I.J. Information Engineering and Electronic Business, **2014, 2, 64-69**
Published Online April 2014 in MECS (http://www.mecs-press.org/)
DOI: 10.5815/ijieeb.2014.02.08

Common Fixed Point Theorem in Fuzzy Metric Spaces using weakly compatible maps

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***Abstract*—**The aim of this paper is to prove a common fixed theorem for four mappings under weakly compatible condition in fuzzy metric space. While proving our results we utilize the idea of weakly compatible maps due to Jungck and Rhoades. Our results substantially generalize and improve a multitude of relevant common fixed point theorems of the existing literature in metric as well as fuzzy metric space.

***Index Terms*—**T-norm, Fuzzy metric space, weakly compatible mappings, Common fixed point theorem.

I. Introduction

In 1965, Zadeh [1] introduced the concept of Fuzzy set as a new way to represent vagueness in our everyday life. However, when the uncertainty is due to fuzziness rather than randomness, as sometimes in the measurement of an ordinary length, it seems that the concept of a fuzzy metric space is more suitable. We can divide them into following two groups: The first group involves those results in which a fuzzy metric on a set *X* is treated as a map where *X* represents the totality of all fuzzy points of a set and satisfy some axioms which are analogous to the ordinary metric axioms. Thus, in such an approach numerical distances are set up between fuzzy objects. On the other hand in second group, we keep those results in which the distance between objects is fuzzy and the objects themselves may or may not be fuzzy. Kramosil and Michalek [2] have introduced the concept of fuzzy metric spaces in different ways.

In 1986, Jungck [3] introduced the notion of compatible maps for a pair of self mappings. However, the study of common fixed points of non-compatible maps is also very interesting. Jungck and Rhoades [4] initiated the study of weakly compatible maps in metric space and showed that every pair of compatible maps is weakly compatible but reverse is not true. In the literature, many results have been proved for weakly compatible maps satisfying some contractive condition in different settings such as probabilistic metric spaces [5, 6, 7]; fuzzy metric spaces [8, 9, 10].

In this paper, we prove a common fixed theorem for four mappings under weakly compatible condition in fuzzy metric space. While proving our results we utilize the idea of weakly compatible maps due to Jungck and Rhoades [4]. Our results substantially generalize and improve a multitude of relevant common fixed point theorems of the existing literature in metric as well as fuzzy metric space.

II. Preliminaries

Definition 2.1.[11] A binary operation * : $[0,1]\times[0,1] \to [0,1]$ is continuous *t*-norm if * satisfies the following conditions:

(i) * is commutative and associative;
(ii) * is continuous;
(iii) a * 1 = a for all $a \in [0,1]$;
(iv) a * b ≤ c * d whenever a ≤ c and b ≤ d for all $a,b,c,d \in [0,1]$.

Kramosil and Michalek [2] introduced the concept of fuzzy metric spaces as follows:

Definition 2.2: [2] The 3-tuple (*X*, *M*, *) is called a fuzzy metric space (shortly, *FM*-space) if *X* is an arbitrary set, * is a continuous *t*-norm and *M* is a fuzzy set in $X^2 \times [0, \infty)$ satisfying the following conditions:

(FM-1) M(x, y, 0) = 0,
(FM-2) M(x, y, t) = 1, for all $t > 0$ if and only if x = y,
(FM-3) M(x, y, t) = M(y, x. t),
(FM-4) M(x, y, t) * M(y, z, s) ≤ M(x, z, t + s) (Triangular inequality)
(FM-5) M(x, y.): [0, 1) → [0, 1] is left continuous for all $x, y, z \in X$ and $s, t > 0$.

Note that *M*(*x*, *y*, *t*) can be thought of as the degree of nearness between *x* and *y* with respect to t.

We can fuzzify examples of metric spaces into fuzzy metric spaces in a natural way:

Let (*X*, *d*) be a metric space. Define *a* * *b* = *a* + *b* for all *a*, *b* in [0,1]. Define

M(*x*, *y*, *t*) = *t* /(*t* + *d*(*x*, *y*)) for all *x*, *y* in *X* and *t* > 0.Then (*X*, *M*, *) is a fuzzy metric space and this fuzzy metric induced by a metric *d* is called the Standard fuzzy metric.

Consider *M* to be a fuzzy metric space with the following condition:

(FM-6) limt→∞M(x, y, t) = 1 for all x, y in X and $t > 0$.

Definition 2.3[2]: Let $(X, M, *)$ be fuzzy metric space. Then

(a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$, $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$

and

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if, for all $t > 0$, $\lim_{n\to\infty} M(x_n, x, t) = 1$.

Definition 2.4[2]: A fuzzy metric space (*X, M,* *) is said to be complete if and only if every Cauchy sequence in *X* is convergent.

Example 2.1[2]: Let $X = \{1/n : n \in N\} \cup \{0\}$ and let * be the continuous *t*-norm and defined by $a*b = ab$ for all $a, b \in [0,1]$. For each $t \in (0, \infty)$ and $x, y \in X$, define *M* by

$$M(x, y, t) = \begin{cases} \dfrac{t}{t + |x - y|}, & t > 0, \\ 0 & t = 0 \end{cases}$$

Clearly, (*X, M,* *) is complete fuzzy metric space.

Definition 2.5[9, 12]: A pair of self-mappings (*A*, *S*) of a fuzzy metric space (*X, M,* *) is said to be commuting if *M* (*ASx, SAx, t*) = 1 for all *x* in *X*.

Definition 2.6[9]: A pair of self-mappings (*A*, *S*) of a fuzzy metric space (*X, M,* *) is said to be weakly commuting if *M* (*ASx, SAx, t*) ≥ *M* (*Ax, Sx, t*) for all *x* in *X* and *t* > 0.

In 1994, Mishra *et al.* [10] introduced the concept of compatible mapping in Fuzzy metric space akin to concept of compatible mapping in metric space as follows:

Definition 2.7[10]: A pair of self-mappings (*A*, *S*) of a fuzzy metric space (*X, M,* *) is said to be compatible if $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ for all *t* > 0, whenever $\{x_n\}$ is a sequence in *X* such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = u$ for some *u* in *X*.

Definition 2.8[1]: Let (*X, M,* *) be a fuzzy metric space. *A* and *S* be self maps on *X*. A point *x* in *X* is called a coincidence point of *A* and *S* iff *Ax* = *Sx*. In this case, *w* = *Ax* = *Sx* is called a point of coincidence of *A* and *S*.

Definition 2.9[4]: A pair of self mappings (*A*, *S*) of a fuzzy metric space (*X, M,* *) is said to be weakly compatible if they commute at the coincidence points *i.e.*, if *Au* = *Su* for some $u \in X$, then *ASu* = *SAu*.

It is easy to see that two compatible maps are weakly compatible but converse is not true.

Lemma 2.1[2]: Let $\{u_n\}$ is a sequence in a fuzzy metric space (*X, M,* *). If there exists a constant $k \in (0,1)$ such that $M(u_n, u_{n+1}, kt) \ge M(u_{n-1}, u_n, t)$, *n* = 1, 2, 3,… then $\{u_n\}$ is a Cauchy sequence in *X*.

III. MAIN RESULTS

Theorem 3.1: Let *A*, *B*, *P* and *Q* be self mappings of a complete fuzzy metric space $(X, M, *)$ satisfying the following:

(3.1) for any x, y in X, and for all $t > 0$ there exists $k \in (0,1)$ such that,

$$M(Px, Qy, kt) \ge \max \left\{ \begin{array}{l} M(Ax, By, t), \\ \dfrac{1}{2}\left(\begin{array}{l} M(Px, Ax, t) \\ + M(Qx, Bx, t) \end{array} \right) \end{array} \right\}$$

(3.2) $P(X) \subset B(X)$ and $Q(X) \subset A(X)$

(3.3) if one of P(X), B(X), Q(X), A(X) is complete subset of X then

(a) *P* and *A* have a coincidence point

(b) Q and B have a coincidence point.

If the pair (*P*, *A*) and (*Q*, *B*) are weakly compatible then *A*, *B*, *P* and *Q* have a unique common fixed point in *X*.

Proof: As $P(X) \subset B(X)$ and $Q(X) \subset A(X)$, so we can define sequences $\{x_n\}$ and $\{y_n\}$ in *X* such that $y_{2n+1} = Px_{2n} = Bx_{2n+1}, y_{2n+2} = Qx_{2n+1} = Ax_{2n+2}$. By (3.1),

$$M(Px_{2n},Qx_{2n+1},kt)\geq\max\left\{\begin{array}{l}M(Ax_{2n},Bx_{2n+1},t),\\ \frac{1}{2}\left(\begin{array}{l}M(Px_{2n},Ax_{2n},t)\\ +M(Qx_{2n},Bx_{2n},t)\end{array}\right)\end{array}\right\}$$

$$M(y_{2n+1},y_{2n+2},kt)\geq\max\left\{\begin{array}{l}M(y_{2n},y_{2n+1},t),\\ \frac{1}{2}\left(\begin{array}{l}M(y_{2n+1},y_{2n},t)\\ +M(y_{2n+1},y_{2n},t)\end{array}\right)\end{array}\right\}$$

$$M(y_{2n+1},y_{2n+2},kt)\geq\max\left\{\begin{array}{l}M(y_{2n},y_{2n+1},t),\\ M(y_{2n+1},y_{2n},t)\end{array}\right\}$$

$$M(y_{2n+1},y_{2n+2},kt)\geq M(y_{2n},y_{2n+1},t)$$

$$M(Px_{2n},Qx_{2n+1},kt)\geq\max\left\{\begin{array}{l}M(Ax_{2n},Bx_{2n+1},t),\frac{1}{2}\\ \left(\begin{array}{l}M(Px_{2n},Ax_{2n},t)+\\ M(Qx_{2n},Bx_{2n},t)\end{array}\right)\end{array}\right\}$$

$$M(y_{2n+1},y_{2n+2},kt)\geq\max\left\{\begin{array}{l}M(y_{2n},y_{2n+1},t),\\ \frac{1}{2}\left(\begin{array}{l}M(y_{2n+1},y_{2n},t)\\ +M(y_{2n+1},y_{2n},t)\end{array}\right)\end{array}\right\}$$

$$M(y_{2n+1},y_{2n+2},kt)\geq\max\left\{\begin{array}{l}M(y_{2n},y_{2n+1},t),\\ M(y_{2n+1},y_{2n},t)\end{array}\right\}$$

$$M(y_{2n+1},y_{2n+2},kt)\geq M(y_{2n},y_{2n+1},t).$$

Similarly, $M(y_{2n+2},y_{2n+3},kt)\geq M(y_{2n+1},y_{2n+2},t)$.

Therefore, in general, $M(y_n,y_{n+1},kt)\geq M(y_{n-1},y_n,t)$.

Hence, by Lemma 2.1, $\{y_n\}$ is Cauchy sequence in *X*. By completeness of X, $\{y_n\}$ converges to some point *z* in *X*.

Therefore, subsequence's $\{y_{2n}\},\{y_{2n+1}\}$, $\{y_{2n+2}\}$ converges to point *z*. i.e.

$$\lim_{n\to\infty}Bx_{2n+1}=\lim_{n\to\infty}Px_{2n}=\lim_{n\to\infty}Qx_{2n+1}=\lim_{n\to\infty}Ax_{2n+2}=z.$$

Now, suppose *A*(*X*) is complete, therefore, let $w\in A^{-1}z$ then $Aw = z$.

Now, consider,

$$M(Pw,Qx_{2n+1},kt)\geq\max\left\{\begin{array}{l}M(Aw,Bx_{2n+1},t),\\ \frac{1}{2}\left(\begin{array}{l}M(Pw,Aw,t)\\ +M(Qw,Bw,t)\end{array}\right)\end{array}\right\}$$

$$M(Pw,y_{2n+2},kt)\geq\max\left\{\begin{array}{l}M(z,y_{2n+1},t),\\ \frac{1}{2}\left(\begin{array}{l}M(Pw,z,t)\\ +M(Qw,Bw,t)\end{array}\right)\end{array}\right\}$$

$n\to\infty$

$$M(Pw,z,kt)\geq\max\left\{\begin{array}{l}M(z,z,t),\\ \frac{1}{2}\left(\begin{array}{l}M(Pw,z,t)\\ +M(Qw,Bw,t)\end{array}\right)\end{array}\right\}$$

$$M(Pw,z,kt)\geq\max\left\{1,\frac{1}{2}\left(\begin{array}{l}M(Pw,z,t)\\ +M(Qw,Bw,t)\end{array}\right)\right\}$$

$$M(Pw,z,kt)\geq 1.$$

This gives, $Pw = z = Aw$. Therefore, *w* is coincidence point of *P* and *A*.
Since, $P(X)\subset B(X)$,
therefore, $z=Pw\in P(X)\subset B(X)$
this gives, $z\in B(X)$, let $v\in B^{-1}z$ *i.e.* $Bv = z$.
By (3.1)

$$M(y_{2n+1},Qv,kt)\geq\max\left\{\begin{array}{l}M(y_{2n},z,t),\\ \frac{1}{2}\left(\begin{array}{l}M(y_{2n+1},y_{2n},t)\\ +M(y_{2n+1},y_{2n},t)\end{array}\right)\end{array}\right\}$$

$n\to\infty$

$$M(z,Qv,kt)\geq\max\left\{\begin{array}{l}M(z,z,t),\\ \frac{1}{2}\left(\begin{array}{l}M(z,z,t)\\ +M(z,z,t)\end{array}\right)\end{array}\right\}$$

$$M(z,Qv,kt)\geq 1.$$

This gives, $Qv = z = Bv$. So, *v* is coincidence point of *Q* and *B*. Since, the pair (*P*, *A*) is weakly compatible, therefore, *P* and *Q* commute at coincidence point i.e. $PAw = APw$, this gives, $Pz = Az$ and as (*Q*, *B*) is weakly compatible, therefore, $QBv = BQv$ this gives, $Qz = Bz$.

Now, we will show that $Pz = z$. By (3.1), we have

$$M(Pz, Qx_{2n+1}, kt) \geq \max\left\{\begin{matrix} M(Az, Bx_{2n+1}, t), \\ \frac{1}{2}\begin{pmatrix} M(Pz, Az, t) \\ +M(Qz, Bz, t) \end{pmatrix} \end{matrix}\right\}$$

$$M(Pz, y_{2n+2}, kt) \geq \max\left\{\begin{matrix} M(Az, y_{2n+1}, t), \\ \frac{1}{2}\begin{pmatrix} M(Az, Az, t) \\ +M(Bz, Bz, t) \end{pmatrix} \end{matrix}\right\}$$

$n \to \infty$

$$M(Pz, z, kt) \geq \max\left\{\begin{matrix} M(Az, z, t), \\ 1 \end{matrix}\right\}$$

$$M(Pz, z, kt) \geq 1.$$

The is gives, $Pz = z = Az$. Similarly, we prove that $Qz = z$. By (3.1),

$$M(Px_{2n}, Qz, kt) \geq \max\left\{\begin{matrix} M(Ax_{2n}, Bz, t), \\ \frac{1}{2}\begin{pmatrix} M(Px_{2n}, Ax_{2n}, t) \\ +M(Qx_{2n}, Bx_{2n}, t) \end{pmatrix} \end{matrix}\right\}$$

$$M(y_{2n+1}, Qz, kt) \geq \max\left\{\begin{matrix} M(y_{2n}, Bz, t), \\ \frac{1}{2}\begin{pmatrix} M(y_{2n+1}, y_{2n+1}, t) \\ +M(y_{2n+1}, y_{2n}, t) \end{pmatrix} \end{matrix}\right\}$$

$n \to \infty$

$$M(z, Qz, kt) \geq \max\left\{\begin{matrix} M(z, Bz, t), \\ \frac{1}{2}\begin{pmatrix} M(z, z, t) \\ +M(z, z, t) \end{pmatrix} \end{matrix}\right\}$$

$$M(z, Qz, kt) \geq \max\left\{\begin{matrix} M(z, Bz, t), \\ 1 \end{matrix}\right\}$$

$$M(z, Qz, kt) \geq 1.$$

This gives, $Qz = z = Bz$. Therefore, z is a common fixed point of P, A, Q and B.

For Uniqueness, let w be another fixed point of P, A, Q and B then by (3.1), we have

$$M(Pz, Qw, kt) \geq \max\left\{\begin{matrix} M(Az, Bw, t), \\ \frac{1}{2}\begin{pmatrix} M(Pz, Az, t) \\ +M(Qz, Bz, t) \end{pmatrix} \end{matrix}\right\}$$

$$M(z, w, kt) \geq \max\left\{\begin{matrix} M(z, w, t), \\ \frac{1}{2}\begin{pmatrix} M(z, z, t) \\ +M(z, z, t) \end{pmatrix} \end{matrix}\right\}$$

$$M(z, w, kt) \geq 1$$

this gives, $z = w$. Hence, z is unique common fixed point of P, A, Q and B.

By choosing P, A, Q and B suitably, one can derive corollaries involving two or more mappings. As a sample, we deduce the following natural result for a pair of self-mappings by setting $P = Q$ in above theorem:

Corollary 3.1: Let A, B and P be self mappings of a complete fuzzy metric space $(X, M, *)$ satisfying the following:

(3.4) for any x, y in X, and for all $t > 0$ there exists $k \in (0,1)$ such that,

$$M(Px, Py, kt) \geq \max\left\{\begin{matrix} M(Ax, By, t), \\ \frac{1}{2}\begin{pmatrix} M(Px, Ax, t) \\ +M(Px, Bx, t) \end{pmatrix} \end{matrix}\right\}$$

(3.5) $P(X) \subset B(X)$ and $P(X) \subset A(X)$

(3.6) if one of P(X), B(X), A(X) is complete subset of X then

(a) P and A have a coincidence point

(b) P and B have a coincidence point.

If the pair *(P, A)* and *(P, B)* are weakly compatible then A, B and P have a unique common fixed point in X.

By taking $A = B = I$ in theorem 3.1, we get

Corollary 3.2. : Let P and Q be self mappings of a complete fuzzy metric space $(X, M, *)$ satisfying the following:

(3.7) for any x, y in X, and for all $t > 0$ there exists $k \in (0,1)$ such that,

$$M(Px, Qy, kt) \geq \max\left\{\begin{matrix} M(x, y, t), \\ \frac{1}{2}\begin{pmatrix} M(Px, x, t) \\ +M(Qx, x, t) \end{pmatrix} \end{matrix}\right\}$$

(3.8) if one of P(X), Q(X) is complete subset of X.

If the pair (P, Q) is weakly compatible then P and Q have a unique common fixed point in X.

Definition 3.1 [6] Two families of self-mappings $\{A_i\}_{i=1}^{m}$ and $\{B_j\}_{j=1}^{n}$ are said to be pairwise commuting if

(a) $A_i A_j = A_j A_i$, $i, j \in \{1, 2, 3, ...m\}$,

(b) $B_i B_j = B_j B_i$, $i, j \in \{1, 2, 3, ...n\}$,

(c) $A_i B_j = B_j A_i$, $i \in \{1, 2, 3, ...n\}$, $j \in \{1, 2, 3, ...n\}$.

As an application of Theorem 3.1, we prove a common fixed point theorem for four finite families of mappings on fuzzy metric spaces. While proving our result, we utilize Definition 3.1 which is a natural extension of commutativity condition to two finite families.

Theorem 3.2: Let $\{A_1, A_2, ..., A_m\}$, $\{B_1, B_2, ..., B_n\}$, $\{P_1, P_2, ..., P_p\}$ and $\{Q_1, Q_2, ..., Q_q\}$ be four finite families of self mappings of a complete fuzzy metric space $(X, M, *)$ such that $A = A_1.A_2.....A_m$, $B = B_1.B_2.....B_n$, $P = P_1.P_2.....P_p$ and $Q = Q_1.Q_2.....Q_q$ satisfying the conditions (3.1), (3.2), (3.3) and (3.9) the pairs of families $(\{A_i\}, \{P_k\})$ and $(\{B_r\}, \{Q_t\})$ commute pairwise.

Then the pairs (A, P) and (B, Q) have a point of coincidence each. Moreover, $\{A_i\}_{i=1}^{m}, \{P_k\}_{k=1}^{p}$, $\{B_r\}_{r=1}^{n}$ and $\{Q_t\}_{t=1}^{q}$ have a unique common fixed point.

Proof: By using (3.9), we first show that $AP = PA$ as

$$\begin{aligned} AP &= (A_1A_2 \ldots A_m)(P_1P_2 \ldots P_p) \\ &= (A_1A_2 \ldots A_{m-1})(A_m P_1P_2 \ldots P_p) \\ &= (A_1A_2 \ldots A_{m-1})(P_1P_2 \ldots P_p A_m) \\ &= (A_1A_2 \ldots A_{m-2})(A_{m-1} P_1P_2 \ldots P_p A_m) \\ &= (A_1A_2 \ldots A_{m-2})(P_1P_2 \ldots P_p A_{m-1} A_m) \\ &= \ldots = A_1(P_1P_2 \ldots P_p A_2 \ldots A_m) \\ &= (P_1P_2 \ldots P_p)(A_1A_2 \ldots A_m) = PA. \end{aligned}$$

Similarly one can prove that $BQ = QB$. And hence, obviously the pairs (A, P) and (B, Q) are weakly compatible. Now using Theorem 3.1, we conclude that A, B, P and Q have a unique common fixed point in X, say z.

Now, one needs to prove that z remains the fixed point of all the component mappings.

For this consider

$$\begin{aligned} A(A_i z) &= ((A_1A_2 \ldots A_m)A_i)z \\ &= (A_1A_2 \ldots A_{m-1})(A_m A_i)z \\ &= (A_1A_2 \ldots A_{m-1})(A_i A_m)z \\ &= (A_1A_2 \ldots A_{m-2})(A_{m-1} A_i A_m)z \\ &= (A_1A_2 \ldots A_{m-2})(A_i A_{m-1} A_m)z \\ &= A_1(A_iA_2 \ldots A_m)z \\ &= (A_1A_i)(A_2 \ldots A_m)z \\ &= (A_i A_1)(A_2 \ldots A_m)z \\ &= A_i (A_1A_2 \ldots A_m)z = A_i Az = A_i z. \end{aligned}$$

Similarly, one can prove that

$$A(P_k z) = P_k (Az) = P_k z,\ P(P_k z) = P_k(Pz) = P_k z,$$
$$P(A_i z) = A_i(Pz) = A_i z,\ B(B_r z) = B_r(Bz) = B_r z,$$
$$B(Q_t z) = Q_t(Bz) = Q_t z,\ Q(Q_t z) = Q_t(Qz) = Q_t z$$

and

$$Q(B_r z) = B_r(Qz) = B_r z,$$

which show that (for all i, r, k and t) $A_i z$ and $P_k z$ are other fixed point of the pair (A, P) whereas $B_r z$ and $Q_t z$ are other fixed points of the pair (B, Q). As A, B, P and Q have a unique common fixed point, so we get

$$z = A_i z = P_k,\quad z = B_r z = Q_t z,$$
for all $i = 1, 2, \ldots, m, \quad k = 1, 2, \ldots, p$
$$r = 1, 2, \ldots, n, \quad t = 1, 2, \ldots, q.$$

which shows that z is a unique common fixed point of $\{A_i\}_{i=1}^{m}, \{P_k\}_{k=1}^{p}$, $\{B_r\}_{r=1}^{n}$ and $\{Q_t\}_{t=1}^{q}$.

Remark 3.1: Theorem 3.2 is a slight but partial generalization of Theorem 3.1 as the commutativity requirements in this theorem are slightly stronger as compared to Theorem 3.1.

IV. Conclusion

In this paper, we prove a common fixed theorem for four mappings under weakly compatible condition in fuzzy metric space. While proving our results we utilize the idea of weakly compatible maps. Our results substantially generalize and improve a multitude of relevant common fixed point theorems of the existing literature in metric as well as fuzzy metric space. As an application of Theorem 3.1, we prove a common fixed point theorem for four finite families of mappings on fuzzy metric spaces (Theorem 3.2).

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How to cite this paper: Saurabh Manro,"Common Fixed Point Theorem in Fuzzy Metric Spaces using weakly compatible maps", IJIEEB, vol.6, no.2, pp.64-69, 2014. DOI: 10.5815/ijieeb.2014.02.08

