint} \xi)^\ast \subseteq \xi\).

Proof. (Necessity) Follows immediately from the above theorem (Sufficiency). Let \(\text{NC} (\text{NCint} \xi)^\ast \subseteq \xi\) then \(\xi^\ast \subseteq \text{NC} (\text{NCint} \xi)^\ast = \text{NCint} \xi^\ast\). from the hypothesis. Hence \(\xi^\ast \subseteq \text{NCLO}(X)\), Thus \(\xi \in \text{NLCC}(X)\).

Corollary 3.7

For a NCTS \((X, \tau)\) with neutrosophic crisp ideal \(L\) on \(X\) the following holds:

(i) The union of neutrosophic crisp \(L\)-closed set and neutrosophic crisp closed set is neutrosophic crisp \(L\)-closed set.

(ii) The union of neutrosophic crisp \(L\)-closed and neutrosophic crisp \(L\)-closed is neutrosophic crisp perfect.

IV. NEUTROSOPHIC CRISP L–CONTINUOUS FUNCTIONS

By utilizing the notion of \(\text{NL} \text{–open sets},\) we establish in this article a class of neutrosophic crisp \(L\)-continuous function. Many characterizations and properties of this concept are investigated.

Definition 4.1

A function \(f : (X, \tau) \to (Y, \sigma)\) with neutrosophic crisp ideal \(L\) on \(X\) is said to be neutrosophic crisp \(L\)-continuous if for every \(\zeta \in \sigma\), \(f^{-1}(\zeta) \subseteq \text{NCLO}(X)\).

Theorem 4.1

For a function \(f : (X, \tau) \to (Y, \sigma)\) with neutrosophic crisp ideal \(L\) on \(X\) the following are equivalent:

(i) \(f\) is neutrosophic crisp \(L\)-continuous. For a neutrosophic crisp point \(p\) in \(X\) and each \(\zeta \in \sigma\) containing \(f(p)\), there exists \(A \in \text{NCLO}(X)\) containing \(p\) such that \(f(A) \subseteq \sigma\).
(ii.) For each neutrosophic crisp point p in X and ζ∈σ containing \( f(p) \), \( f^{-1}(\zeta) \) is neutrosophic crisp nbd of p.

(iii.) The inverse image of each neutrosophic crisp closed set in Y is neutrosophic crisp L–closed.

**Proof**

(i) \( \Rightarrow \) (ii). Since ζ∈σ containing \( f(p) \), then by (i), \( f^{-1}(\zeta) \in \text{NCLO}(X) \), hence A = \( f^{-1}(\zeta) \) which containing p, we have \( f(A) \subseteq \sigma \).

(ii) \( \Rightarrow \) (iii). Let ζ∈σ containing \( f(p) \). Then by (ii) there exists A ∈ NCLO(X) containing p such that \( f(A) \subseteq \sigma \), so p ∈ A ⊆ NCint(NCA^*) so NCint \( f^{-1}(\zeta)^* \subseteq (f^{-1}(\zeta))^* \). Hence \( (f^{-1}(\zeta))^* \) is neutrosophic crisp nbd of p.

(iii) \( \Rightarrow \) (i) Let ζ∈σ, since \( f^{-1}(\zeta) \) is neutrosophic crisp nbd of any point \( f^{-1}(\zeta) \), every point x, ∈ \( (f^{-1}(\zeta))^* \) is a neutrosophic crisp interior point of \( f^{-1}(\zeta)^* \). Then \( f^{-1}(\zeta) \subseteq \text{NCint} \text{ NC} \) \( f^{-1}(\zeta)^* \) and hence \( f \) is neutrosophic crisp L–continuous

\( (i) \Rightarrow (iv) \) Let ξ∈η be a neutrosophic crisp closed set. Then \( f^{-1}(\xi) \) is neutrosophic crisp open set, by \( f^{-1}(\zeta) = (f^{-1}(\zeta))^* \in \text{NCLO}(X) \). Thus \( f^{-1}(\zeta) \) is neutrosophic crisp L–closed set.

The following theorem establish the relationship between neutrosophic crisp L–continuous and neutrosophic crisp continuous by using the previous neutrosophic crisp notions.

**Theorem 4.2**

Given \( f : (X,\tau) \rightarrow (Y,\sigma) \) is a function with a neutrosophic crisp ideal L on X then we have. If f is neutrosophic crisp L–continuous of each neutrosophic crisp*– perfect set in X, then \( f \) is neutrosophic crisp continuous.

**Proof:** Obvious.

**Corollary 4.1**

Given a function \( f : (X,\tau) \rightarrow (Y,\sigma) \) and each member of X is neutrosophic crisp NC*–dense – in – itself. Then we have every neutrosophic crisp continuous function is neutrosophic crisp NCL–continuous.

**Proof:** It’s clear.

We define and study two different types of neutrosophic crisp functions.

**Definition 4.2**

A function \( f : (X,\tau) \rightarrow (Y,\sigma) \) with neutrosophic crisp ideal L on Y is called neutrosophic crisp L-open (resp. neutrosophic crisp NCL–closed), if for each A ∈ L (resp. A is neutrosophic crisp closed in X), \( f(A) \in \text{NCLO}(Y) \) (resp. \( f(A) \) is NCL–closed).

**Theorem 4.3**

Let a function \( f : (X,\tau) \rightarrow (Y,\sigma) \) with neutrosophic crisp ideal L on Y. Then the following are equivalent:

(i.) \( f \) is neutrosophic crisp L-open.
(ii.) For each p ∈ X and each neutrosophic crisp nbd A of p, there exists a neutrosophic crisp L–open set \( B \in Y \) containing \( f(p) \) such that \( B \subseteq f(A) \).

**Proof:** Obvious.

**Theorem 4.4**

A neutrosophic crisp function \( f : (X,\tau) \rightarrow (Y,\sigma) \) with neutrosophic crisp ideal L on Y be a neutrosophic crisp L-open (resp.neutrosophic crisp L-closed), if A in Y and B in X is a neutrosophic crisp closed (resp. neutrosophic crisp open ) set C in Y containing A such that \( f^{-1}(C) \subseteq B \).

**Proof**

Assume that \( A = y \setminus \{ f(\{x \setminus B\}) \} \), since \( f^{-1}(C) \subseteq B \) and \( A \subseteq C \) then C is neutrosophic crisp L-closed and \( f^{-1}(C) = y \setminus f^{-1}(f(\{x \setminus A\})) \subseteq B \).

**Theorem 4.5**

If a function \( f : (X,\tau) \rightarrow (Y,\sigma) \) with neutrosophic crisp ideal L on Y is a neutrosophic crisp L-open, then \( f^{-1}(\text{NC}(\text{NCint}(A))^\#) \subseteq \text{NC}(f^{-1}(A)^\#) \) such that \( f^{-1}(A) \) is neutrosophic crisp*–dense–in–itself and A in Y.

**Proof**

Since A in Y, \( \text{NC}(f^{-1}(A)^\#) \) is neutrosophic crisp closed in X containing \( f^{-1}(A) \). \( f \) is neutrosophic crisp L-open then by using Theorem 4.4 there is a neutrosophic crisp L-closed set \( A \subseteq B \) suchthat, \( \{f^{-1}(A)\} \supseteq f^{-1}(B) \supseteq f^{-1}(\text{NCint}(B))^\# \supseteq f^{-1}(\text{NCint}(\mu))^\# \).
Corollary 4.2

For any bijective function \( f : (X, \tau) \rightarrow (Y, \sigma) \) with neutrosophic crisp ideal \( L \) on \( Y \), the following are equivalent:

(i.) \( f^{-1} : (Y, \sigma) \rightarrow (X, \tau) \) is neutrosophic crisp \( L \)-continuous.

(ii.) \( f \) is neutrosophic crisp \( L \)-open.

(iii.) \( f \) is neutrosophic crisp \( L \)-closed.

Proof: Follows directly from Definitions.

V. CONCLUSION

In our work, we have put forward some new concepts of neutrosophic crisp open set and neutrosophic crisp continuity via neutrosophic crisp ideals. Some related properties have been established with example. It’s hoped that our work will enhance this study in neutrosophic set theory.

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How to cite this paper: A. A. Salama, Said Broumi, Florentin Smarandache,"Neutrosophic Crisp Open Set and Neutrosophic Crisp Continuity via Neutrosophic Crisp Ideals", IJIEEB, vol.6, no.3, pp.1-8, 2014. DOI: 10.5815/ijieeb.2014.03.01